Problem 1

\[ u = 0.6 \, c, \quad \tau = \frac{1}{\sqrt{1 - u^2/c^2}} = \frac{1}{\sqrt{1 - 0.36}} = 1.25 \]

(a) From turn B, 1 year has passed. That is, the proper time, \( \Delta t_p \).

The time on earth is

\[ \Delta t = \tau \Delta t_p = 1.25 \text{ years}. \]

So turn B is 21.25 years old when she lights up the candles according to clocks on the earth.

(b) After 1.25 years, the spaceship has traveled a distance

\[ d = 0.6 \, c \times 1.25 \text{ years}. \]

The light from the candle of turn B travels towards turn A at speed c. It reaches turn A after a time

\[ t = \frac{d}{c} = \frac{0.6 \, c \times 1.25 \text{ years}}{c} = 0.75 \text{ years}. \]

So the age of turn A when the light first reaches him is

\[ \text{age}(A) = 20 \text{ years} + 1.25 \text{ years} + 0.75 \text{ year} = 22 \text{ years}. \]

(c) Because of the principle of relativity, the answer must be the same for turn B.

So, \( \text{age}(B) = 22 \text{ years} \) when the light from A reaches her.
Problem 1 (cont.)
Alternative way to solve in part (c):

Turn a light candle after 1 year.
Light travels time $t$ until it reaches spaceship.
So it travels distance $D = ct$.

Space ship has traveled a time $= 1 \text{ year} + t$, at speed $0.6c$.
So $D = (1 \text{ year} + t) \times 0.6c$. Equating, we find $t$

$$(1 \text{ year} + t) \times 0.6c = dt$$

$$(1 \text{ year} + t) \times 0.6c = 0.6c + 0.6t = t$$

$(1 \text{ year} + t) \times 0.6c = 0.6c + 0.6t = t$

$0.4t = 0.6 \text{ year}$

$t = 1.5 \text{ years}$

So according to the earth reference frame, the
light from A reaches B after a time $1 \text{ year} + t =$

$= 1 \text{ year} + 1.5 \text{ years} = 2.5 \text{ years}$ since the 20th
birthday.

But the time for B is the proper time, so

it is \( \frac{2.5 \text{ years}}{0.8} = \frac{2.5}{1.25} \text{ years} \)

So the light from A reaches B when B is

22 years old, as we had found earlier.
Problem 2

hen lays egg: \( x_1' = 100 \text{ m}, \ t_1' = 0 \)

chicken born: \( x_2' = 0, \ t_2' = +0.1 \mu\text{s} = 10^{-7} \text{s} \)

\[ t_1 = \gamma (t_1' + \frac{v}{c^2} x_1') = \frac{v}{c^2} x_1' \]

\[ t_2 = \gamma (t_2' + \frac{v}{c^2} x_2') = \frac{v}{c^2} t_2' \]

We want \( t_1 = t_2 \Rightarrow \frac{v}{c^2} x_1' = t_2' \Rightarrow v = \frac{c^2 t_2'}{x_1'} \Rightarrow \]

\[ u = c \cdot \frac{t_2'}{x_1'} \cdot c = c \cdot \frac{10^{-7} \text{s} \cdot 3 \times 10^8 \text{ m}}{100 \text{ m}} = 0.3 c \]

So \( u = 0.3 c \)

For \( u > 0.3c \), chicken is born before hen lays egg in ground ref. frame.

(b) \( (\Delta s')^2 = (c \Delta t')^2 - (\Delta x')^2 = (3 \times 10^8 \times 10^{-7})^2 \text{ m}^2 - 100^2 \text{ m}^2 = \]

\[ = 30^2 \text{ m}^2 - 100 \text{ m}^2 = \boxed{-9,100 \text{ m}^2} \] Space-like since it is negative.

(c) Since spacetime interval is invariant, \( (\Delta s)^2 = (c \Delta t)^2 - (\Delta x)^2 \)

\( \Delta t = 0, \) since \( t_1 = t_2. \) \( \Delta x = \frac{L_p}{\delta} = L_p \sqrt{1 - \frac{v^2}{c^2}}, \) with \( L_p = 100 \text{ m} \)

\[ (\Delta s)^2 = -(\Delta x)^2 = -\frac{L_p^2}{\delta^2} = \frac{10,000 \text{ m}^2 \times (1 - (0.3)^2)}{} \]

\[ = -10,000 \text{ m}^2 \times 0.91 = \boxed{-9100 \text{ m}^2} \]
Problem 3

\[ \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.8^2}} = \frac{1}{0.6} = 1.6667 \text{ in ship relative to ground.} \]

(b) Length of ship as measured from ground:

\[ L = \frac{L_0}{\gamma} = \frac{200 \text{ m}}{1.667} = 120 \text{ m} \]

(a) Relative speed:

\[ m_x = -0.8c = \text{speed of B with respect to ground} \]
\[ u = 0.8c = \text{speed of frame } S' = \text{reference frame of ship A} \]

Find \( m'_x = \text{speed of B relative to frame } S' \):

\[ m'_x = \frac{m_x - u}{1 - \frac{um_x}{c^2}} = \frac{-0.8c - 0.8c}{1 - 0.8 \times 0.8} = \frac{-1.6c}{1 + 0.64} = -0.9756c \]

So relative speed of B with respect to A is \( 0.9756c \)

(c) The \( \gamma \) in this relative speed is:

\[ \gamma = \frac{1}{\sqrt{1 - 0.9756^2}} = 4.555 \]

So the length contraction is:

\[ L = \frac{L_0}{\gamma} = \frac{200 \text{ m}}{4.555} = 43.9 \text{ m} \]