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(a) $\vec{E}$ has units of N/C or V/m, so $\vec{E}$ will have the largest value where the voltage is changing most rapidly with respect to distance or, equivalently, where the equipotentials are most closely packed. In this case, this is on the $\boxed{\text{positive x-axis.}}$

(b) In this region, the electric potential is decreasing in the $\boxed{\text{negative x-direction}}$, so this is the direction of $\vec{E}$.

(c) Again, the units of $\vec{E}$ are V/m. On the positive x-axis, in the negative x-direction, the potential is dropping at a rate of $\boxed{10\ \text{V/m}}$, so this is the magnitude of $\vec{E}$ in this region.

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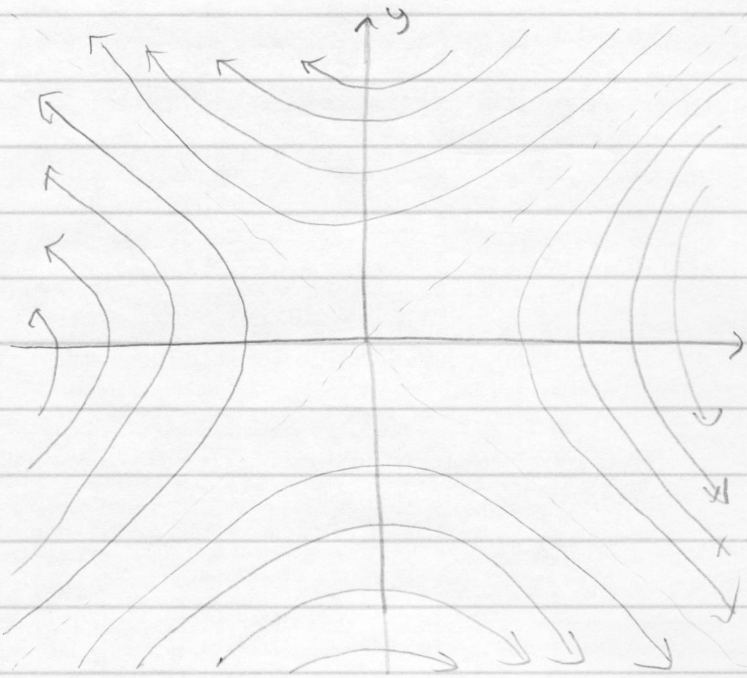
(a) $V = axy$, what is $\vec{E}$?

$\vec{E} = E_x \hat{\imath} + E_y \hat{\jmath}$ where $E_x = -\frac{dV}{dx}$ and $E_y = -\frac{dV}{dy}$

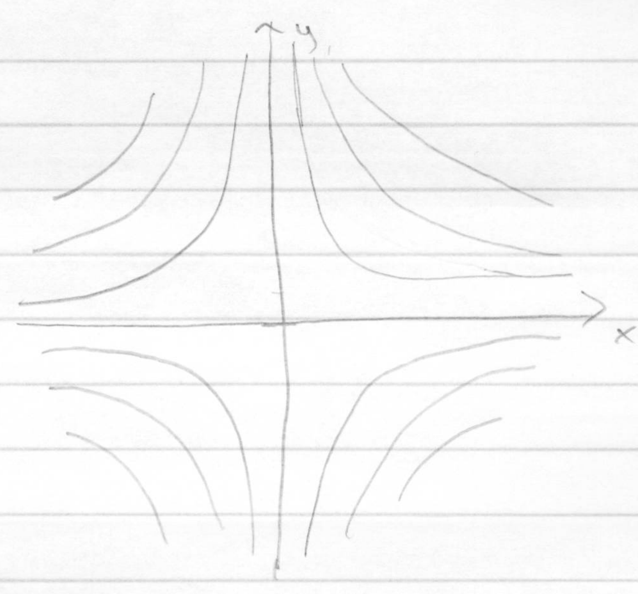
$$\left.\begin{aligned} E_x &= -\frac{\partial}{\partial x}(axy) = -ay \\ E_y &= -\frac{\partial}{\partial y}(axy) = -ax \end{aligned}\right\} \boxed{\vec{E} = (-ay)\hat{\imath} + (-ax)\hat{\jmath}}$$

(b)

$\vec{E}$



V = constant



Note: the $\vec{E}$ lines and the equipotential lines are perpendicular everywhere

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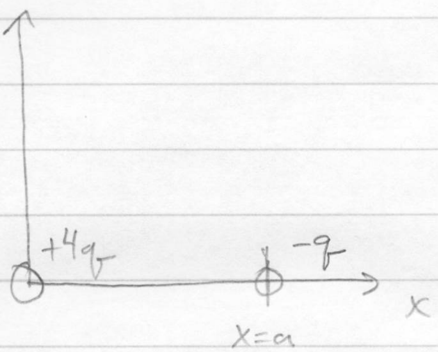

What is $V(x)$ for $x>a$?

$$V = \sum_i \frac{kq_i}{r_i} = \boxed{\frac{k(4q)}{x} + \frac{k(-q)}{(x-a)}}$$

Where is $V=0$ in this region?

$$0 = \frac{4kq}{x} - \frac{kq}{(x-a)}$$

$$0 = 4kq(x-a) - kqx$$

$$0 = 4x - 4a - x \Rightarrow \boxed{x = \frac{4}{3}a}$$

What is $E(x)$ for $x>a$?

$$E_x = -\frac{d}{dx}\left(V(x)\right) = -\frac{d}{dx}\left(\frac{4kq}{x} - \frac{kq}{(x-a)}\right)$$

$$= +\frac{4kq}{x^2} - \frac{kq}{(x-a)^2}$$

Note: you get one minus from the definition and another from the derivative

$$E(x) = E_x \hat{\imath} = \boxed{kq\left(\frac{4}{x^2} - \frac{1}{(x-a)^2}\right)\hat{\imath}}$$

Where is $E(x)=0$ for $x>a$?

$$0 = kq\left(\frac{4}{x^2} - \frac{1}{(x-a)^2}\right)$$

$$\Rightarrow \frac{4}{x^2} = \frac{1}{(x-a)^2} \Rightarrow 4(x-a)^2 = x^2$$

$$\Rightarrow 4\left[x^2 - 2ax + a^2\right] = x^2 \Rightarrow 3x^2 - 8ax + 4a^2 = 0$$

$$x = \frac{8a \pm \sqrt{64a^2 - 4(3)4a^2}}{2(3)} = \frac{4}{3}a \pm \frac{2}{3}a \Rightarrow \boxed{x = 2a} \text{ (for } x>a\text{)}$$

What is the potential and electric field at the surface of a conducting sphere with radius R and total charge Q?

Like the electric field in the previous chapter, outside a spherically symmetric charge distribution, the potential looks like that of a point charge with the same total charge located at its center.

$$\boxed{V = \frac{kQ}{R}, \quad \vec{E} = \frac{kQ}{R^2}\hat{r}}$$

Now triple the the charge and radius, How does this change V and $\vec{E}$?

$Q' \to 3Q$, $R' \to 3R$

$\boxed{V' = \frac{k(3Q)}{3R} = V}$ but $\boxed{\vec{E}' = \frac{k(3Q)\hat{r}}{(3R)^2} = \frac{kQ\hat{r}}{3R^2} = \frac{1}{3}\vec{E}}$

Note: here the prime indicates the values for the larger sphere.

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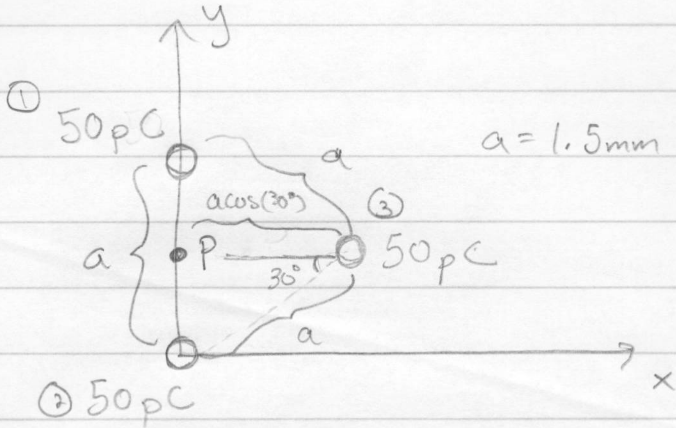

We are free to choose our coordinate system and the zero of our potential V, which will be taken as $V(\infty) = 0$.

How much work would it take to bring a proton to P?

First, calculate the potential at point P:

$$V(P) = \sum_i \frac{k q_i}{r_i} = \frac{kq}{a/2} + \frac{kq}{a/2} + \frac{kq}{a\cos(30°)} = \frac{kq}{a}\left(4 + \frac{2}{\sqrt{3}}\right)$$

The work required to bring the proton in from infinity (where $V=0$) is simply the charge on the proton times $V(P)$.

$$W = eV(P) = \frac{kqe}{a}\left(4 + \frac{2}{\sqrt{3}}\right) = \boxed{2.47 \times 10^{-16}\,J}$$

Note: $k = 9 \times 10^{9}\, \frac{Nm^2}{C^2}$, $q = 50 \times 10^{-12}\, C$

$e = 1.6 \times 10^{-19}\, C$, $a = 1.5 \times 10^{-3}\, m$

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From the previous problem, replacing $q \to -q$ in one of the terms for $V(P)$, we have:

$$V(P) = \frac{kq}{a}\left(4 - \frac{2}{\sqrt{3}}\right)$$

and $$W = eV(P) = \frac{keq}{a}\left(4 - \frac{2}{\sqrt{3}}\right) = \boxed{1.37 \times 10^{-16}\ \mathrm{J}}$$

The presence of the negative charge means less work is required to bring the proton into position P.

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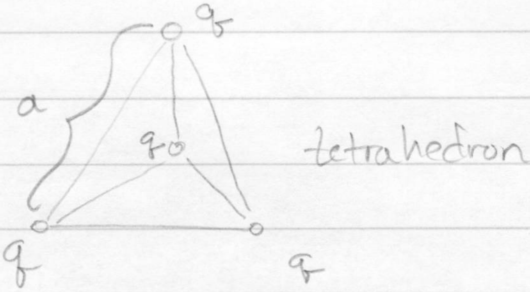

What is U for this configuration?

In other words, what work is required to assemble this distribution?

For two point charges seperated by r_{12}:

$$U = \frac{kq_1 q_2}{r_{12}}$$

Put the first charge in place. There is no work required because no other charges are present.

The second charge gives:

$$U_{12} = \frac{kqq}{a}$$

The third:

$$U_{123} = \underbrace{\frac{kqq}{a}}_{\text{from before}} + \underbrace{\frac{kqq}{a} + \frac{kqq}{a}}_{\text{interacts w/ existing two charges}}$$

The fourth

$$U_{1234} = \underbrace{\frac{3kqq}{a}}_{\text{from before}} + \underbrace{\frac{kqq}{a} + \frac{kqq}{a} + \frac{kqq}{a}}_{\text{3 existing charges}} = \boxed{\frac{6kq^2}{a}}$$

Equivalently, you can sum over all unique pairs of charges. In other words, don't count both U_{12} and U_{21}.

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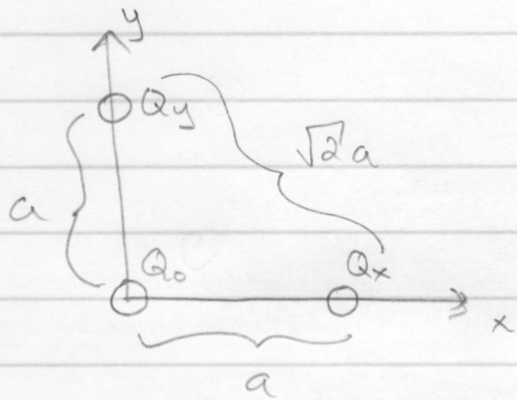

First Q_x is brought in

$$U_{0x} = \frac{k Q_0 Q_x}{a}$$

$$U_{y0} + U_{yx} = \frac{k Q_y Q_0}{a} + \frac{k Q_y Q_x}{\sqrt{2}\, a}$$

and we are told:

$$2U_{0x} = U_{y0} + U_{yx}$$

$$\Rightarrow \quad \frac{2kQ_0 Q_x}{a} = \frac{k Q_y Q_0}{a} + \frac{k Q_y Q_x}{\sqrt{2} a}$$

What is Q_y in terms of Q_x and Q_0?

$$Q_y = \frac{2k Q_0 Q_x}{a}\left(\frac{k Q_0}{a} + \frac{k Q_x}{\sqrt{2} a}\right)^{-1}$$

substitute $Q_x = 2Q_0$

$$Q_y = \frac{4k Q_0^2}{a}\left(\frac{k Q_0}{a} + \frac{2k Q_0}{\sqrt{2} a}\right)^{-1} = \boxed{4(-1+\sqrt{2})\, Q_0}$$

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From equation 26-2:

$$U = W = \frac{d}{2\varepsilon_0 A} Q^2$$

$$\Rightarrow \quad d = \frac{W(2\varepsilon_0 A)}{Q^2}$$

$$= \frac{6.3\,J\,(2\varepsilon_0\,(\pi \times (15\times10^{-2}\,m)^2))}{(45\times10^{-6}\,C)^2}$$

$$= \boxed{3.89\times10^{-3}\,m}$$

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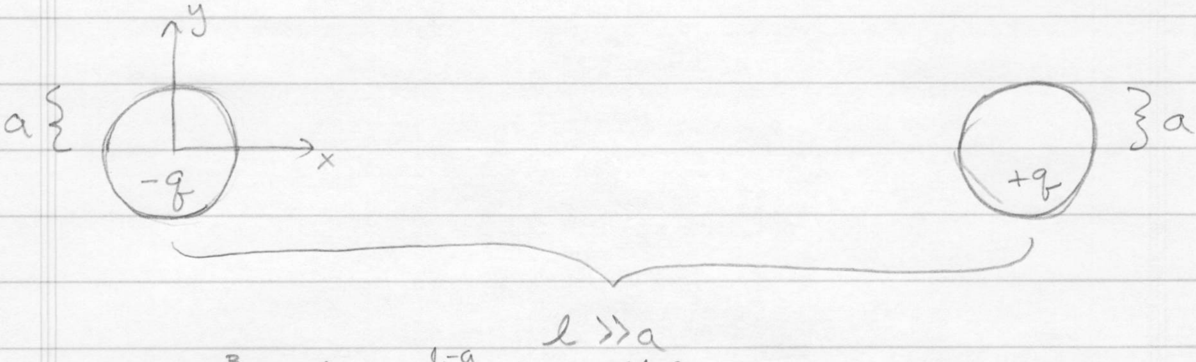

$$V = -\int_A^B \vec{E}\cdot d\vec{l} = -\int_a^{l-a} E_x\,dx = -\int_a^{l-a} \frac{k(-q)}{x^2} - \frac{kq}{(x-l)^2}\,dx$$

Notes: In between the spheres, the fields are in the same $(-\hat{i})$ direction. I integrated from the surface of the $-q$ sphere $(x=a)$ to the surface of the $+q$ sphere $(x=l-a)$. I then simplified and got rid of terms without l because $l \gg a$.

$$= kq\left[\frac{1}{l-x} - \frac{1}{x}\right]_a^{l-a}$$

$$= kq\left[\frac{1}{a} - \frac{1}{l-a} - \frac{1}{l-a} + \frac{1}{a}\right]$$

$$= kq\left(\frac{2}{a} - \frac{2}{l-a}\right)$$

$$= kq\left(\frac{2(l-a)-2a}{a(l-a)}\right)$$

$$= kq\left(\frac{2l-4a}{al-a^2}\right)$$

$$\simeq kq\left(\frac{2}{a}\right) = \boxed{\frac{2kq}{a}}$$

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(b) $dW = V\,dq = \frac{2kq}{a}\,dq$

(c) $W = \int_0^Q \frac{2kq}{a}\,dq = \left[\frac{kq^2}{a}\right]_0^Q = \boxed{\frac{kQ^2}{a}}$

"Assume both are initially uncharged" means the lower limit for your integral is zero.

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From equation 26-3:

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 \quad \text{, electric energy density}$$

and the expression for the electric field above a conductor's surface $E = \dfrac{\sigma}{\varepsilon_0}$

we get $\sigma = \varepsilon_0 \sqrt{\dfrac{2u_E}{\varepsilon_0}}$, plus we know $q = \sigma A$

$$\Rightarrow q = \varepsilon_0 \sqrt{\frac{2u_E}{\varepsilon_0}}\, A = \varepsilon_0 \sqrt{\frac{2 \times 4.5\times10^{3}\ \text{J/m}^3}{\varepsilon_0}}\,(10\times10^{-2}\ \text{m})^2$$

$$= \boxed{2.82 \times 10^{-6}\ \text{C}}$$

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Look at Example 26-3. Setting $R_2 = R$ and $R_1 = \infty$ (because you are assembling the sphere from previously separated charges) yields $U = \frac{kQ^2}{2}\left(\frac{1}{R} - 0\right)$

$$= \boxed{\frac{kQ^2}{2R}}$$

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From equation 26-6:

$$C = \frac{\varepsilon_0 A}{d}, \text{ parallel plate capacitor}$$

we get $A = \frac{Cd}{\varepsilon_0} = \frac{Q}{V}\frac{d}{\varepsilon_0} = \frac{2.3\times10^{-6}\,C \times 1.1\times10^{-3}\,m}{150\,V \times \varepsilon_0}$

$$= \boxed{1.9\,m^2}$$

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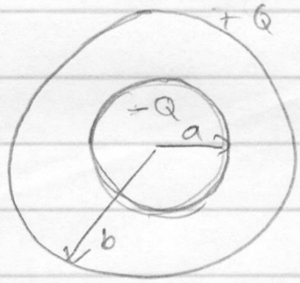

$$V = -\int \vec{E} \cdot d\vec{l} = -\int_a^b E_r \, dr = +\int_a^b \frac{kQ}{r^2} \, dr$$

$$= \left[\frac{-kQ}{r} \right]_a^b = \left(\frac{kQ}{a} - \frac{kQ}{b} \right)$$

$$C = \frac{Q}{V} = \frac{Q}{kQ\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{1}{k\left(\frac{1}{a} - \frac{1}{b}\right)} = \boxed{\frac{ab}{k(b-a)}}$$

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$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

in general   parallel plate capacitor

You can imagine slicing the middle plate in half (so that $A \to 2A$, but it is half as thick) and "unfolding the taco" to create a larger, normal geometry, parallel plate capacitor. Send $A' \to 2A$ in the above expression to get:

$$\boxed{C' = \frac{\varepsilon_0 (2A)}{d}}$$

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(a) when you put the slab in, charges will be induced on its surfaces so that the Electric field will be zero. This then acts like two capacitors in series.

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}, \quad C = \frac{\varepsilon_0 A}{d}$$

$$\frac{1}{C'} = \frac{(.2)d}{\varepsilon_0 A} + \frac{(.2)d}{\varepsilon_0 A} = \frac{0.4d}{\varepsilon_0 A}$$

$\Rightarrow \boxed{C' = 2.5\,C}$ the capacitance goes up

(b) $U = \frac{Q^2}{2C} \Rightarrow U' = \frac{Q^2}{2(2.5C)} = \frac{Q^2}{5C}$ for each capacitor

The total energy stored is twice that

$\Rightarrow$ the energy stored is $\boxed{40\%}$ of the orginal value

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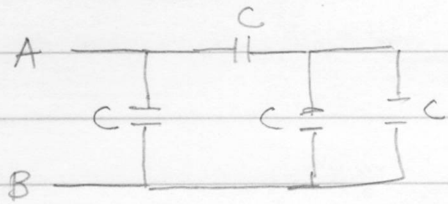
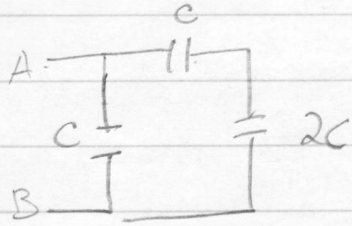
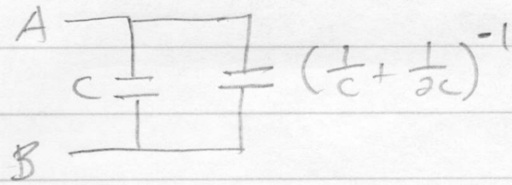
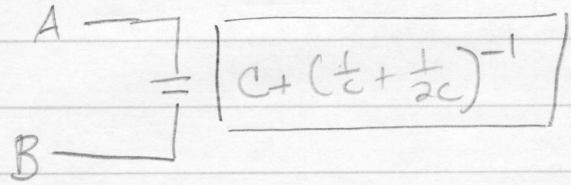


$\Leftrightarrow$ 

$\Leftrightarrow$ 

$\Leftrightarrow$ 

or $\frac{5}{3}C$

Note: in parallel $C_{12} = C_1 + C_2$

in series $C_{12} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$

apply repeatedly until you have one capacitor

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Before glass is inserted:

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 (76\times10^{-4}\,m^2)}{1.2\times10^{-3}\,m} = \boxed{5.6\times10^{-11}\,F}$$

$V_0 = 900V$ (given)

$$U_0 = \tfrac{1}{2} C_0 V_0^2 = \tfrac{1}{2}(5.6\times10^{-11}\,F)(900\,V)^2 = \boxed{2.27\times10^{-5}\,J}$$

After the glass is inserted:

$$C = \kappa C_0 = (3.4)(5.6\times10^{-11}\,F) = \boxed{1.9\times10^{-10}\,F}$$

$$V = Q/C = C_0 V_0 / C = \boxed{265\,V}$$ (Q is constant)

$$U = \tfrac{1}{2} C V^2 = \tfrac{1}{2}(1.9\times10^{-10}\,F)(265\,V)^2 = \boxed{6.67\times10^{-6}\,J}$$