→ Phase space flow incompressible (Liouville Thm.)

→ Derive Vlasov Eqn. from:

- Liouville Eqn.
- $N = \sum_i \delta(\underline{x}-\underline{x}_i)\,\delta(\underline{v}-\underline{v}_i)$ 〉 Klimontovich Eqn.

  "peas"
- hierarchy, with $f(\underline{X}_1, \underline{X}_2, t) =$

  "crushed pea soup" ← $f(\underline{X}_1, t)\, f(\underline{X}_2, t) + g(\underline{X}_1, \underline{X}_2, t)$

  and $1/n\lambda_D^3 \ll 1 \Rightarrow g \ll f^2$ etc.

(Return in Fluctuations Discussion)

IV.) Collective Response in Collisionless Plasma

→ Waves in Vlasov Plasma (1D)

- $\omega, kv \gg \nu \Rightarrow$

$$f = \langle f \rangle + \tilde{f}$$

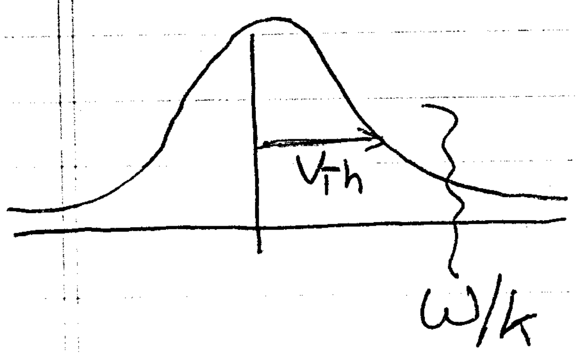
$$\langle f \rangle = \left(1/\sqrt{2\pi}\,v_{Th}\right) \exp\left(-v^2/2v_{Th}^2\right)$$

(Maxwellian)

i.e. $\langle f \rangle$ established on long-time scale

- seek contact with Langmuir Wave (ions stationary)

$\Rightarrow \omega > k V_{th}$ (Heuristic)



Then, linearize:

$$\frac{\partial \tilde{f}}{\partial t} + v \frac{\partial \tilde{f}}{\partial x} = -\frac{q}{m} \tilde{E} \frac{\partial \langle f \rangle}{\partial v}$$

$$\nabla^2 \tilde{\phi} = -4\pi n_0 q \int \tilde{f} \, dv$$

$$f = \sum_{k,\omega} f_{k,\omega} \, e^{i(kx - \omega t)}$$

$$\Rightarrow -i(\omega - kv) \tilde{f}_{k,\omega} = \frac{q}{m} i k \tilde{\phi}_{k,\omega} \, \partial \langle f \rangle / \partial v$$

$$+ k^2 \tilde{\phi}_{k,\omega} = 4\pi n_0 q \int \tilde{f}_{k,\omega} \, dv$$

$$\tilde{f}_{k,\omega} = -k \frac{q}{m} \frac{\tilde{\phi}_{k,\omega} \, \partial \langle f \rangle / \partial v}{(\omega - kv)}$$

so $$k^2 \tilde{\phi}_{k,\omega} = -\omega_p^2 k \int dv \, \frac{\partial \langle f \rangle / \partial v}{(\omega - kv)} \tilde{\phi}_{k,\omega}$$

Thus, $\epsilon(k,\omega) = 1 + \frac{\omega_p^2}{k}\int dv \frac{\partial\langle f\rangle/\partial v}{(\omega - kv)}$

- dielectric function for Vlasov Plasma

$\boxed{\text{? How Handle Pole at } \omega = kv \text{ ?}}$

- Recall V. E. derived in limit $\gamma \to 0$

$$1/\omega - kv = \lim_{\epsilon\to 0} 1/\omega - kv + i\epsilon$$

Concepts
- wave-particle resonance
- collisionless damping

- Alternatively, causality requires: $\tilde{\phi} \underset{t\to -\infty}{\to} 0$

$$\phi \sim e^{-i\omega t} \Rightarrow \phi \sim e^{-i(\omega + i\epsilon)t}$$

(i.e. formally IVP)

$$1/\omega - kv = \lim_{\epsilon \to 0} 1/\omega - kv + i\epsilon$$

$$= \frac{P}{\omega - kv} - i\pi\delta(\omega - kv)$$

(Plemelj Formulae)

$$\epsilon(k,\omega) = 1 + \frac{\omega_p^2}{k}\int dv\, \frac{\partial\langle f\rangle/\partial v}{\omega - kv}$$

$$= 1 + \frac{\omega_p^2}{k}\int dv\, \frac{P}{\omega - kv}\frac{\partial\langle f\rangle}{\partial v}$$

$$-i\pi\,\frac{\omega_p^2}{k|k|}\frac{\partial\langle f\rangle}{\partial v}\bigg|_{\omega/k} \rightarrow \text{physical content!?}$$

i.e.

$$\delta(\omega - kv) = \frac{1}{|k|}\,\delta(v - \omega/k)$$

Further: $\frac{\partial\langle f\rangle}{\partial v} = -\frac{v}{v_{Th}}\langle f\rangle$

$$kv_{Th} < \omega \Rightarrow \frac{P}{\omega - kv} = \frac{1}{\omega}\left(1 + \frac{kv}{\omega} + \left(\frac{kv}{\omega}\right)^2 + \left(\frac{kv}{\omega}\right)^3 + \dots\right)$$

$$\epsilon_r(k,\omega) = 1 - \frac{\omega_p^2}{k v_{Th}^2}\int \frac{\langle f\rangle v}{\omega}\left(1 + \frac{kv}{\omega} + \left(\frac{kv}{\omega}\right)^2 + \left(\frac{kv}{\omega}\right)^3 + \dots\right)$$

$$= 1 - \frac{\omega_p^2}{\omega^2} - \frac{3\omega_p^2 v_{Th}^2 k^2}{\omega^4}$$

$$\epsilon_r(k,\omega) = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + 3\frac{k^2 V_{Th}^2}{\omega^2}\right)$$

so

$$\epsilon = \epsilon_R + i\,\epsilon_{IM}$$

$$\epsilon_R = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + 3\frac{k^2 V_{Th}^2}{\omega^2}\right)$$

$$\epsilon_{IM} = -\pi \frac{\omega_p^2}{k|k|}\left.\frac{\partial \langle f \rangle}{\partial v}\right|_{\omega/k}$$

$\rightarrow \epsilon_R = 0 \Rightarrow$ collective resonance / wave

- as ϵ derived via $(kv/\omega) < 1$ expansion, need determine $\omega(k)$ iteratively

$$\epsilon_r = 0 = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + 3\frac{k^2}{\omega^2}V_{Th}^2\right)$$

Lowest order: $\omega^{(0)} = \omega_p$

$$\Rightarrow \epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + 3\frac{k^2 V_{Th}^2}{\omega_p^2}\right)$$

$$\therefore \omega^2 = \omega_p^2(1 + 3k^2\lambda_D^2)$$ $\longrightarrow$ structure agrees with fluid mdl.

$\hookrightarrow$ contrast fluid

— Distribution function determines equation of state

i.e. # 3 $\leftrightarrow \int v^4 \langle f \rangle$

Contrast K+T : $\begin{cases} P = P_0 (\rho/\rho_0)^\gamma & \gamma = 3 \\ \gamma = 3 \leftrightarrow \text{Maxwellian} \end{cases}$

— Structure of dispersion relation identical to warm fluid model
$\leftrightarrow k v_{Th} < \omega$,

$\rightarrow \epsilon_{IM}$.

$$\epsilon_{IM} = -\frac{\pi \omega_p^2}{k|k|} \left. \frac{\partial \langle f \rangle}{\partial v} \right|_{\omega/k}$$

$Q = \omega \, \epsilon_{IM} \left(|E|^2 / 8\pi \right)$ $\rightarrow$ dissipated energy

$\Rightarrow$

$$Q = -\omega_k \frac{\pi \omega_p^2}{k|k|} \left. \frac{\partial \langle f \rangle}{\partial v} \right|_{\omega_k/k} |E|^2/8\pi$$

Now,
$$\frac{\partial W_{\underline{k}}}{\partial t} + \nabla \cdot S_{\underline{k}} + Q_{\underline{k}} = 0$$

$$\Rightarrow \gamma_{\underline{k}} = -Q_{\underline{k}} / W_{\underline{k}} \qquad W_{\underline{k}} = \omega_{\underline{k}} \frac{\partial \epsilon_r}{\partial \omega}\bigg|_{\omega_{\underline{k}}} \frac{|E|^2}{8\pi}$$

$$\therefore \quad \gamma_{\underline{k}} = \left(\frac{\pi \omega_p^2}{k|k|} \frac{\partial \langle f \rangle}{\partial v}\bigg|_{\frac{\omega_k}{k}} \right) \Big/ \left(\frac{\partial \epsilon_r}{\partial \omega}\bigg|_{\omega_{\underline{k}}} \right)$$

Alternatively:

$$\epsilon = \epsilon_R(\underline{k}, \omega) + i\, \epsilon_{IM}(\underline{k}, \omega)$$

$$\omega = \omega_{\underline{k}} + i \gamma_{\underline{k}} \qquad \gamma \ll \omega_{\underline{k}}$$

$$\begin{aligned} \epsilon &= \epsilon_R(k, \omega_{\underline{k}} + i\gamma_{\underline{k}}) + i\, \epsilon_{IM}(k, \omega_{\underline{k}}) \\ &\cong \epsilon_R(k, \omega_{\underline{k}}) + i \gamma_{\underline{k}} \frac{\partial \epsilon_R}{\partial \omega}\bigg|_{\omega_{\underline{k}}} + i\, \epsilon_{IM}(k, \omega_{\underline{k}}) \end{aligned}$$

$$\gamma_{\underline{k}} = -\epsilon_{IM}(k, \omega_{\underline{k}}) \Big/ (\partial \epsilon_R / \partial \omega)\big|_{\omega_{\underline{k}}}$$

agrees above.

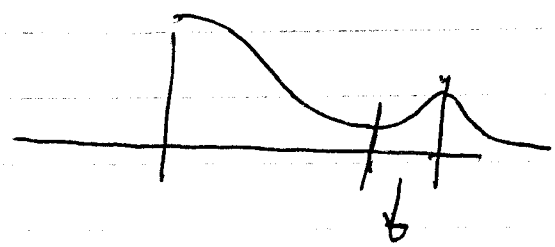
Thus → $\partial\langle f\rangle/\partial v|_{\omega_k/k} < 0$

⇒ damping (Landau damping)

→ $\partial\langle f\rangle/\partial v|_{\omega/k} > 0$

⇒ growth

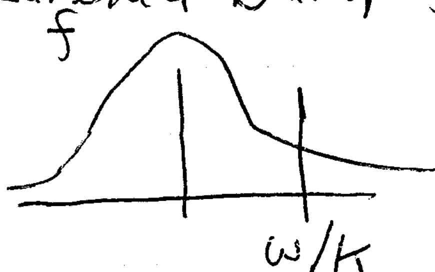
i.e. 'Bump on Tail'



$\omega/k \sim v$ grows as $\partial\langle f\rangle/\partial v > 0$

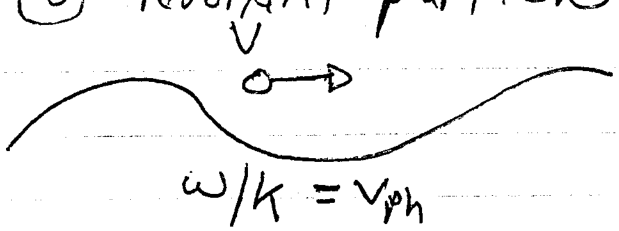
→ Physics of Landau Damping

Consider



→ Landau damping occurs due wave particle resonance $\omega/k \sim v$

→ intuitively, consider wave interaction with ≈ resonant particle



Resonant particle 'sees' ≈ DC field

$$\frac{dv}{dt} = \frac{q}{m} E \cos(kx - \omega t)$$

$$= \frac{q}{m} E \cos(k(x - v_{ph} t))$$

if boost to frame at particle velocity V

$$x' = x - Vt$$
$$v' = v - V$$
$$a' = a$$

$\Rightarrow$

$$\frac{dv}{dt} = \frac{q}{m} E \cos(k(x + (V - v_{ph})t))$$

$\therefore$

- secular (in time) interaction at $V \sim V_{ph}$ resonance
- $v \lesssim \omega/k \Rightarrow$ wave accelerates particles, loses energy

  $v \gtrsim \omega/k \Rightarrow$ wave decelerates particles, gains energy

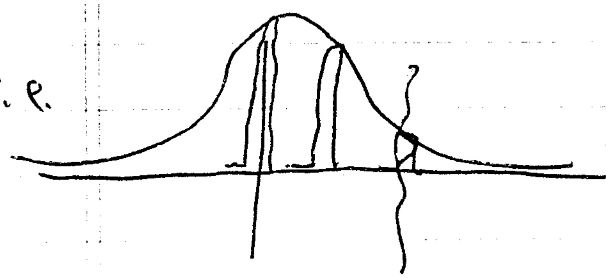
$$Q = \#\text{ accelerated} - \#\text{ decelerated}$$
$$\sim (\partial f/\partial v)\Big|_{\omega/k}$$

▷ Quantitatively :

— as $Q = \langle \underline{E}^* \cdot \underline{J} \rangle$

seek $\bar{q} = \langle q v \underline{E} \rangle$ → time averaged work on resonant 'beam'

i.e.



⇒ plasma distribution as superposition of beams

then $Q = \int dv \, \bar{q}$

— $v = v_0 + \delta v$

$\mapsto$ perturbations induced by wave

$x = x_0 + \delta x$

so $$\frac{d\,\delta v}{dt} = \frac{q}{m} E \Big|_{x_0, v_0}$$

$$\frac{d\delta x}{dt} = \delta v$$

$\bar{q} = q \langle v E \rangle$

$v = v_0 + \delta v$

$E = E(t, x = x_0 + \delta x)$

$\cong E(t, x_0) + \delta x \, \partial E / \partial x \big|_{x_0}$

DC $\quad$ osc $\quad$ osc $\quad$ both osc.

$$\bar{q} = q \left\langle (V_0 + \delta V)\left(E(t, x_0) + \delta x \, \partial E/\partial x \Big|_{x_0, t}\right) \right\rangle$$ 46.

$$\bar{q} = q V_0 \left\langle \delta x \, \partial E/\partial x \Big|_{x_0, t} \right\rangle + q \left\langle \delta V \, E(t, x_0) \right\rangle$$

Now,
$$\frac{d\delta V}{dt} = \frac{q}{m} E(t, x_0) \qquad x_0 = x_0' + V_0 t$$

$$= \frac{q}{m} E_0 e^{ikx_0'} e^{ik(V_0 - \omega/k)t} e^{\delta t}$$

$x_0' = 0$ (convenience)

$\omega/k = v_{ph}$

$\delta > 0 \Rightarrow \delta v \to 0$ as $t \to -\infty$

$\therefore$

$$\frac{d\delta V}{dt} = \frac{q}{m} E_0 \exp\left(ik\left(V_0 - \omega/k - i\delta\right)t\right)$$

$$\delta V = \frac{q}{m} \frac{E_0 e^{ik(V_0 - \omega/k - i\delta)t}}{i\left(k(V_0 - V_{ph}) - i\delta\right)} \Bigg|_{-\infty}^{t}$$

$$\Rightarrow \delta v = \frac{q}{m} E(t, x_0) \Big/ ik(V_0 - V_{ph}) + \delta$$

$$\delta x = \frac{q}{m} E(t, x_0) \Big/ \left(ik(V_0 - V_{ph}) + \delta\right)^2$$

Thus

$$\bar{q} = q v_0 \left\langle \delta x \frac{\partial E}{\partial x} \right\rangle + q \left\langle \delta v\, E \right\rangle$$

$$= q v_0 \left\langle -ik E^*(t,x_0) \frac{q}{m} \frac{E(t,x_0)}{(ik(v_0-v_p)+\delta)^2} \right\rangle$$

$$+ q \left\langle E^*(t,x_0) \frac{q}{m} \frac{E(t,x_0)}{(ik(v_0-v_p)+\delta)} \right\rangle$$

note! $E^* E$ gives DC beat

$\Rightarrow$

$$\bar{q} = \frac{d}{dv_0} \left\{ \frac{q^2}{2m} |E|^2 \frac{v_0}{ik(v_0-v_p)+\delta} \right\}$$

$$= \frac{d}{dv_0} \left\{ \frac{q^2}{2m} |E|^2 \frac{-i v_0}{k(v_0-v_p)-i\delta} \right\}$$

note! '2' from $\cos^2$

real part $\Rightarrow$

$$\bar{q} = \frac{d}{dv_0} \left\{ \frac{q^2}{2m} |E|^2 \frac{v_0 \pi}{|k|} \delta(v_0 - v_p) \right\}$$

$$Q = n\int dv_0\ \bar{q}(v_0)\left\langle f(v_0)\right\rangle$$

$$= \int dv_0 \left\langle f(v_0)\right\rangle \frac{d}{dv_0}\left\{\frac{nq^2}{2m}|E|^2 \frac{v_0\pi}{|k|}\delta(v_0 - v_{ph})\right\}$$

$$= -\frac{\pi\omega_p^2}{|k|}\frac{\omega}{k}\frac{\partial\left\langle f(v)\right\rangle}{\partial v}\Big|_{\omega/k}\left(|E|^2/8\pi\right)$$

$\Rightarrow$

$$Q = -\pi\frac{\omega_p^2}{|k|}\frac{\omega}{k}\frac{\partial\langle f\rangle}{\partial v}\Big|_{\omega/k}\left(|E|^2/8\pi\right)$$

→ agrees with previous result

→ establishes Landau damping mechanism as collisionless heating, due secularity at wave-particle resonance.

→ Fate of energy:

$$\frac{\partial W_k}{\partial t} + \cancel{\nabla\cdot S_k} + Q_k = 0$$

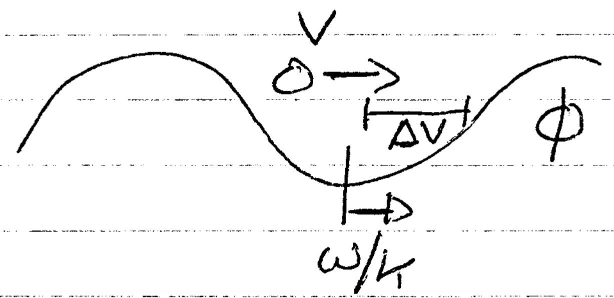
$$\frac{\partial W_k}{\partial t} = -Q_k \qquad \Rightarrow \text{L.D.} \leftrightarrow \text{wave energy dissipated}$$

but clearly resonant particles heated

so $\frac{\partial RPKED}{\partial t} + \frac{\partial}{\partial t} W_{\underline{k}} = 0$

$\therefore$ Landau damping heats resonant piece of distribution at expense of wave energy.

→ Clearly, linear theory of Landau damping only valid for times less than bounce time in trough of wave:



$\Delta V \sim \sqrt{2q\phi/m}$

$1/\tau_b = k \Delta V$

Then $\gamma_{\underline{k}} = \gamma_{\underline{k}}^{(0)}$ for $t < \tau_b$, only.

> Formal Theory of Landau Damping

Consider initial value problem:

$$f(t=0) = \langle f(v) \rangle + \tilde{f}(0, v, x)$$

Evolution of ϕ?

(i.) Landau Solution

$$\frac{\partial \tilde{f}}{\partial t} + v \frac{\partial \tilde{f}}{\partial x} = -\frac{q}{m} \tilde{E} \frac{\partial \langle f \rangle}{\partial v}$$

$$\nabla^2 \tilde{\phi} = -4\pi n_0 q \int \tilde{f} dv$$

$$\frac{\partial \tilde{f}_k}{\partial t} + ikv \tilde{f}_k = ik \tilde{\phi}_k \frac{q}{m} \frac{\partial \langle f \rangle}{\partial v}$$

$$k^2 \tilde{\phi}_k = 4\pi n_0 q \int \tilde{f}_k dv$$

Laplace Transform: $\phi_{k,\omega} = \int_0^\infty e^{i\omega t} \phi_k(t)$

$\operatorname{Im} \omega > 0$

$$\phi_k(t) = \int_{-\infty + i\epsilon}^{+\infty + i\epsilon} e^{-i\omega t} \phi_{k,\omega} \frac{d\omega}{2\pi}$$

then: $\int_0^\infty e^{i\omega t} \frac{\partial \tilde{f}_k}{\partial t} = -\tilde{f}_k(v,0) - i\omega \int_0^\infty e^{i\omega t} \tilde{f}_k$

$= -\tilde{f}_k(v,0) - i\omega \tilde{f}_{k,\omega}$

$$-\tilde{f}_k(v,0) - i(\omega - kv)\tilde{f}_{k,\omega} = i\frac{q}{m} k \tilde{\phi}_{k,\omega} \frac{\partial \langle f \rangle}{\partial v}$$

$$\tilde{f}_{k,\omega} = i\frac{\tilde{f}_k(v,0)}{(\omega - kv)} - \frac{q}{m}\frac{k}{(\omega - kv)} \phi_{k,\omega} \frac{\partial \langle f \rangle}{\partial v}$$

$\therefore$

$$k^2 \phi_{k,\omega} = 4\pi n_0 q \int dv \left\{ -\frac{q}{m} \frac{k}{\omega - kv} \partial\langle f \rangle / \partial v \, \phi_{k,\omega} + i \frac{\tilde{f}_k(v,0)}{\omega - kv} \right\}$$

$\Rightarrow$

$$\epsilon(k,\omega)\phi_{k,\omega} = \frac{4\pi n_0 q i}{k^2} \int dv \frac{\tilde{f}_k(v,0)}{\omega - kv}$$

$$\epsilon(k,\omega) = 1 + \frac{\omega_p^2}{k} \int dv \frac{\partial \langle f \rangle / \partial v}{\omega - kv}$$

$$\therefore \quad \phi_{k,\omega} = \frac{4\pi n_0 q}{k^2 \epsilon(k,\omega)}\, i \int dv\, \frac{\tilde{f}_k(v,0)}{\omega - kv}$$

Then,

$$\phi_k(t) = \int_{-\infty+i\epsilon}^{+\infty+i\epsilon} d\omega\, \frac{4\pi n_0 q}{k^2 \epsilon(k,\omega)} \left(i \int dv\, \frac{\tilde{f}_k(v,0)}{\omega - kv} \right) e^{-i\omega t}$$

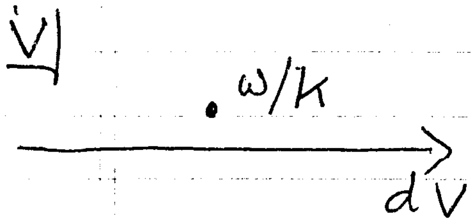
$\phi_k(t)$ determined by analytic structure of integrand

$\Rightarrow$ Singularities $\int dv\, \frac{\tilde{f}_k(v,0)}{\omega - kv}$

$\Rightarrow \left\{ \begin{matrix} \text{zeroes} \\ \text{singularities} \end{matrix} \right\} \epsilon(k,\omega)$

Now: $\to \omega = \omega + i\epsilon \Rightarrow v = v - i\epsilon$

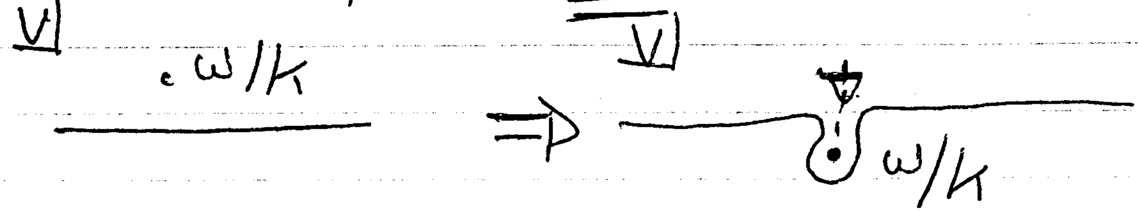
$\therefore$ so v integration along contour <u>below</u> pole at ω/k



If consider case of damped mode,

analytically continue by deforming contour so pole above ct

i.e.



∴ → singularities $\int dv\, \tilde{f}_k(v,0) \Big/ (\omega - kv) \Big|_{\text{analytic continuation}}$

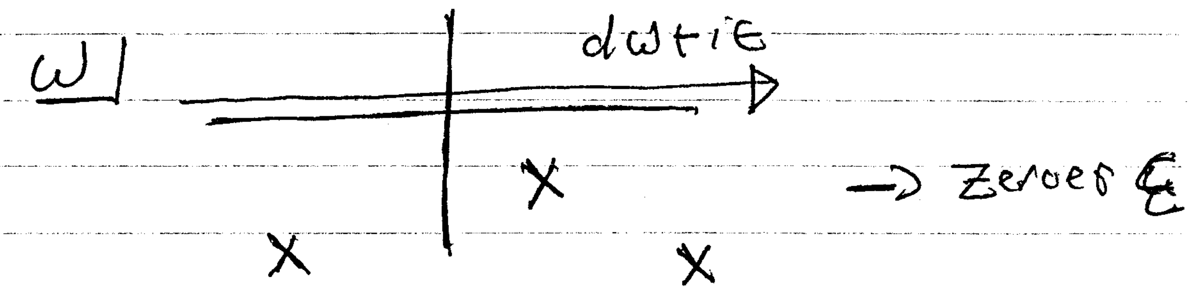
only at singularities $\tilde{f}_k(v,0)$

∴ → assuming $\tilde{f}_k(v,0)$ entire function (no singularity at finite v) and normalizable

∴ $\int dv \, \dfrac{\tilde{f}_k(v,0)}{\omega - kv}$ → entire function ω

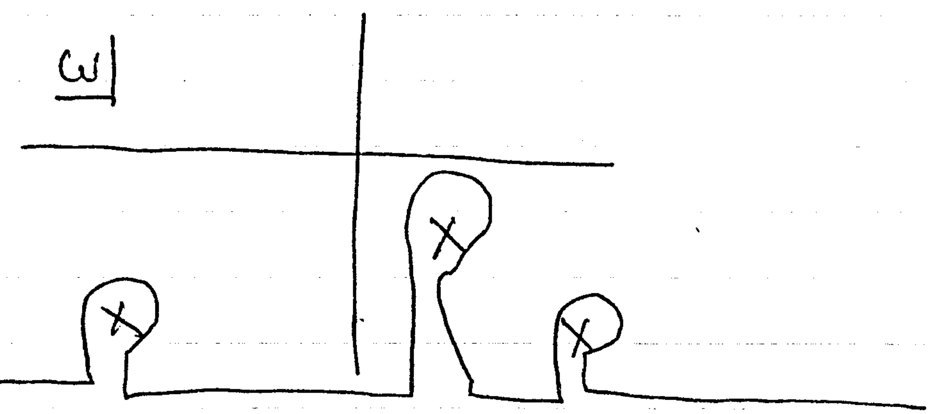
$\epsilon(k,\omega)$ → entire function (same argument)

∴ only singularities of integrand at zeroes $\epsilon(k,\omega)$



⇒ deform ω contour downward till encircles zeroes.

i.e.



Then;

$$\phi_k(t) = \sum_j \phi_k^j e^{-i\omega_{k,r}^j t} e^{-\omega_{k,I}^j t}$$

↳ residue of j th mode

$\underline{\underline{\text{so}}}$ long time response dominated by least damped mode..

iii) Case - Van Kampen Solution (Schematic)

Aside: General solution of IVP

→ determine complete set of normal modes of system

→ evolution as normal modes with IVData + Normal Mode Evolution

i.e. Plucked string 

→ Fourier series with IVD ⇒ coefficients

→ Laplace Transform

For Vlasov Plasma ↔ – Continuum of Singular Modes of f
– L.D. as phase mixing

For modes:

$$\frac{\partial \tilde{f}_k}{\partial t} + ikv\tilde{f}_k = i\frac{q}{m}k\phi_k \frac{\partial \langle f \rangle}{\partial v}$$

$$k^2 \phi_k = 4\pi n_0 q \int \tilde{f}_k \, dv$$

$$\frac{\partial f_k}{\partial t} + ikv f_k = i\frac{\omega_p^2}{k}\frac{\partial\langle f\rangle}{\partial V}\int dV\, f_k(V)$$

$$\Rightarrow \begin{cases} \dfrac{\partial f_k}{\partial t} + ikv f_k = -ik\eta(v)\displaystyle\int_{-\infty}^{+\infty} dv'\, f_k(v') \\ \eta(v) = -\dfrac{\omega_p^2}{k^2}\dfrac{\partial\langle f\rangle}{\partial V} \end{cases}$$

$$f_k = f_{k,\omega}\, e^{-i\omega t}$$

$$(V - \omega/k)\, f_{\omega/k}(v) = -\eta(v)\int_{-\infty}^{+\infty} dv'\, f_{\frac{\omega}{k}}(v')$$

$f = f(v, r)$

$r \equiv \omega/k$

$$(V - r)\, f_r(v) = -\eta(v)\int_{-\infty}^{+\infty} dv'\, f_r(v')$$

with normalization $\displaystyle\int_{-\infty}^{+\infty} dv\, f_r(v) = 1$

$$f_r(v) = -\frac{P\,\eta(v)}{v - r} + \lambda(r)\,\delta(v - r)$$

i.e. $(v-r)\,\delta(v-r) = 0$

$$1 = \int_{-\infty}^{+\infty} dv \left(-\frac{P\eta(v)}{v-r} + \lambda(r)\,\delta(v-r) \right)$$

Normalization

$$\therefore \quad \lambda(r) = 1 + \int_{-\infty}^{+\infty} dv \, \frac{P\eta(v)}{v-r}$$

so, normal modes f:

$$\rightarrow \quad f_r(v) = -\frac{P\eta(v)}{v-r} + \lambda(r)\,\delta(v-r)$$

$$\lambda(r) = 1 + \int_{-\infty}^{+\infty} dv \, \frac{P\eta(v)}{v-r}$$

$$\eta(v) = -\frac{\omega_p^2}{k^2} \frac{\partial \langle f \rangle}{\partial v}$$

→ Modes $\left\{ \begin{array}{l} \text{undamped} \\ \text{singular} \end{array} \right.$ ⇒ correspond to ballistic modes (particle streams)

→ Complete, Orthogonal Set (Case Ann. Phys. $\underline{7}$ 349 1959)

Can super-pose to show equivalence to Landau solution; Damping via Phase-Mixing i.e. $\int e^{-v^2/v_T^2} e^{-ikvt} = \int dv\, e^{-\left(\frac{v}{v_T} + \frac{ikt v_T}{2}\right)^2} + e^{-k^2 v_T^2 t^2/4}$

$\Downarrow$

undamped ballistic mode

Mathematical Note:

$$\epsilon = 1 + \frac{\omega_p^2}{k} \int dv \, \frac{\partial \langle f \rangle / \partial v}{\omega - kv}$$

$$= 1 - \frac{\omega_p^2}{k v_{Th}} \int dv \, \frac{\langle f \rangle}{(\omega - kv)} \left(\frac{v k}{v_{Th} k} - \omega + \omega \right)$$

$$= 1 + \frac{\omega_p^2}{(k v_{Th})^2} \int dv \langle f \rangle + \frac{\omega}{k} \frac{\omega_p^2}{(k v_{Th})^2} \int dv \, \frac{\langle f \rangle}{v - \frac{\omega}{k}}$$

$$= 1 + \frac{1}{k^2 \lambda_D^2} \left(1 + \frac{\omega}{k v_{Th}} \int d\varepsilon \, \frac{e^{-\varepsilon^2}}{\varepsilon - \omega / k v_{Th}} \right)$$

$$Z(\omega / k v_{Th}) = \int d\varepsilon \, e^{-\varepsilon^2} / \varepsilon - \omega / k$$

$\Downarrow$

Plasma Dispersion Function

(Tabulated)