Finish Chapter 2 of H&M

- November revolution, charm and beauty
- CP symmetry and violation
- Simple example
- Unitarity matrix for leptons and quarks

Beginning of Neutrino Physics
We’ll skip some stuff in Chapter 2

- Magnetic moment of proton etc.
- November revolution
  - Charm
  - Beauty
  - OZI suppression
- I encourage you to read up on this in chapter 2 of H&M
CP Symmetry

- So far, we talked about charge conjugation as the symmetry between particle and antiparticle.
- Well, that was good enough for QCD, but makes no sense in weak interactions.
- For weak interactions we need a simultaneous flip of Charge and Parity, or CP conjugation.
- We will discuss this in detail next quarter. Today, we simply introduce some basics, and introduce CP violation.
Simple Example

\[ \text{CP (} \overline{B^0} \rightarrow K^-\pi^+ \text{)} = B^0 \rightarrow K^+\pi^- \]

\( b \) anti-\( d \) Meson \hspace{1cm} \text{anti-} b \) d Meson

Theory: If the partial widths for these two decays are not the same, then CP is violated.

Experiment: if the branching fractions for these two decays are not the same then CP is violated.
Breaking CP is easy

⇒ Add complex coupling to Lagrangian.
⇒ Allow 2 or more channels
⇒ Add CP symm. Phase, e.g. via dynamics.

\[ T = |T| e^{-i(\delta - \gamma)} \quad \overline{T} = |T| e^{-i(\delta + \gamma)} \]

\[ \delta = \text{strong phase shift} \quad \gamma = \text{difference in weak phase} \]

\[ \text{CP } \gamma = -\gamma \quad \text{CP } \delta = +\delta \]

\[ A_{cp} = \frac{\mathcal{B}(B^0 \to K^+\pi^-) - \mathcal{B}(\overline{B}^0 \to K^-\pi^+)}{\mathcal{B}(B^0 \to K^+\pi^-) + \mathcal{B}(B^0 \to K^-\pi^+)} = \frac{P + T e^{-i(\delta - \gamma)}}{P + T e^{-i(\delta + \gamma)}} \]

\[ = -\frac{2|TP| \sin \gamma \sin \delta}{|T|^2 + |P|^2 + 2|TP| \cos \gamma \cos \delta} \]

The rest is simple algebra.
CP Violation in Standard Model

Note:
- \( V_{\text{CKM}} \) is a 3x3 unitary matrix of couplings.
- It provides the complex coupling in the Lagrangian to allow for CP violation.

We’ll get back to the details next quarter.
Breaking CP in Standard Model

• Where does the CP violating phase come from?
  – 3x3 unitary matrix => 3 angles + 6 phases
    • 2N² parameters, N² constraints from unitarity
  – 6 spinors with arbitrary phase convention
    • Only relative phase matters because only |M|² is physical.
    ⇒ Only 5 phases can be used to define a convention.
    ⇒ One phase left in 3x3 matrix that has physical consequences.

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
c_x c_z & s_x c_z & s_z e^{-i\phi} \\
-s_x c_y - c_x s_y s_z e^{i\phi} & c_x c_y - s_x s_y s_z e^{i\phi} & s_y c_z \\
s_x s_y - c_x c_y s_z e^{i\phi} & -c_x s_y - s_x c_y s_z e^{i\phi} & c_y c_z
\end{pmatrix}
\]

x, y, z are Euler angles. c=cos, s=sin.

Note: sin(z) = 0 <=> NO CP violating phase left !!!
CP violation summary

• CP violation is easy to add in field theory:
  – Complex coupling in Lagrangian
  – Interference of channels with:
    • Different CP violating phase
    • Different CP conserving phase

• Standard Model implements this via:
  – CP violating phase in charged current coupling across 3 families
  – CP conserving phase via:
    • Dynamics, e.g. Breit Wigner resonance lineshape
    • Flavor Mixing & oscillation in neutrino or quark sector
  – The scale of CP violation allowed in the standard model quark sector is well measured, and not sufficient for cosmology.

Let’s look at neutrino sector in some detail!
Aside: Scale of CP violation

- Standard model allows for CP violation only in the quark sector.
- The existence of neutrino oscillations requires extending the standard model to include CP violation in the lepton sector.
- In hw4 problem 1c you will calculate what is sometimes called the “Jarlskog Invariant”. You will show that the scale of CP violation allowed depends on the product of 4 sin terms:
  - The three family mixing angles
  - The CP violating phase
Mixing in Standard Model

• Weak eigenstates not equal mass eigenstates.
  – Mass eigenstates responsible for propagation in time.
    • Eigenstates of the Hamiltonian (excluding decay) are the mass eigenstates.
    – Weak eigenstates responsible for production and/or decay.

⇒ Oscillation between weak eigenstates as a function of time.

⇒ Discuss this in detail for Neutrino sector now.
Neutrino mixing in vacuum

• At the W vertex an electron-neutrino is created together with a positron.
• That electron-neutrino is a superposition of mass eigenstates:
  \[ |\nu_e(t)\rangle = \sum_{i=1}^{3} U_{ei}^{*} |\nu_i(t)\rangle \]
• The time evolution of the mass eigenstate can be described either in its rest-frame or in the labframe:
  \[ |\nu_i(t)\rangle = e^{-i\tilde{m}_it_i} |\nu_i(0)\rangle = e^{-i(E_it-p_iL)} |\nu_i(0)\rangle \]
• For interference among the mass eigenstates to be possible, they all have to have the same E because experimentally we average over time.
Interference for same energy states

- A neutrino beam does of course include a spectrum of neutrino energies.
- Each neutrino acquires a phase factor according to its energy:
  \[ e^{-iE_j t} \]
- Two neutrinos with differing energies will thus have a relative phase factor of:
  \[ e^{-i(E_j - E_k) t} \]
- As the time between production and detection of the neutrino has a large spread, this phase factor leads to no observable interference.
- As a result, only states of equal energy contribute to the interference effect that is observed.
Oscillation Amplitude

\[
Amp(\nu_\mu \rightarrow \nu_\tau) = \langle \nu_\tau | e^{-iEt} \sum_{i=1}^{3} e^{i p_i L} U^{*}_{\mu i} | \nu_i \rangle
\]

\[
Amp(\nu_\mu \rightarrow \nu_\tau) = e^{-iEt} \sum_{i,j=1}^{3} e^{i p_{ij} L} U^{*}_{\mu i} U_{\tau j} \langle \nu_j | \nu_i \rangle
\]

\[
Amp(\nu_\mu \rightarrow \nu_\tau) = e^{-iEt} \sum_{i=1}^{3} e^{i p_i L} U^{*}_{\mu i} U_{\tau i}
\]

Next we taylor expand \( p_i \) using:

\[
p_i = \sqrt{E^2 - m_i^2} = E - \frac{m_i^2}{2E} + ...
\]
Oscillation Probability

\[
Amp(\nu_\mu \rightarrow \nu_\tau) = e^{-iE(t-L)} \sum_{i=1}^{3} e^{-i \frac{m_i^2}{2E} L} U_{\mu i}^* U_{\tau i}
\]

\[
Prob(\nu_\mu \rightarrow \nu_\tau) = \left| Amp(\nu_\mu \rightarrow \nu_\tau) \right|^2 = \left| \sum_{i=1}^{3} e^{-i \frac{m_i^2}{2E} L} U_{\mu i}^* U_{\tau i} \right|^2
\]

In homework, you do this for the general case of \( N \) flavors. Here we do it for the simpler case of 2 flavors only. Note: For 2 flavors the \( U_{jk} \) are real, not imaginary.
Simple math aside

$$|1 - e^{ix}|^2 = (1 - [\cos x + i \sin x])(1 - [\cos x - i \sin x])$$

$$= [1 - \cos x]^2 + \sin^2 x$$

$$= 2(1 - \cos x)$$

We’ll need this in a second.
2 flavor oscillation probability

\[ |U_{11}U_{21}e^{-im_1^2 \frac{L}{2E}} + U_{12}U_{22}e^{-im_2^2 \frac{L}{2E}}|^2 = |U_{11}U_{21} + U_{12}U_{22}e^{i(m_1^2-m_2^2) \frac{L}{2E}}|^2 \]

\[ = |-\cos \theta \sin \theta + \cos \theta \sin \theta e^{i(m_1^2-m_2^2) \frac{L}{2E}}|^2 = \cos^2 \theta \sin^2 \theta |1 - e^{i(m_1^2-m_2^2) \frac{L}{2E}}|^2 \]

\[ = \cos^2 \theta \sin^2 \theta \left[ (1 - \cos \Delta)^2 + \sin^2 \Delta \right] = 2 \cos^2 \theta \sin^2 \theta \left[ 1 - \cos \Delta \right] \]

\[ = \frac{1}{2} \sin^2 2\theta \left[ 2 \sin^2 \frac{\Delta}{2} \right] \]

\[ \Delta = (m_1^2 - m_2^2) \frac{L}{2E} \]

This is a bit simplistic, as it ignores matter effects. We’ll discuss those in next lecture.
Discussion of Oscillation Equation

\[ \text{Prob}(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 \left( \frac{(m_1^2 - m_2^2)L}{4E} \right) \]

• Depends on difference in mass squared.
  – No mixing if masses are identical
  – Insensitive to mass scale
  – Insensitive to mass hierarchy because \( \sin^2 x = \sin^2(-x) \)

• Depends on \( \sin^2(2\theta) \)
  – Need large mixing angle to see large effect

• Depends on \( L/4E \)
  – Exp. with unfortunate \( L/E \) won’t see any effect.
  – Exp. with variable \( L/E \) can measure both angle and mass squared difference.
  – Exp. with \( \Delta m^2 L/4E >> 1 \) and some energy spread average over \( \sin^2 \rightarrow 1/2 \)
Experimental situation

- **Sources of electron neutrinos**
  - Sun
  - Reactors

- **Sources of muon neutrinos**
  - From charged pion beams
    - Protons on target gives charged pions.
  - From charged pion decay in atmosphere
Atmospheric neutrinos

• Expect $\nu_\mu$ anti-$\nu_\mu$ in equal numbers
• Expect $\nu_e$ half as many as $\nu_\mu$ or anti-$\nu_\mu$
• Can change $L$ as a function of Zenith angle. ($L \sim 15$km to $L \sim 13,000$km)
• $\nu_e$ Oscillation to $\nu_\mu$
  => See excess of $\nu_\mu$ vs zenith angle
• $\nu_\mu$ Oscillation to $\nu_e$
  => See excess of $\nu_e$ vs zenith angle
• $\nu_e$ Oscillation to $\nu_\tau$
  => Deficit of $\nu_e$ vs zenith angle
• $\nu_\mu$ Oscillation to $\nu_\tau$
  => Deficit of $\nu_\mu$ vs zenith angle

Let’s walk through these arguments one by one!
Atmospheric neutrinos

• Expect $\nu_\mu$ anti-$\nu_\mu$ in equal numbers
  - There are equal number of $\pi^+$ and $\pi^-$ produced in hadronic collisions in the atmosphere.
    • $\pi^+$ decays to $\mu^+ \nu_\mu$
    • $\pi^-$ decays to $\mu^- \text{anti-}\nu_\mu$

• Expect $\nu_e$ half as many as $\nu_\mu$ or anti-$\nu_\mu$
  • $\pi^+$ decays to $\mu^+ \nu_\mu$
  • $\mu^+$ decays to anti-$\nu_\mu + e^+ + \nu_e$
  • And accordingly for CP conjugate

For every $\nu_e$ there is one $\nu_\mu$ and one anti-$\nu_\mu$ because of anti-muon decay chain.

For every anti-$\nu_e$ there is one $\nu_\mu$ and one anti-$\nu_\mu$ because of muon decay chain.
Atmospheric neutrinos

- Can change L as a function of Zenith angle. (L ~ 15km to L ~ 13,000km)
Atmospheric neutrinos

• $\nu_e$ Oscillation to $\nu_\mu$
  $\Rightarrow$ See excess of $\nu_\mu$ vs zenith angle
  $\Rightarrow$ $L$ depends on zenith angle.
  $\Rightarrow$ only specific $L$ gives you maximal interference effect for a given $E$
  $\Rightarrow$ deficit of $\nu_e$ and excess of $\nu_\mu$ at the appropriate zenith angle.

• $\nu_\mu$ Oscillation to $\nu_e$
  $\Rightarrow$ See excess of $\nu_e$ vs zenith angle

• $\nu_e$ Oscillation to $\nu_\tau$
  $\Rightarrow$ Deficit of $\nu_e$ vs zenith angle but no excess of $\nu_\mu$

• $\nu_\mu$ Oscillation to $\nu_\tau$
  $\Rightarrow$ Deficit of $\nu_\mu$ vs zenith angle but no excess of $\nu_e$
Super Kamiokande Results

Left 2 plots: $\nu_e$ as expected

Right 2 plots: $\nu_\mu$ deficit largest at large L

$\cos \theta = -1$ up-going, $L \approx 13,000$ km

$\cos \theta = 1$ down-going, $L \approx 15$ km
$\nu_\mu \rightarrow \nu_\tau$

i.e. 23 mixing.

Latest Result from MINOS

Neutrinos from the Sun

• Many mechanisms, all leading to electron neutrinos with varying energies.
  – Expect: $0.5 \sin^2(2\theta)$ of solar model flux convolved with energy dependent efficiency if electron neutrino oscillations exist.

• Neutrino energy too low to produce muons or taus.
  – For decades, all experiments could measure was a neutrino flux half that of the solar neutrino model prediction.
    • People did not trust the model nor experiments as both are quite complicated!
  – Super Kamiokande was first to have pointing accuracy, and thus show that flux came from sun.
  – SNO showed that total neutrino flux agrees with solar model, and electron neutrino flux is short by factor 2.
Solar Model is Quite Complex

\[
p^+ + p^+ \rightarrow 2H + e^+ + \nu_e \\
p^+ + e^- + p^+ \rightarrow 2H + \nu_e \\
^2H + p^+ \rightarrow ^3He + \gamma \\
^3He + p^+ \rightarrow ^4He + e^+ + \nu_e \\
^3He + ^4He \rightarrow ^7Be + \gamma \\
^7Be + e^- \rightarrow ^7Li + \nu_e \\
^7Be + p^+ \rightarrow ^8B + \gamma \\
^8B \rightarrow ^8Be^* + e^+ + \nu_e \\
^8Be^* \rightarrow ^4He + ^4He
\]
Neutrino Energies are quite small
Very Challenging Experimentally for many decades
Super-Kamiokande showed that deficit in low energy neutrinos is due to neutrinos that come from the sun.

$0.465 \pm 0.005^{+0.016}_{-0.015}$ of expectation
SNO allowed CC and NC, and was thus sensitive to all neutrino flavors \(\rightarrow\) measures solar flux and electron neutrino flux.

\[
\begin{align*}
\nu_e + d &\rightarrow p + p + e^- & (CC), \\
\nu_x + d &\rightarrow p + n + \nu_x & (NC), \\
\nu_x + e^- &\rightarrow \nu_x + e^- & (ES).
\end{align*}
\]

Interpreted as \(\nu_e \rightarrow \nu_\mu\)

i.e. 12 mixing
Reactor Experiments
All except KamLAND had $L$ that is too small!
⇒ Only KamLAND saw oscillations !!!
⇒ KamLAND established $\nu_e$ disappearance
Interpretation

• Atmospheric must be $\nu_\mu \rightarrow \nu_\tau$
  – Though tau appearance has never been seen.
    • New experiment called Opera at Gran Sasso designed to observe tau appearance.
  – However, electron appearance is ruled out.
  – The state that is far in mass from the other two must have very little electron neutrino content!
Two Possible Mass Hierarchies

Normal

Inverted

\( \nu_e [ | U_{ei} |^2 ] \)

\( \nu_\mu [ | U_{\mu i} |^2 ] \)

\( \nu_\tau [ | U_{\tau i} |^2 ] \)
Things we have not discussed yet.

- Majorana Neutrinos -> see homework
- “Size of CP violation” -> see homework
- Getting well collimated E via off-axis -> see homework
- Reactor neutrinos and syntheta13 -> see homework
- Resolving the mass hierarchy -> Next lecture.