Physics 214 UCSD/225a UCSB

Lecture 14

- Schedule for remaining Quarter
- Selected topics from chapters 8&9
  - inelastic scattering
  - deep inelastic scattering
    - parton density distributions

Schedule for remaining Quarter

- Week of 11/16 - 11/20
  - Mo lecture
  - Tuesday lecture
  - This will probably conclude the material out of H&M that we will cover this quarter.
- Week of 11/23 - 11/27
  - Mo: seminar talks
  - Tue: seminar talks
  - Friday November 27th: Take home final available on website
- Week of 11/30 - 12/04
  - Mo: Lecture on Comphep (you'll probably need this for the take home final !!!)
  - Tue: either remaining seminars, or something else. We'll decide on this one once we get closer.
  - Friday December 4th, 10am, take home Final is due.

Logic of what we are doing:

- Electron - muon scattering in lab frame
  - Show what spin 1/2 on spin 1/2 scattering looks like for point particles.
- Elastic electron - proton scattering
  - Introduce the concept of form factors
  - Show how the charge radius of proton is determined
- Inelastic electron - proton scattering
  - Parameterize cross section instead of amplitude
- Deep inelastic electron - proton scattering
  - Introduce partons and parton density function
  - Discuss parton density function of proton
- Construct "parton-parton luminosity" for pp and ppbar
  - Explain the excitement about the LHC

e-proton vs e-muon scattering

- What's different?
- If proton was a spin 1/2 point particle with magnetic moment e/2M then all one needs to do is plug in the proton mass instead of muon mass into:

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{4\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

- However, magnetic moment differs, and we don't have a point particle !!!

Proton Current

$$J^{\mu} = -e\overline{u}(p')\left[\gamma^{\mu}F_1\left(q^2\right) + i\sigma^{\mu\nu}q_{\nu}\frac{\kappa}{2M}F_2\left(q^2\right)\right]u(p)e^{i(p'-p)x}$$

We determine F1, F2, and kappa experimentally, with the constraint that F1(0)=1=F2(0) in order for kappa to have the meaning of the anomalous magnetic moment.

The two form factors F1 and F2 parameterize our ignorance regarding the detailed structure of the proton.

Cross Section in labframe

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{4\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left[\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}(F_1+\kappa F_2)^2\sin^2\frac{\theta}{2}\right]$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{4\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left[\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} - 2\tau G_M^2\sin^2\frac{\theta}{2}\right]$$

$$G_E = F_1 - \kappa\tau F_2$$

$$G_M = F_1 + \kappa F_2$$

$$\tau = -\frac{q^2}{4M^2}$$

Introduce G_E and G_M to avoid terms like F_1F_2 which are harder to fit for experimentally.

Also, for $q^2 \ll M^2$ G_E and G_M are the Fourier transforms of the proton's charge and magnetic moment respectively.

Introduced tau to save some writing.

Experimentally: G_E , G_M have ~ same q^2 dependence.

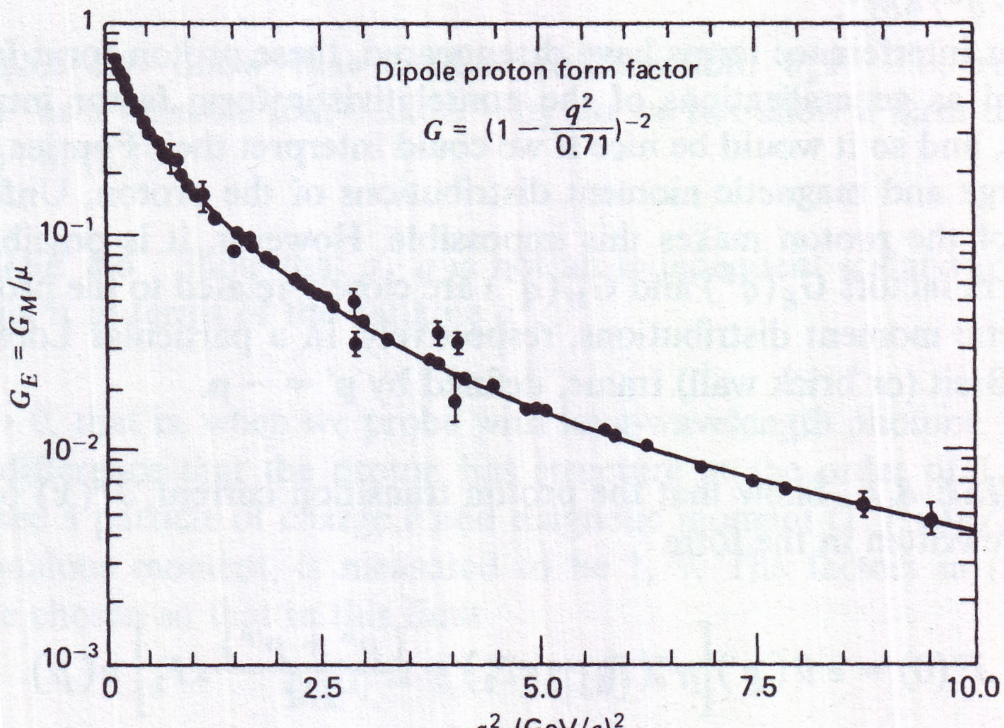


Fig. 8.4 The proton form factors as a function of q^2.

$$G = \frac{1}{\left(1 - \frac{q^2}{0.71}\right)^2}$$

(in units of GeV)

We can show (H&M Ex. 8.4), this implies a charge distribution of:

e^{-mr}

For a mean radius of:

$$\langle r^2 \rangle = 0.8\, fm$$

Size of the proton ~ 0.8fm

Inelastic Scattering

- If we wanted to work at the amplitude level, we would have to produce a current that includes a sum over all possible final state multiplicities.
  - Not a very appealing formalism!
- Instead, we go back to the cross section in terms of the product of electron and muon tensor, and generalize the muon tensor, rather than the muon current !!!

Inelastic Cross section

$$d\sigma \propto L_{electron}{}^{\mu\nu} L^{muon}{}_{\mu\nu}$$

$$d\sigma \propto L_{electron}{}^{\mu\nu} L^{"proton"}{}_{\mu\nu}$$

$$L^{"proton"\,\mu\nu} \equiv W^{\mu\nu}$$

The W_i ; i=1,2,3,4 are called proton structure functions.

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2}\left(p^\mu q^\nu + q^\mu p^\nu\right)$$

Note the omission of W_3 . It is reserved for a parity-violating structure function that is present in neutrino-proton scattering. Here the virtual photon is replaced by a virtual W or Z that interacts with the proton, or its constituents.

Note: antisymmetric pq-qp vanishes because electron tensor is symmetric.

Not all W_i are independent

$$d\sigma \propto L_{electron}{}^{\mu\nu} W_{\mu\nu}$$

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2}\left(p^\mu q^\nu + q^\mu p^\nu\right)$$

$$\left.\begin{aligned} W_5 &= -\frac{pq}{q^2} W_2 \\ W_4 &= \left(\frac{pq}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1 \end{aligned}\right\}$$

Current conservation
See Exercise 8.10 in H&M

$$\Rightarrow \boxed{W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{W_2}{M^2}\left(p^\mu - \frac{pq}{q^2} q^\mu\right)\left(p^\nu - \frac{pq}{q^2} q^\nu\right)}$$

W_i are functions of Lorentz scalars

- Unlike elastic scattering, there are two Lorentz scalars in inelastic scattering (after all, M after collision is not fixed):

$$q^2 = (k - k')^{\mu}(k - k')_{\mu}$$

This notation is just to write less -> $\nu \equiv \frac{p \cdot q}{M}$

- These are more commonly replaced by x,y defined as follows:

$$x = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2M\nu}$$

- We'll get back to x,y later.

$$y = \frac{p \cdot q}{p \cdot k}$$

Two comments on x,y

- The physical region for x,y is within [0,1]
- Both x,y depend only on measurement of:
  - Incoming electron 3-vector in lab
  - Outgoing electron 3-vector in lab
  - Incoming 3-vector of proton in lab

$$x = \frac{-q^2}{2p \cdot q}$$

$$y = \frac{p \cdot q}{p \cdot k}$$

Note:
$q^2 = -2kk' < 0$
$pq > 0$ and thus $x > 0$.
$M'^2 = (p+q)^2 = M^2 + 2pq + q^2$
As $M' > M$, pq must be >0 and thus $x < 1$.

$q^2 = -2kk' < 0$ For small scattering angle this is certainly true.

$M'^2 = (p+q)^2 = M^2 + 2pq + q^2$

$M'^2 - M^2 = 2pq + q^2 > 0$

M' = mass after collision is larger than M before collision because relative motion of pieces against each other can only increase the mass.

As $q^2<0$ we get $2pq > 0$

We thus know that $x > 0$.

$$x = \frac{-q^2}{2p \cdot q}$$

$2pq + q^2 > 0 \Rightarrow 2pq > -q^2$

And thus $x < 1$.

Next look at y.

$$y = \frac{p \cdot q}{p \cdot k}$$

$q = k - k'$

In proton restframe, $p = (M, 0)$

$\Rightarrow y = M(E-E')/ME = (E-E')/E$

Energy conservation tells you that $E > E'$ because $M'>M$, and the best you can have is $p' = 0$ to maximize E'.

$\Rightarrow$ $y > 0$ and $y < 1$.

Cross section for inelastic electron proton scattering

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{4\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left[W_2\cos^2\frac{\theta}{2} + 2W_1\sin^2\frac{\theta}{2}\right]}$$

$$W_i = W_i\left(\nu, q^2\right)$$

$$q^2 = \left(k - k'\right)^\mu \left(k - k'\right)_\mu$$

$$\nu \equiv \frac{p \cdot q}{M}$$

The structure functions depend on two scalars: q^2 and ν

See H&M chapter 8.3 for details.

Deep inelastic scattering

- Intuitively, it seems obvious that small wavelength, i.e. large $-q^2$, virtual photons ought to be able to probe the charge distribution inside the proton.
- If there are pointlike spin 1/2 particles, i.e. “quarks” inside, then we ought to be able to measure their charge via electron-proton scattering at large $-q^2$.
- *Within the formalism so far, this means that we measure W_1 and W_2 to have a form that indicates pointlike spin 1/2 particles.*

What’s that form?

Let’s compare e-mu, elastic, and inelastic scattering.

Electron muon:

$W_1 = -(q^2/4m^2)\,\delta[\nu + (q^2/2m)]$
$W_2 = \delta[\nu + (q^2/2m)]$

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{lab} = \frac{4\alpha^2 E'^2}{q^4}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right]\delta\left(\nu + \frac{q^2}{2m}\right)$$

Elastic electron proton:

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{lab} = \frac{4\alpha^2 E'^2}{q^4}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} - 2\tau G_M^2 \sin^2\frac{\theta}{2}\right]\delta\left(\nu + \frac{q^2}{2M}\right)$$

Inelastic electron proton:

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{lab} = \frac{4\alpha^2 E'^2}{q^4}\left[W_2\cos^2\frac{\theta}{2} + 2W_1\sin^2\frac{\theta}{2}\right]$$

Aside

- Let's replace $-q^2$ by Q^2 in order to always have a positive Q^2 value in all our expressions.

- Mathematical aside: $\delta[ax] = \delta[x]/a$

- Let's introduce the dimensionless variable ω

$$\omega = \frac{2q \cdot p}{Q^2}$$

$W_1 = -(q^2/4m^2)\,\delta[\nu + (q^2/2m)]$
$W_1 = (Q^2/4m^2)\,\delta[\nu - (Q^2/2m)]$

$$2mW_1 = \frac{Q^2}{2m}\delta\left(\nu - \frac{Q^2}{2m}\right) \Rightarrow 2mW_1 = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\nu \equiv \frac{p \cdot q}{m}$$

$$\frac{Q^2}{2m\nu} = \frac{Q^2}{2p \cdot q} = \frac{1}{\omega}$$

ω is a dimensionless variable

$$\boxed{2mW_1 = \frac{1}{\omega}\delta\left(1 - \frac{1}{\omega}\right)}$$

The proton structure function thus depends only on a dimensionless variable if there are spin 1/2 point particles that make up the proton.

$W_2 = \delta[\nu + (q^2/2m)]$
$W_2 = \delta[1 + (1/\omega)]/\nu$

=> $$\nu W_2 = \delta\left(1 - \frac{1}{\omega}\right)$$

W_1, W_2 for point particles in proton

$$\boxed{\begin{aligned} 2mW_1 &= \frac{1}{\omega}\delta\left(1-\frac{1}{\omega}\right) \\ vW_2 &= \delta\left(1-\frac{1}{\omega}\right) \end{aligned}}$$

Both of these structure functions are now functions of only one dimensionless variable !!!

$\Rightarrow$ ***Bjorken Scaling***

Both of these structure functions are obviously related.
In this case, there is only one F(x). More on this in a sec.

Scaling as characteristic of point particles inside the proton

- To understand why the scale independence itself is the important characteristics of having point particles inside the proton, compare W_i for e-mu with elastic e-proton:

$$2mW_1 = \frac{1}{\omega}\delta\left(1-\frac{1}{\omega}\right) \qquad 2MW_1^{elastic} = \boxed{G(Q^2)}\frac{1}{\omega}\delta\left(1-\frac{1}{\omega}\right)$$

$$\nu W_2 = \delta\left(1-\frac{1}{\omega}\right) \qquad \nu W_2^{elastic} = \boxed{G(Q^2)}\delta\left(1-\frac{1}{\omega}\right)$$

- For elastic scattering, there is an explicit Q^2 dependence. The 0.71GeV mass scale in the pole of G sets a size cut-off below which the proton is more likely to disintegrate than scatter elastically.
- Not having such a cut-off, i.e. scaling, is a signature of constituents that are point particles.

$$G = \frac{1}{\left(1+\frac{Q^2}{0.71}\right)^2}$$

Bjorken Scaling

$$\omega = \frac{2q \cdot p}{Q^2}$$

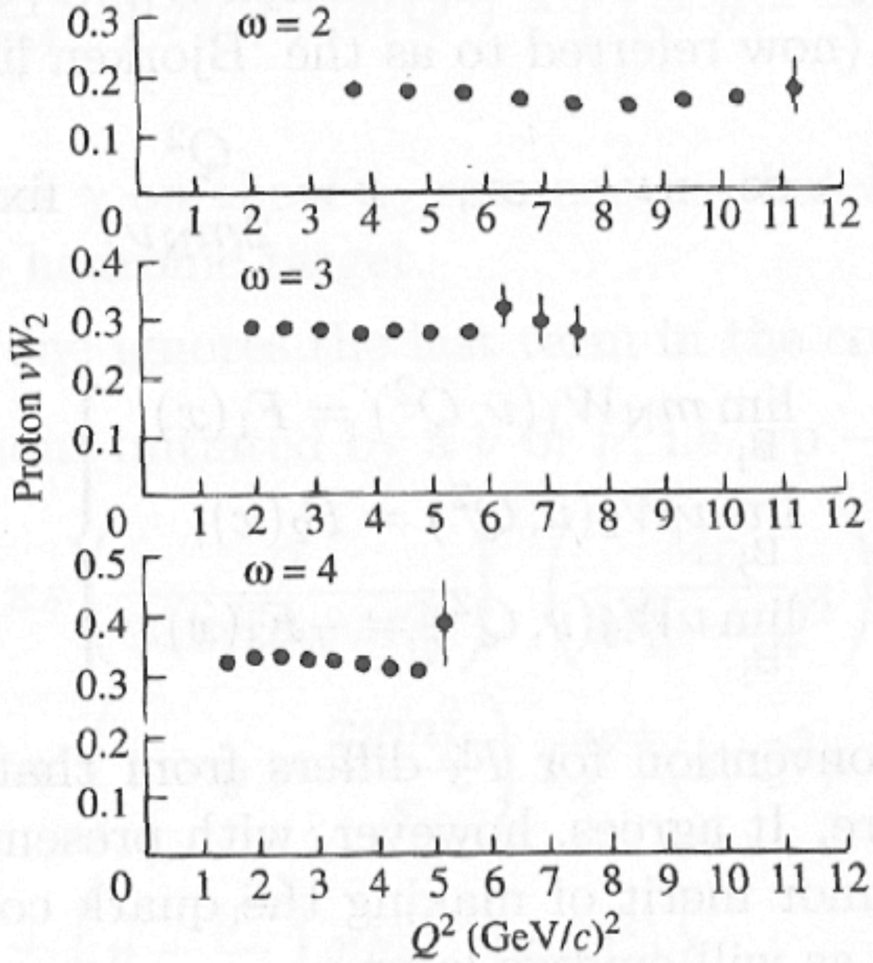


Fig. 15.9. Scaling behaviour of electromagnetic structure function νW_2 at various ω values. There is virtually no variation with Q^2. (From Panofsky, 1968.)

The parton picture of the proton

- Proton is made up of some set of partons.
  - Some of which are charged
  - Others aren't.
- Each parton carries a fraction, x, of the momentum.

	Proton	Parton
Energy	E	xE
Momentum	p_L	x p_L
	$p_T = 0$	$p_T = 0$
Mass	M	xM

- All fractions add up to 1:

$$\sum_i \int dx \quad x f_i(x) = 1$$

What weirdo frame is that?

- The only way you can have the kinematic assignments from the previous page is if

$$|p| >> m, M$$

- Everything is thus highly relativistic, all partons move at ~ c, and the kinematics described makes sense.
- Relativistic time dilation in this “frame” leads to partons being free particles, i.e. the dt during which the virtual photon interaction takes place is << than the time for the partons to interact with each other.
  - ⇒We can add probabilities for interacting with each parton, rather than the amplitudes.
  - ⇒This is referred to as the *incoherence assumption*, and implicit in our use of $f_i(x)$: $\sum_i \int dx \quad x f_i(x) = 1$

Recap of parton structure function

- There is only one F(x).
- It is made out of the incoherent sum of *probabilities for finding a given type i of parton at a given x* in the proton:

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 x f_i(x)$$

- The experimental problem is thus to *extract $f_i(x)$ from a large variety of measurements*.
- For deep inelastic e-proton, the gluon structure function can be obtained from the requirement that it all adds up. Gluons are the leftovers.

Simple Example for determining structure function for quarks.

- Compare e-proton with e-neutron deep inelastic scattering.
  - $\Rightarrow$This gives us F^{ep} and F^{en} structure function.

$$\frac{1}{x}F^{ep}=\left(\frac{2}{3}\right)^2\left(u^p(x)+\overline{u}^p(x)\right)+\left(\frac{1}{3}\right)^2\left(d^p(x)+\overline{d}^p(x)\right)+\left(\frac{1}{3}\right)^2\left(s^p(x)+\overline{s}^p(x)\right)$$

$$\frac{1}{x}F^{en}=\left(\frac{2}{3}\right)^2\left(u^n(x)+\overline{u}^n(x)\right)+\left(\frac{1}{3}\right)^2\left(d^n(x)+\overline{d}^n(x)\right)+\left(\frac{1}{3}\right)^2\left(s^n(x)+\overline{s}^n(x)\right)$$

We then assume that all sea quark contributions are the same for ep and en. And the valence quark ones are related by isospin.

We then assume that all sea quark contributions are the same for ep and en. And the valence quark ones are related by isospin.

$u^p = d^n = u(x)$
$d^p = u^n = d(x)$
$s^p = s^n = s(x)$
$u - ubar = u_v$
$d - dbar = d_v$

=>

$$\frac{1}{x}F_2^{ep}(x) = \frac{1}{9}\left[4u_v(x) + d_v(x)\right] + \frac{12}{9}S(x)$$

$$\frac{1}{x}F_2^{en}(x) = \frac{1}{9}\left[u_v(x) + 4d_v(x)\right] + \frac{12}{9}S(x)$$

Here S(x) refers generically to sea quarks, while 12/9 accounts for the sum of e^2 for u,d,s and their anti-quarks in the sea.

Note: charm and beauty is ignored in this discussion.

Some observations

- Since gluons create the sea q-qbar pairs, one should expect a momentum spectrum at low x similar to bremsstrahlung:
  => $S(x) \to 1/x$ as $x \to 0$ at fixed Q^2 .

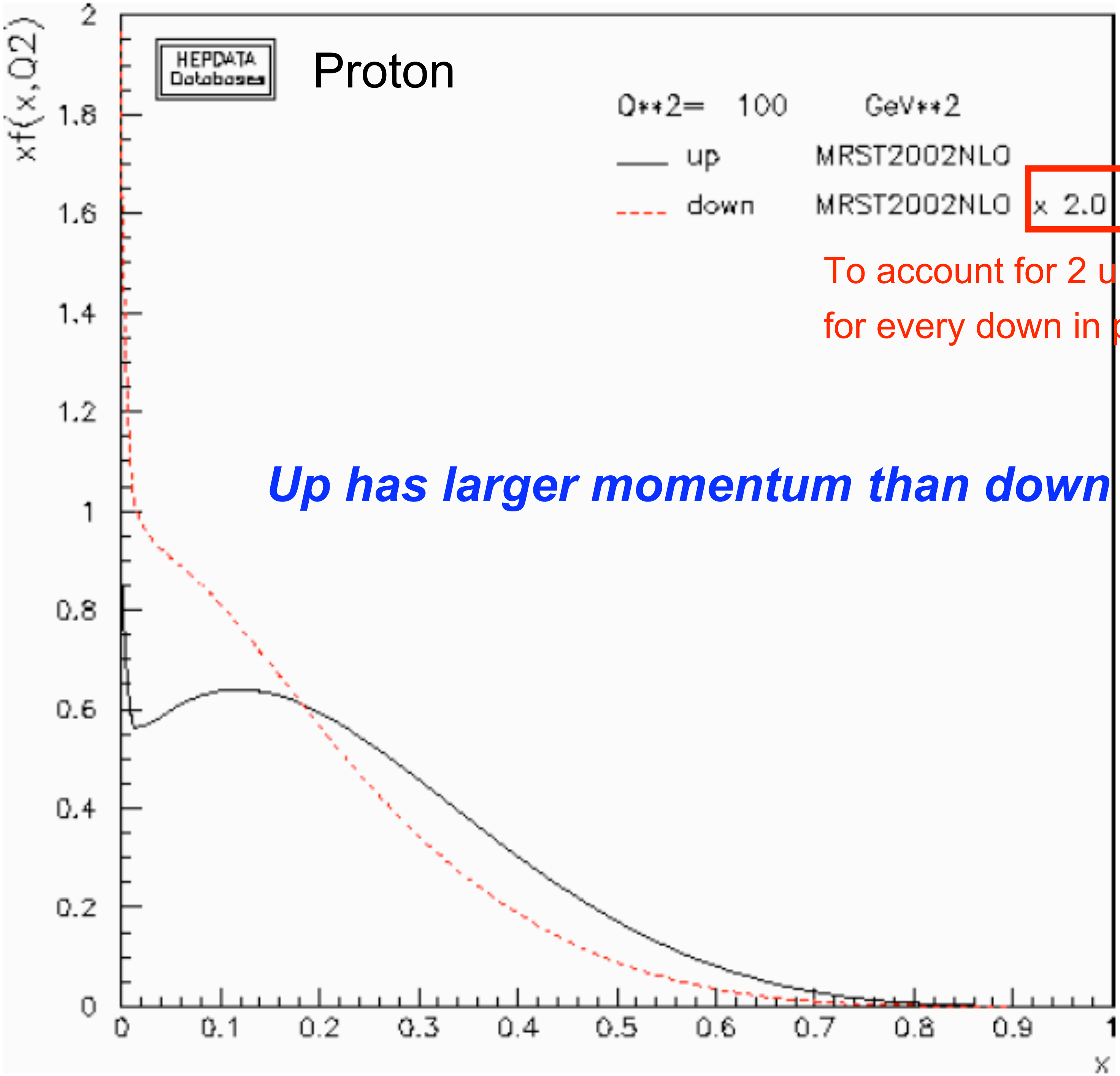
  => $F^{en}/F^{ep} \to 1$ as $x \to 0$
  => $F^{en}/F^{ep} \to (u_v + 4d_v)/(4u_v + d_v)$ as $x \to 1$

- Experimentally, we observe:
  $F^{en}/F^{ep} \to 1$ as $x \to 0$ as expected.
  $F^{en}/F^{ep} \to 0.25$ as $x \to 1$ => u_v appears to dominate at high x.

- This means that up quarks dominate in proton while down quarks dominate in neutron at large x.
- The dominant valence quark dominates at large x.

- Fitting structure functions of proton and anti-proton is an industry. There are 3 independent groups doing it, using a large number of independent measurements including ep,en, neutrino-p, neutrino-n, photon cross section, DY, W forward-backward asymmetry etc. etc. etc.

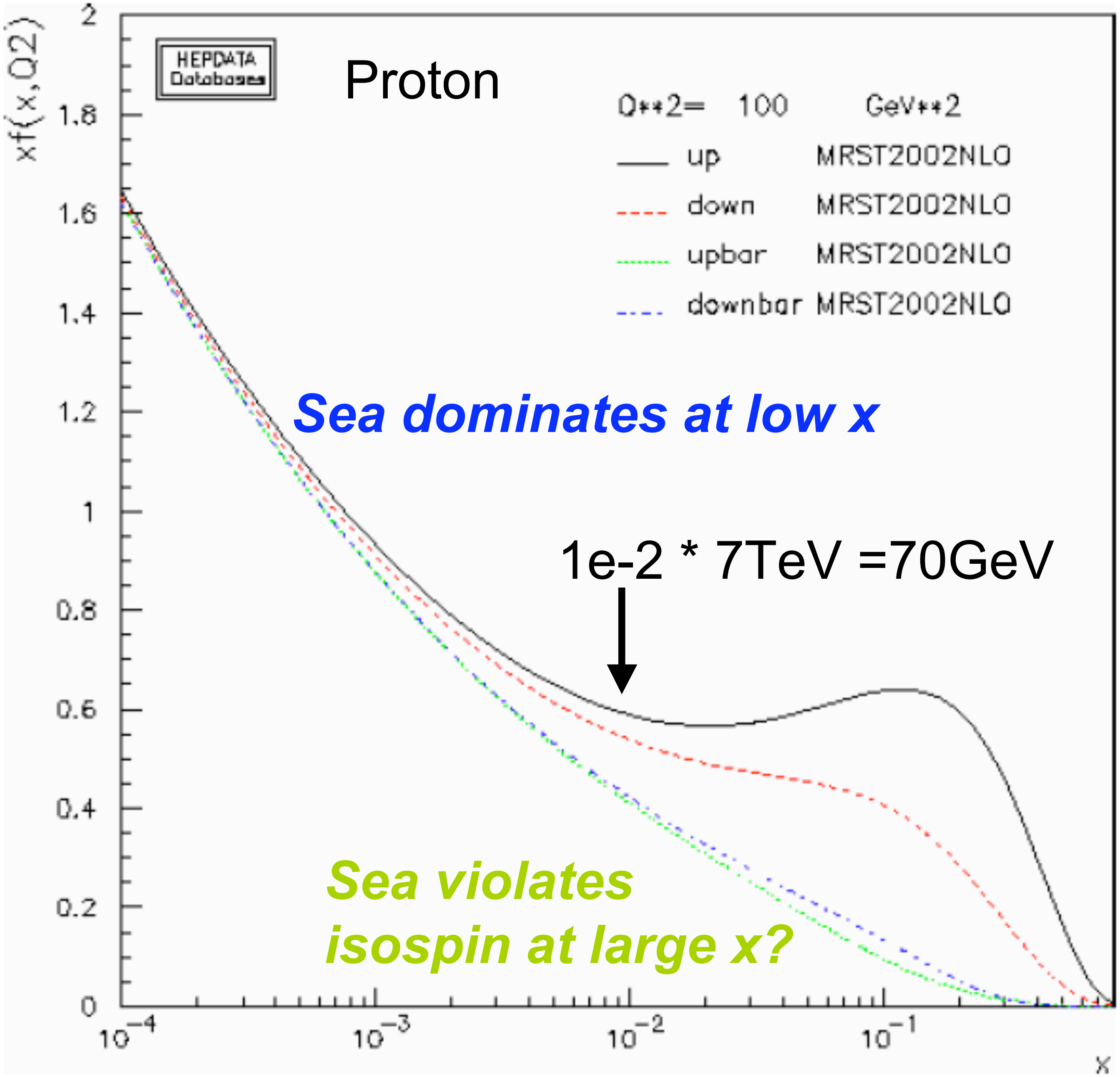
- This is very important “engineering” work for the LHC !!!



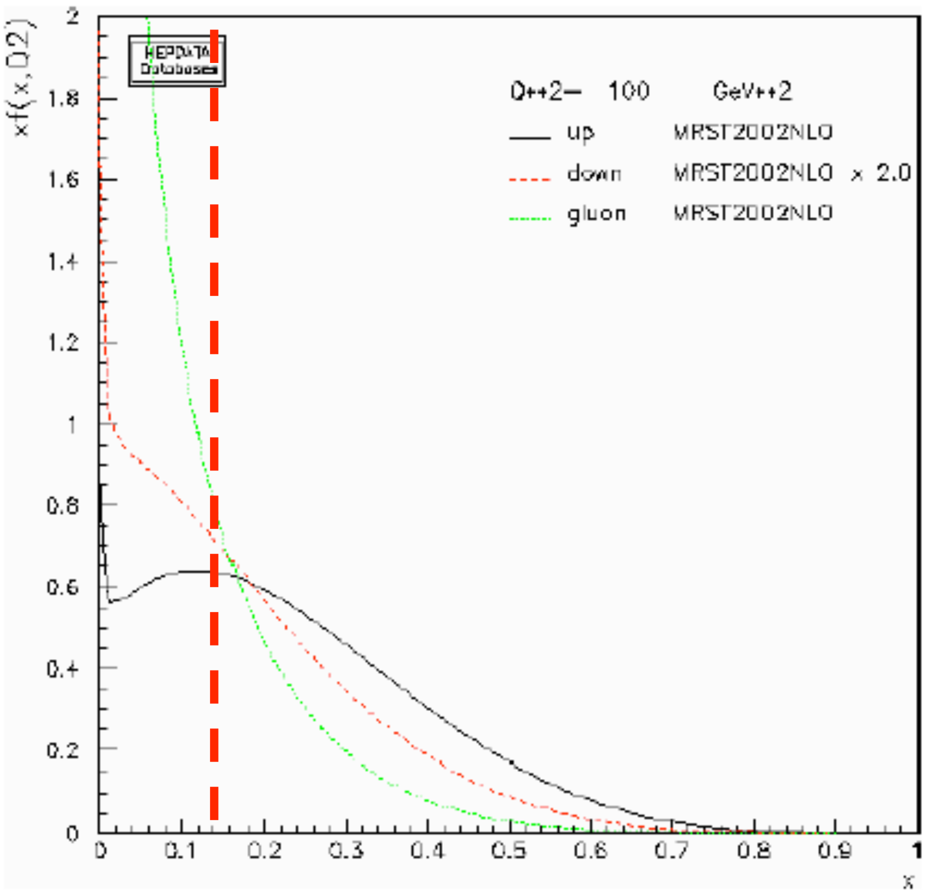
To account for 2 up
for every down in proton !!!

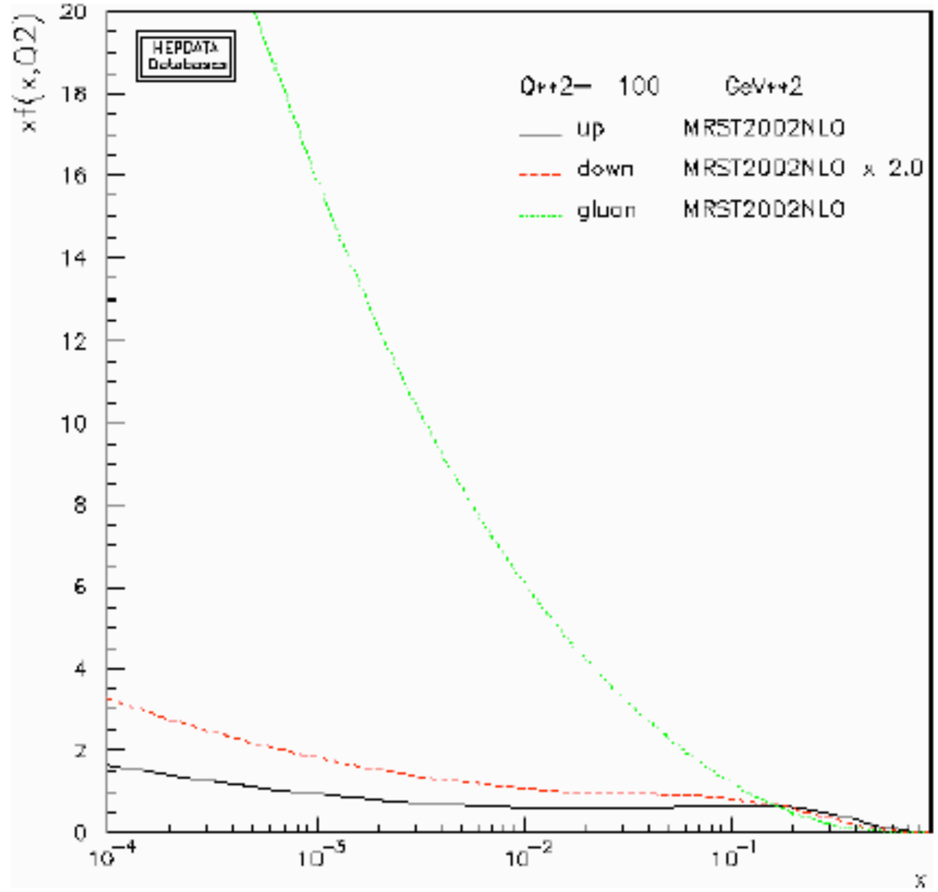
Up has larger momentum than down quark.

PDFs from http://durpdg.dur.ac.uk/hepdata/pdf3.html



A 14TeV collider can be pp instead of ppbar !!!





Gluons dominate at low x .

To set the scale, x = 0.14 at LHC is 0.14 * 7TeV = 1TeV

=> The LHC is a gluon collider !!!

Parton Model and Bjorken Scaling

We introduced two definitions for "x".

One from e-p scattering:

$$x = \frac{-q^2}{2p \cdot q} = \frac{1}{\omega}$$

And one from the parton model momentum fraction.

Section 9.2 in H&M shows that these are actually the same.

$$f_2^{\,i}(\omega) = \delta\left(1 - \frac{1}{x\omega}\right)$$

Is the F_2 structure function for the ith parton, that has a momentum fraction x.

As we sum over all partons: $F_2(\omega) = \sum_i \int dx e_i^2 f_i(x) x \delta\left(x - \frac{1}{\omega}\right)$

The δ-function here means that the virtual photon must have just the right x to be absorbed by a parton with momentum fraction,x, of the proton.