[1] Evaluate all cases of \{A_i, A_j\}, where
\[
A_1 = \frac{1}{4}(x^2 + p_x^2 - y^2 - p_y^2) \quad A_3 = \frac{1}{2}(x p_y - y p_x)
\]
\[
A_2 = \frac{1}{2}(x y + p_x p_y) \quad A_4 = x^2 + y^2 + p_x^2 + p_y^2.
\]

[2] Determine the generating function $F_3(p, Q)$ which produces the same canonical transformation as the generating function $F_2(q, P) = q^2 \exp(P)$.

[3] Show explicitly that the canonical transformation generated by an arbitrary $F_1(q, Q, t)$ preserves the symplectic structure of Hamilton’s equations. That is, show that
\[
M_{aj} \equiv \frac{\partial \xi_a}{\partial \xi_j}
\]
is symplectic. Hint: Start by writing $p_\sigma = \frac{\partial F_1}{\partial q_\sigma}$ and $P_\sigma = -\frac{\partial F_1}{\partial Q_\sigma}$, and then evaluate the differentials $dp_\sigma$ and $dP_\sigma$.

[4] Consider the small oscillations of an anharmonic oscillator with Hamiltonian
\[
H = \frac{p^2}{2m} + \frac{1}{2}m \omega^2 q^2 + \alpha q^3 + \beta q p^2
\]
under the assumptions $\alpha q \ll m \omega^2$ and $\beta q \ll \frac{1}{m}$.

(a) Working with the generating function
\[
F_2(q, P) = qP + a q^2 P + b P^3
\]
find the parameters $a$ and $b$ such that the new Hamiltonian $\tilde{H}(Q, P)$ does not contain any anharmonic terms up to third order (i.e. no terms of order $Q^3$ nor of order $QP^2$).

(b) Determine $q(t)$.

[5] A particle of mass $m$ moves in one dimension subject to the potential
\[
U(x) = \frac{k}{\sin^2(x/a)}.
\]

(a) Obtain an integral expression for Hamilton’s characteristic function.
(b) Under what conditions may action-angle variables be used?

(c) Assuming that action-angle variables are permissible, determine the frequency of oscillation by the action-angle method.

(d) Check your result for the oscillation frequency in the limit of small oscillations.