PHYSICS 200A : CLASSICAL MECHANICS
PROBLEM SET #6

[1] Evaluate all cases of $\{A_i\,,\,A_j\}$, where

$$A_1 = \tfrac{1}{4}\big(x^2 + p_x^2 - y^2 - p_y^2\big) \qquad\qquad A_3 = \tfrac{1}{2}\big(x\,p_y - y\,p_x\big)$$

$$A_2 = \tfrac{1}{2}\big(x\,y + p_x\,p_y\big) \qquad\qquad A_4 = x^2 + y^2 + p_x^2 + p_y^2\ .$$

[2] Determine the generating function $F_3(p,Q)$ which produces the same canonical transformation as the generating function $F_2(q,P) = q^2\,\exp(P)$.

[3] Show explicitly that the canonical transformation generated by an arbitrary $F_1(q,Q,t)$ preserves the symplectic structure of Hamilton's equations. That is, show that

$$M_{aj} \equiv \frac{\partial \Xi_a}{\partial \xi_j}$$

is symplectic. *Hint : Start by writing* $p_\sigma = \frac{\partial F_1}{\partial q_\sigma}$ *and* $P_\sigma = -\frac{\partial F_1}{\partial Q_\sigma}$, *and then evaluate the differentials* dp_σ *and* dP_σ.

[4] Consider the small oscillations of an anharmonic oscillator with Hamiltonian

$$H = \frac{p^2}{2m} + \tfrac{1}{2} m\,\omega^2\,q^2 + \alpha\,q^3 + \beta\,q\,p^2$$

under the assumptions $\alpha\,q \ll m\,\omega^2$ and $\beta\,q \ll \frac{1}{m}$.

(a) Working with the generating function

$$F_2(q,P) = qP + a\,q^2\,P + b\,P^3\ ,$$

find the parameters a and b such that the new Hamiltonian $\tilde{H}(Q,P)$ does not contain any anharmonic terms up to third order (*i.e.* no terms of order Q^3 nor of order QP^2).

(b) Determine $q(t)$.

[5] A particle of mass m moves in one dimension subject to the potential

$$U(x) = \frac{k}{\sin^2(x/a)}\ .$$

(a) Obtain an integral expression for Hamilton's characteristic function.

(b) Under what conditions may action-angle variables be used?

(c) Assuming that action-angle variables are permissible, determine the frequency of oscillation by the action-angle method.

(d) Check your result for the oscillation frequency in the limit of small oscillations.