PHYSICS 200A : CLASSICAL MECHANICS
PROBLEM SET #2

[1] Consider a three-dimensional one-particle system whose potential energy in cylindrical polar coordinates $\{\rho, \phi, z\}$ is of the form $V(\rho, k\phi + z)$, where k is a constant.

(a) Find a symmetry of the Lagrangian and use Noether's theorem to obtain the constant of the motion associated with it.

(b) Write down at least one other constant of the motion.

(c) Obtain an explicit expression for the vector field,

$$\frac{d}{dt} = \dot{q}_\sigma \frac{\partial}{\partial q_\sigma} + \ddot{q}_\sigma \frac{\partial}{\partial \dot{q}_\sigma}$$

and use it to verify that the functions found in (a) and (b) are indeed constants of the motion.

[2] Derive the equations of motion for the Lagrangian

$$L = e^{\gamma t} \left[\frac{1}{2} m \dot{q}^2 - \frac{1}{2} k q^2 \right] \; ,$$

where $\gamma > 0$. Compare with known systems. Rewrite the Lagrangian in terms of the new variable $Q \equiv q \, \exp(\gamma t/2)$, and from this obtain a constant of the motion.

[3] A bead of mass m slides frictionlessly along a wire curve $z = x^2/2b$, where $b > 0$. The wire rotates with angular frequency ω about the $\hat{\boldsymbol{z}}$ axis.

(a) Find the Lagrangian of this system.

(b) Find the Hamiltonian.

(c) Find the effective potential $U_{\text{eff}}(x)$.

(d) Show that the motion is unbounded for $\omega^2 > \omega_{\text{c}}^2$ and find the critical value ω_{c}.

(e) Sketch the phase curves for this system for the cases $\omega^2 < \omega_{\text{c}}^2$ and $\omega^2 > \omega_{\text{c}}^2$.

(f) Find an expression for the period of the motion when $\omega^2 < \omega_{\text{c}}^2$.

(g) Find the force of constraint which keeps the bead on the wire.

[4] A particle of mass m is embedded, a distance b from the center, in a uniformly dense disk of mass M. The mass of the disk plus the inclusion is thus $M + m$. The disk rolls without slipping along a plane inclined at an angle α with respect to the horizontal, under the influence of gravity. The axis of the disk remains horizontal throughout the motion.

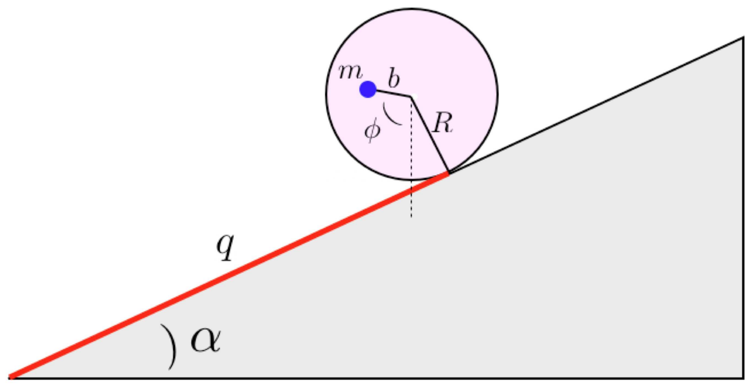

Figure 1: A cylinder of radius R with an inclusion rolls along an inclined plane.

(a) Let q be the distance from the disk's point of contact to the bottom corner of the wedge, as shown in figure fig. 1. Let ϕ be the angle the inclusion makes with respect to the vertical. Find the position $(x_{\mathrm{C}}, y_{\mathrm{C}})$ of the geometrical center of the disk in terms of q and α. Find also the position (x, y) of the mass inclusion as a function of q, α, and ϕ. Show that q and ϕ are related by a holonomic constraint.

(b) Find the Lagrangian $L(\phi, \dot{\phi}, t)$.

(c) Find the equations of motion.

(d) Under what conditions does a stable equilibrium exist?

(e) Find the frequency of small oscillations about the equilibrium.