PHYSICS 200A : CLASSICAL MECHANICS
PROBLEM SET #1

[1] Minimize the functional

$$F\big[y(x)\big] = \int\limits_0^{\ln 2} dx \left(\frac{1}{2}y'^2 + \frac{1}{2}y^2 + y\right)$$

when the values of y are not specified at the endpoints.

[2] Find the extrema of the functional

$$F\big[y(x), z(x)\big] = \int\limits_0^{\frac{\pi}{2}} dx \left(y'^2 + z'^2 + 2yz\right)$$

subject to the boundary conditions

$$y(0) = z(0) = 0 \qquad , \qquad y(\tfrac{\pi}{2}) = z(\tfrac{\pi}{2}) = 1 \ .$$

[3] Find the extrema of the functional

$$F\big[y(x)\big] = \int\limits_0^1 dx \left(y'^2 + x^2\right)$$

subject to the boundary conditions

$$y(0) = 0 \qquad , \qquad y(1) = 1 \qquad , \qquad \int\limits_0^1 dx\, y^2 = 2 \ .$$

[4] Consider the functional

$$E\big[y(x)\big] = \frac{1}{2}\int\limits_0^L dx \left(y''^2 + 2ay'^2 + b^2\, y^2\right) ,$$

with the boundary conditions

$$y(0) = y_0 \quad , \quad y(L) = y_L \quad , \quad y'(0) = y_0' \quad , \quad y'(L) = y_L' \ .$$

(a) Find $K_1(x) = \frac{\delta E}{\delta y(x)}$.

(b) Solve $\frac{\delta E}{\delta y(x)} = 0$.

(c) Find $K_2(x,x') = \frac{\delta^2 E}{\delta y(x)\,\delta y(x')}$.

(d) What is the condition which determines the eigenvalues of $K_2(x,x')$?