

## Chapter 27

# Quantum Physics

### Answers to Even Numbered Conceptual Questions

4. Measuring the position of a particle implies having photons reflect from it. However, collisions between photons and the particle will alter the velocity of the particle.
6. Light has both wave and particle characteristics. In Young's double-slit experiment, light behaves as a wave. In the photoelectric effect, it behaves like a particle. Light can be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time, light can be characterized as a stream of photons, each carrying a
10. Ultraviolet light has a shorter wavelength and higher photon energy than visible light.
16. The red beam. Each photon of red light has less energy (longer wavelength) than a photon of blue light, so the red beam must contain more photons to carry the same total energy.

## Problem Solutions

27.3 The wavelength of maximum radiation is given by

$$\lambda_{\text{max}} = \frac{0.2898 \times 10^{-2} \text{ m} \cdot \text{K}}{5800 \text{ K}} = 5.00 \times 10^{-7} \text{ m} = \boxed{500 \text{ nm}}$$

27.4 The energy of a photon having wavelength  $\lambda$  is  $E_\gamma = hf = hc/\lambda$ . Thus, the number of photons delivered by each beam must be:

Red Beam:

$$n_R = \frac{E_{\text{total}}}{E_{\gamma,R}} = \frac{E_{\text{total}} \lambda_R}{hc} = \frac{(2500 \text{ eV})(690 \times 10^{-9} \text{ m})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})} \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = \boxed{1.39 \times 10^3}$$

Blue Beam: 
$$n_B = \frac{E_{\text{total}}}{E_{\gamma,B}} = \frac{E_{\text{total}} \lambda_B}{hc} = \frac{(2500 \text{ eV})(420 \times 10^{-9} \text{ m})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})} \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = \boxed{845}$$

27.8 The energy entering the eye each second is

$$P = I \cdot A = (4.0 \times 10^{-11} \text{ W/m}^2) \left[ \frac{\pi}{4} (8.5 \times 10^{-3} \text{ m})^2 \right] = 2.3 \times 10^{-15} \text{ W}$$

The energy of a single photon is

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{500 \times 10^{-9} \text{ m}} = 3.98 \times 10^{-19} \text{ J}$$

so the number of photons entering the eye in  $\Delta t = 1.00 \text{ s}$  is

$$N = \frac{\Delta E}{E_\gamma} = \frac{P \cdot (\Delta t)}{E_\gamma} = \frac{(2.3 \times 10^{-15} \text{ J/s})(1.00 \text{ s})}{3.98 \times 10^{-19} \text{ J}} = \boxed{5.7 \times 10^3}$$

27.11 (a) From the photoelectric effect equation, the work function is

$$\phi = \frac{hc}{\lambda} - KE_{\text{max}}, \text{ or}$$

$$\phi = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{350 \times 10^{-9} \text{ m}} \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) - 1.31 \text{ eV}$$

$$\phi = \boxed{2.24 \text{ eV}}$$

$$(b) \lambda_c = \frac{hc}{\phi} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{2.24 \text{ eV}} \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) = \boxed{555 \text{ nm}}$$

$$(c) f_c = \frac{c}{\lambda_c} = \frac{3.00 \times 10^8 \text{ m/s}}{555 \times 10^{-9} \text{ m}} = \boxed{5.41 \times 10^{14} \text{ Hz}}$$

27.19 Assuming the electron produces a single photon as it comes to rest, the energy of that photon is  $E_\gamma = (\kappa E)_i = eV$ . The accelerating voltage is then

$$V = \frac{E_\gamma}{e} = \frac{hc}{e\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})\lambda} = \frac{1.24 \times 10^{-6} \text{ V} \cdot \text{m}}{\lambda}$$

$$\text{For } \lambda = 1.0 \times 10^{-8} \text{ m}, V = \frac{1.24 \times 10^{-6} \text{ V} \cdot \text{m}}{1.0 \times 10^{-8} \text{ m}} = \boxed{1.2 \times 10^2 \text{ V}}$$

$$\text{and for } \lambda = 1.0 \times 10^{-13} \text{ m}, V = \frac{1.24 \times 10^{-6} \text{ V} \cdot \text{m}}{1.0 \times 10^{-13} \text{ m}} = \boxed{1.2 \times 10^7 \text{ V}}$$

27.25 The interplanar spacing in the crystal is given by Bragg's law as

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.140 \text{ nm})}{2\sin 14.4^\circ} = \boxed{0.281 \text{ nm}}$$

**27.26** The scattering angle is given by the Compton shift formula as

$$\theta = \cos^{-1} \left( 1 - \frac{\Delta\lambda}{\lambda_c} \right) \text{ where the Compton wavelength is}$$

$$\lambda_c = \frac{h}{m_e c} = 0.00243 \text{ nm}$$

$$\text{Thus, } \theta = \cos^{-1} \left( 1 - \frac{1.50 \times 10^{-3} \text{ nm}}{2.43 \times 10^{-3} \text{ nm}} \right) = \boxed{67.5^\circ}$$

**27.34** The de Broglie wavelength of a particle of mass  $m$  is  $\lambda = h/p$  where the momentum is given by  $p = \gamma m v = m v / \sqrt{1 - (v/c)^2}$ . Note that when the particle is not relativistic, then  $\gamma \approx 1$ , and this relativistic expression for momentum reverts back to the classical expression.

(a) For a proton moving at speed  $v = 2.00 \times 10^4 \text{ m/s}$ ,  $v \ll c$  and  $\gamma \approx 1$  so

$$\lambda = \frac{h}{m_p v} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^4 \text{ m/s})} = \boxed{1.98 \times 10^{-11} \text{ m}}$$

(b) For a proton moving at speed  $v = 2.00 \times 10^7 \text{ m/s}$

$$\begin{aligned} \lambda &= \frac{h}{\gamma m_p v} = \frac{h}{m_p v} \sqrt{1 - (v/c)^2} \\ &= \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})}{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^7 \text{ m/s})} \sqrt{1 - \left( \frac{2.00 \times 10^7 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2} = \boxed{1.98 \times 10^{-14} \text{ m}} \end{aligned}$$

**27.36** After falling freely with acceleration  $a_y = -g = -9.80 \text{ m/s}^2$  for 50.0 m, starting from rest, the speed of the ball will be

$$v = \sqrt{v_0^2 + 2a_y(\Delta y)} = \sqrt{0 + 2(-9.80 \text{ m/s}^2)(-50.0 \text{ m})} = 31.3 \text{ m/s}$$

so the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{m v} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(0.200 \text{ kg})(31.3 \text{ m/s})} = \boxed{1.06 \times 10^{-34} \text{ m}}$$

**27.43** From the uncertainty principle, the minimum uncertainty in the momentum of the electron is

$$\Delta p_x = \frac{h}{4\pi(\Delta x)} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(0.10 \times 10^{-9} \text{ m})} = 5.3 \times 10^{-25} \text{ kg}\cdot\text{m/s}$$

so the uncertainty in the speed of the electron is

$$\Delta v_x = \frac{\Delta p_x}{m} = \frac{5.3 \times 10^{-25} \text{ kg}\cdot\text{m/s}}{9.11 \times 10^{-31} \text{ kg}} = 5.8 \times 10^5 \text{ m/s or } \sim 10^6 \text{ m/s}$$

Thus, if the speed is on the order of the uncertainty in the speed, then  $v \sim 10^6 \text{ m/s}$

**27.45** With  $\Delta x = 5.00 \times 10^{-7} \text{ m}$ , the minimum uncertainty in the speed is

$$\Delta v_x = \frac{\Delta p_x}{m_e} \geq \frac{h}{4\pi m_e (\Delta x)} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(9.11 \times 10^{-31} \text{ kg})(5.00 \times 10^{-7} \text{ m})} = 116 \text{ m/s}$$