

# Chapter 20

## Induced Voltages and Inductance

### Problem Solutions

20.1 The magnetic flux through the area enclosed by the loop is

$$\Phi_B = BA \cos \theta = B(\pi r^2) \cos 0^\circ = (0.30 \text{ T}) \left[ \pi (0.25 \text{ m})^2 \right] = \boxed{5.9 \times 10^{-2} \text{ T} \cdot \text{m}^2}$$

20.5 (a) Every field line that comes up through the area  $A$  on one side of the wire goes back down through area  $A$  on the other side of the wire. Thus, the net flux through the coil is zero

(b) The magnetic field is parallel to the plane of the coil, so  $\theta = 90.0^\circ$ . Therefore,  
 $\Phi_B = BA \cos \theta = BA \cos 90.0^\circ = \boxed{0}$

20.8 
$$|\mathcal{E}| = \frac{|\Delta \Phi_B|}{\Delta t} = \frac{(\Delta B) A \cos \theta}{\Delta t} = \frac{(1.5 \text{ T} - 0) \left[ \pi (1.6 \times 10^{-3} \text{ m})^2 \right] \cos 0^\circ}{120 \times 10^{-3} \text{ s}} = 1.0 \times 10^{-4} \text{ V} = \boxed{0.10 \text{ mV}}$$

20.11 The magnitude of the induced emf is 
$$|\mathcal{E}| = \frac{|\Delta \Phi_B|}{\Delta t} = \frac{|\Delta(B \cos \theta)| A}{\Delta t}$$

If the normal to the plane of the loop is considered to point in the original direction of the magnetic field, then  $\theta_i = 0^\circ$  and  $\theta_f = 180^\circ$ . Thus, we find

$$|\mathcal{E}| = \frac{|(0.20 \text{ T}) \cos 180^\circ - (0.30 \text{ T}) \cos 0^\circ| \pi (0.30 \text{ m})^2}{1.5 \text{ s}} = 9.4 \times 10^{-2} \text{ V} = \boxed{94 \text{ mV}}$$

20.13 The required induced emf is  $|\mathcal{E}| = IR = (0.10 \text{ A})(8.0 \Omega) = 0.80 \text{ V}$ .

$$\text{From } |\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \left(\frac{\Delta B}{\Delta t}\right)NA \cos\theta$$

$$\frac{\Delta B}{\Delta t} = \frac{|\mathcal{E}|}{NA \cos\theta} = \frac{0.80 \text{ V}}{(75)[(0.050 \text{ m})(0.080 \text{ m})] \cos\theta} = \boxed{2.7 \text{ T/s}}$$

20.16 The magnitude of the average emf is

$$\begin{aligned} |\mathcal{E}| &= \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{NBA[\Delta(\cos\theta)]}{\Delta t} \\ &= \frac{200(1.1 \text{ T})(100 \times 10^{-4} \text{ m}^2)(\cos 0^\circ - \cos 180^\circ)}{0.10 \text{ s}} = 44 \text{ V} \end{aligned}$$

Therefore, the average induced current is  $I = \frac{|\mathcal{E}|}{R} = \frac{44 \text{ V}}{5.0 \Omega} = \boxed{8.8 \text{ A}}$

20.18 From  $\mathcal{E} = B\ell v$ , the required speed is

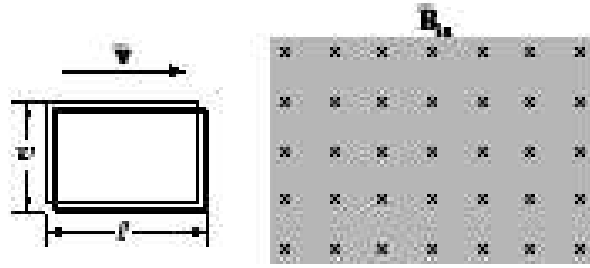
$$v = \frac{\mathcal{E}}{B\ell} = \frac{IR}{B\ell} = \frac{(0.500 \text{ A})(6.00 \Omega)}{(2.50 \text{ T})(1.20 \text{ m})} = \boxed{1.00 \text{ m/s}}$$

20.19  $\mathcal{E} = B_\perp \ell v$ , where  $B_\perp$  is the component of the magnetic field perpendicular to the velocity  $\mathbf{v}$ . Thus,

$$\mathcal{E} = [(50.0 \times 10^{-6} \text{ T}) \sin 58.0^\circ](60.0 \text{ m})(300 \text{ m/s}) = \boxed{0.763 \text{ V}}$$

- 20.23 (a) To oppose the motion of the magnet, the magnetic field generated by the induced current should be directed to the right along the axis of the coil. The current must then be **left to right** through the resistor.
- (b) The magnetic field produced by the current should be directed to the left along the axis of the coil, so the current must be **right to left** through the resistor.

- 20.25 (a) After the right end of the coil has entered the field, but the left end has not, the flux through the area enclosed by the coil is directed into the page and is increasing in magnitude. This increasing flux induces an emf of magnitude



$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{NB(\Delta A)}{\Delta t} = NBwv$$

in the loop. Note that in the above equation,  $\Delta A$  is the area enclosed by the coil that enters the field in time  $\Delta t$ . This emf produces a counterclockwise current in the loop to oppose the increasing inward flux. The magnitude of this current is  $I = \mathcal{E}/R = NBwv/R$ . The right end of the loop is now a conductor, of length  $Nw$ , carrying a current toward the top of the page through a field directed into the page. The field exerts a magnetic force of magnitude

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \text{ directed } \boxed{\text{toward the left}}$$

on this conductor, and hence, on the loop.

- (b) When the loop is entirely within the magnetic field, the flux through the area enclosed by the loop is constant. Hence, there is no induced emf or current in the loop, and the field exerts  $\boxed{\text{zero}}$  force on the loop.
- (c) After the right end of the loop emerges from the field, and before the left end emerges, the flux through the loop is directed into the page and decreasing. This decreasing flux induces an emf of magnitude  $|\mathcal{E}| = NBwv$  in the loop, which produces an induced current directed clockwise around the loop so as to oppose the decreasing flux. The current has magnitude  $I = \mathcal{E}/R = NBwv/R$ . This current flowing upward, through conductors of total length  $Nw$ , in the left end of the loop, experiences a magnetic force given by

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \text{ directed } \boxed{\text{toward the left}}$$

- 20.27 Since the magnetic force,  $F_m = qvB\sin\theta$ , on a positive charge is directed toward the top of the bar when the velocity is to the right, the right hand rule says that the magnetic field is directed  $\boxed{\text{into the page}}$ .

- 20.29** When the switch is closed, the current from the battery produces a magnetic field directed toward the left along the axis of both coils.
- As the current from the battery, and the leftward field it produces, increase in magnitude, the induced current in the leftmost coil opposes the increased leftward field by flowing **right to left** through  $R$  and producing a field directed toward the right along the axis.
  - As the variable resistance is decreased, the battery current and the leftward field generated by it increase in magnitude. To oppose this, the induced current is **right to left** through  $R$ , producing a field directed toward the right along the axis.
  - Moving the circuit containing  $R$  to the left decreases the leftward field (due to the battery current) along its axis. To oppose this decrease, the induced current is **left to right** through  $R$ , producing an additional field directed toward the left along the axis.
  - As the switch is opened, the battery current and the leftward field it produces decrease rapidly in magnitude. To oppose this decrease, the induced current is **left to right** through  $R$ , generating additional magnetic field directed toward the left along the axis.

- 20.31** Note the similarity between the situation in this problem and a generator. In a generator, one normally has a loop rotating in a constant magnetic field so the flux through the loop varies sinusoidally in time. In this problem, we have a stationary loop in an oscillating magnetic field, and the flux through the loop varies sinusoidally in time. In both cases, a sinusoidal emf  $\mathcal{E} = \mathcal{E}_{\max} \sin \omega t$  where  $\mathcal{E}_{\max} = NBA\omega$  is induced in the loop.

The loop in this case consists of a single band ( $N = 1$ ) around the perimeter of a red blood cell with diameter  $d = 8.0 \times 10^{-6} \text{ m}$ . The angular frequency of the oscillating flux through the area of this loop is  $\omega = 2\pi f = 2\pi(60 \text{ Hz}) = 120\pi \text{ rad/s}$ . The maximum induced emf is then

$$\mathcal{E}_{\max} = NBA\omega = B \left( \frac{\pi d^2}{4} \right) \omega = \frac{(1.0 \times 10^{-3} \text{ T}) \pi (8.0 \times 10^{-6} \text{ m})^2 (120\pi \text{ s}^{-1})}{4} = \boxed{1.9 \times 10^{-11} \text{ V}}$$

- 20.34** (a) Using  $\mathcal{E}_{\max} = NBA\omega$ ,

$$\mathcal{E}_{\max} = 1000(0.20 \text{ T})(0.10 \text{ m}^2) \left[ \left( 60 \frac{\text{rev}}{\text{s}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \right] = 7.5 \times 10^3 = \boxed{7.5 \text{ kV}}$$

- (b)  $\mathcal{E}_{\max}$  occurs when the flux through the loop is changing the most rapidly. This is when the plane of the loop is parallel to the magnetic field.

$$20.37 \quad |\mathcal{E}_{\text{av}}| = L \left| \frac{\Delta I}{\Delta t} \right| = (3.00 \times 10^{-3} \text{ H}) \left( \frac{1.50 \text{ A} - 0.20 \text{ A}}{0.20 \text{ s}} \right) = 2.0 \times 10^{-2} \text{ V} = \boxed{20 \text{ mV}}$$

$$20.39 \quad (a) \quad L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (400)^2 [\pi (2.5 \times 10^{-2} \text{ m})^2]}{0.20 \text{ m}}$$

$$= 2.0 \times 10^{-3} \text{ H} = \boxed{2.0 \text{ mH}}$$

$$(b) \quad \text{From } |\mathcal{E}| = L(\Delta I/\Delta t), \quad \frac{\Delta I}{\Delta t} = \frac{|\mathcal{E}|}{L} = \frac{75 \times 10^{-3} \text{ V}}{2.0 \times 10^{-3} \text{ H}} = \boxed{38 \text{ A/s}}$$