## PHYSICS 1B - Fall 2009



## Electricity \& <br> Magnetism



Professor Brian Keating
SERF Building. Room 333

# Chapter 16 Electrical Potential 

Electrical potential energy
Electrical potential

## Potential Energy of a system of charges and masses (the field is uniform, constant )


work done by Electric field

work done by Gravitational field

Change in PE =-work done by the field

## Potential Energy of a system of charges


$\Delta P E=-q E d$

Work done by the Electric field decreases the PE of the system

$$
W=F d
$$

$\Delta P E=-W=-F d=-q E d$

## Potential, V

$$
V=\frac{\Delta P E}{q}
$$

## Relation between E and V

$$
\begin{aligned}
& V=E d \\
& E=\frac{V}{d}
\end{aligned}
$$

## $E$ has units of $\mathrm{V} / \mathrm{m}$

## Potential, V

$$
\begin{gathered}
V=\frac{\Delta P E}{q} \\
\text { Units } \frac{\text { Joules }}{\text { Coulomb }}=\text { Volt }(\mathrm{V})
\end{gathered}
$$

Relation between E and V

$$
\begin{aligned}
& V=E d \\
& E=\frac{V}{d}
\end{aligned}
$$

$E$ has units of $\mathrm{V} / \mathrm{m}$

## Difference between Potential Energy and Potential

Potential-Depends only position in the field. Units (V)
Potential Energy- Depends on the interaction of the field with a charge. Units (J)
Related by

$$
\Delta \mathrm{PE}=\mathrm{q} \Delta \mathrm{~V}
$$

Both PE and V are relative.
Only differences/changes in Potential Energy and Voltage ( $\triangle \mathrm{PE}$ and $\triangle \mathrm{V}$ ) are important.

## Potential



The potential field is a property of the space due to charges

## Potential Energy



The potential energy is due to the charge interacting with the potential field.

A parallel plate capacitor has a constant electric field of $1000 \mathrm{~V} / \mathrm{m}$. The distance between the plates is 5 cm . Find the potential difference between the two plates.

| $\mathrm{d}=5 \mathrm{~cm}$ |
| :---: |
| $E=1000 \mathrm{j} / \mathrm{C}$ |

A parallel plate capacitor has a constant electric field of $1000 \mathrm{~V} / \mathrm{m}$. The distance between the plates is 5 cm . Find the potential difference between the two plates.


$$
\begin{aligned}
& \Delta V=\frac{\Delta P E}{q}=\frac{q E d}{q}=E d \\
& \Delta V=1000(0.05)=50 \mathrm{~V}
\end{aligned}
$$

## Potential Energy = Voltage? (T) True <br> (F) False

An molecular ion $\mathrm{CO}^{+}$is accelerated from rest across a potential difference of 1000 V . Find the final velocity of the ion. Mass $=4.7 \times 10^{-26} \mathrm{~kg}$


An molecular ion $\mathrm{CO}^{+}$is accelerated from rest across a potential difference of 1000 V . Find the final velocity of the ion. Mass $=4.7 \times 10^{-26} \mathrm{~kg}$


An molecular ion $\mathrm{CO}^{+}$is accelerated from rest across a potential difference of 1000 V . Find the final velocity of the ion. Mass $=4.7 \times 10^{-26} \mathrm{~kg}$

$$
+\xlongequal{+}\left|\begin{array}{c}
\Delta \mathrm{PE}=\mathrm{q} \Delta \mathrm{~V} \\
\text { Conservation of Energy } \\
\Delta \mathrm{PE}+\Delta \mathrm{KE}=0 \\
\Delta K E=-\Delta P E=-q \Delta V=\frac{1}{2} m v^{2} \\
\Delta \mathrm{~V}=-1000 \mathrm{~V}
\end{array}\right|
$$

An molecular ion $\mathrm{CO}^{+}$is accelerated from rest across a potential difference of 1000 V . Find the final velocity of the ion. Mass $=4.7 \times 10^{-26} \mathrm{~kg}$
$+\quad\left[\left.\begin{array}{c}\Delta \mathrm{PE}=\mathrm{q} \Delta \mathrm{V} \\ \text { Conservation of Energy } \\ \Delta \mathrm{PE}+\Delta \mathrm{KE}=0 \\ \Delta K E=-\Delta P E=-q \Delta V=\frac{1}{2} m v^{2} \\ \Delta \mathrm{~V}=-1000 \mathrm{~V}\end{array} \right\rvert\, \begin{array}{l}v=\sqrt{\frac{2 q(-\Delta V)}{m}}=\sqrt{\frac{2\left(1.6 \times 10^{-19}\right)(1000)}{4.7 \times 10^{-26}}} \\ v=8.25 \times 10^{4} \mathrm{~m} / \mathrm{s}\end{array}\right.$

## Potential due to a point charge

$E$ field is not constant

$$
E=\frac{k_{e} q}{r^{2}}
$$

E gets smaller with distance
The potential

$$
V=\frac{k_{e} q}{r}
$$

$$
\mathrm{V}=0 \text { at } \quad r=\infty
$$

## Dimensional arguments

$\mathrm{V}=$ Electric field x length e.g. for constant field

## $V=E d$

For point charge

$$
\begin{aligned}
& E=\frac{k_{e} q}{r^{2}} \\
& V=\frac{k_{e} q}{r}
\end{aligned}
$$

$V$ has the appropriate units of $E$ times length

## Potential and E field due to positive point charge




## Potential and E field due to positive point charge




## Potential and E field due to positive point charge




## E and V due to a negative point charge



## E and V due to a negative point charge



## $E$ and $V$ due to a negative point charge



## Potential energy of 2 point charges

$$
P E=q_{1} V_{21}=q_{2} V_{12}
$$

$\mathrm{V}_{21}$ is the potential due to charge2 at the position of charge1.

$$
\begin{gathered}
P E=\frac{k_{e} q_{1} q_{2}}{r_{12}} \\
P E=0 \quad \text { at } \mathrm{r}=\infty
\end{gathered}
$$

Potential energy and Potential are Scalar (not Vector) quantities

In a crystal of $\mathrm{Na}^{+} \mathrm{Cl}^{-}$the distance between the ions is 0.24 nm . Find the potential due to $\mathrm{Cl}^{-}$at the position of the $\mathrm{Na}^{+}$. Find the electrostatic energy of the $\mathrm{Na}^{+}$ due to the interaction with $\mathrm{Cl}^{-}$.
$r=0.24 \mathrm{~nm}$

at the position of $\mathrm{Na}+$

In a crystal of $\mathrm{Na}^{+} \mathrm{Cl}^{-}$the distance between the ions is 0.24 nm . Find the potential due to $\mathrm{Cl}^{-}$at the position of the $\mathrm{Na}^{+}$. Find the electrostatic energy of the $\mathrm{Na}^{+}$ due to the interaction with $\mathrm{Cl}^{-}$.
$r=0.24 \mathrm{~nm}$


$$
V=\frac{k_{e} q}{r}=\frac{9 \times 10^{9}\left(-1.6 \times 10^{-19}\right)}{\left(0.24 \times 10^{-9}\right)}=-6.0 \mathrm{~V} \text { at the position of } \mathrm{Na}+
$$

In a crystal of $\mathrm{Na}^{+} \mathrm{Cl}^{-}$the distance between the ions is 0.24 nm . Find the potential due to $\mathrm{Cl}^{-}$at the position of the $\mathrm{Na}^{+}$. Find the electrostatic energy of the $\mathrm{Na}^{+}$ due to the interaction with $\mathrm{Cl}^{-}$.
$r=0.24 \mathrm{~nm}$


$$
V=\frac{k_{e} q}{r}=\frac{9 \times 10^{9}\left(-1.6 \times 10^{-19}\right)}{\left(0.24 \times 10^{-9}\right)}=-6.0 \mathrm{~V} \text { at the position of } \mathrm{Na}+
$$

$P E=q V$

In a crystal of $\mathrm{Na}^{+} \mathrm{Cl}^{-}$the distance between the ions is 0.24 nm . Find the potential due to $\mathrm{Cl}^{-}$at the position of the $\mathrm{Na}^{+}$. Find the electrostatic energy of the $\mathrm{Na}^{+}$ due to the interaction with $\mathrm{Cl}^{-}$.
$r=0.24 \mathrm{~nm}$


$$
V=\frac{k_{e} q}{r}=\frac{9 \times 10^{9}\left(-1.6 \times 10^{-19}\right)}{\left(0.24 \times 10^{-9}\right)}=-6.0 \mathrm{~V} \text { at the position of } \mathrm{Na}+
$$

$$
P E=q V=1.6 \times 10^{-19} x-6.0=-9.6 \times 10^{-19} \mathrm{~J}
$$

In a crystal of $\mathrm{Na}^{+} \mathrm{Cl}^{-}$the distance between the ions is 0.24 nm . Find the potential due to $\mathrm{Cl}^{-}$at the position of the $\mathrm{Na}^{+}$. Find the electrostatic energy of the $\mathrm{Na}^{+}$ due to the interaction with $\mathrm{Cl}^{-}$.
$\mathrm{r}=0.24 \mathrm{~nm}$


$$
V=\frac{k_{e} q}{r}=\frac{9 \times 10^{9}\left(-1.6 \times 10^{-19}\right)}{\left(0.24 \times 10^{-9}\right)}=-\frac{-6.0 \mathrm{~V}}{\text { at the position of } \mathrm{Na}+}
$$

$P E=q V=1.6 \times 10^{-19} x-6.0=-9.6 \times 10^{-19} \mathrm{~J}$
ELECTRON VOLT (convenient unit for atomic physics)
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$P E=-6.0 \mathrm{eV}$
(energy in eV is V times the charge in electron units)

## PHYSICS 1B - Fall 2009



## Electricity \& <br> Magnetism



Professor Brian Keating
SERF Building. Room 333

Hydrogen Bond
$\mathrm{N}-\mathrm{H} \quad \mathrm{O}-\mathrm{C} \longrightarrow \mathrm{N}-\mathrm{H}-\mathrm{C}$
The hydrogen bond energy can be estimated by partial charges



DNA

Hydrogen Bond
$\mathrm{N}-\mathrm{H}$

$$
\mathrm{O}-\mathrm{C}
$$

$\qquad$

The hydrogen bond energy can be estimated by partial charges
$-0.3 e \quad+0.3 e \quad-0.4 e+0.4 e$

bond energy $=$ sum $\frac{\mathrm{kq}_{\mathrm{q}}}{\mathrm{r}_{\mathrm{ij}}}$ (scalar sum)


DNA

Hydrogen Bond
$\mathrm{N}-\mathrm{H}$
$\mathrm{O}-\mathrm{C}$ $\qquad$ $\mathrm{N}-\mathrm{H} \quad \mathrm{O}-\mathrm{C}$

The hydrogen bond energy can be estimated by partial charges

$$
\begin{array}{cc}
-0.3 e^{+0.3 e} & -0.4 \mathrm{e}+0.4 \mathrm{e} \\
\mathrm{~N}-\mathrm{H} & \mathrm{O}-\mathrm{C} \\
0.1 & 0.2 \\
0.25
\end{array} \mathrm{~nm}
$$

bond energy $=\operatorname{sum} \frac{\mathrm{kq} \mathrm{q}_{\mathrm{j}}}{\mathrm{r}_{\mathrm{ij}}}$ (scalar sum)


DNA
$\Delta \mathrm{PE}=\frac{\mathrm{ke}^{2}}{10^{-9}}\left(\frac{(-.3)(-.4)}{.1+.2}+\frac{-.3(.4)}{.1+.2+.25}+\frac{+.3(-.4)}{.2}+\frac{.3(.4)}{.2+.25}\right)$

Hydrogen Bond
$\mathrm{N}-\mathrm{H}$
$\mathrm{O}-\mathrm{C}$ $\qquad$ $\mathrm{N}-\mathrm{H} \quad \mathrm{O}-\mathrm{C}$

The hydrogen bond energy can be estimated by partial charges

$$
\begin{array}{cc}
-0.3 e^{+0.3 e} \\
\mathrm{~N}-\mathrm{H} & \begin{array}{l}
-0.4 \mathrm{e}+0.4 \mathrm{e} \\
0.1 \\
0.2 \\
0.25
\end{array} \mathrm{Cm}
\end{array}
$$

bond energy $=$ sum $\frac{\mathrm{kq} q_{\mathrm{j}}}{\mathrm{r}_{\mathrm{ij}}}$ (scalar sum)


DNA
$\Delta \mathrm{PE}=\frac{\mathrm{ke}^{2}}{10^{-9}}\left(\frac{(-.3)(-.4)}{.1+.2}+\frac{-.3(.4)}{.1+.2+.25}+\frac{+.3(-.4)}{.2}+\frac{.3(.4)}{.2+.25}\right)=-3.49 \times 10^{-20} \mathrm{~J}$

Hydrogen Bond
$\mathrm{N}-\mathrm{H}$
$\mathrm{O}-\mathrm{C}$ $\longrightarrow \mathrm{N}-\mathrm{H}^{-} \mathrm{C}$

The hydrogen bond energy can be estimated by partial charges

$$
\begin{array}{cc}
-0.3 e^{+0.3 e} & -0.4 \mathrm{e}+0.4 \mathrm{e} \\
\mathrm{~N}-\mathrm{H} & \mathrm{O}-\mathrm{C} \\
0.1 & 0.2 \\
0.25 & \mathrm{~nm}
\end{array}
$$

bond energy $=\operatorname{sum} \frac{\mathrm{kq} q_{\mathrm{j}}}{\mathrm{r}_{\mathrm{ij}}}$ (scalar sum)


DNA
$\Delta \mathrm{PE}=\frac{\mathrm{ke}^{2}}{10^{-9}}\left(\frac{(-.3)(-.4)}{.1+.2}+\frac{-.3(.4)}{.1+.2+.25}+\frac{+.3(-.4)}{.2}+\frac{.3(.4)}{.2+.25}\right)=-3.49 \times 10^{-20} \mathrm{~J}$ $\Delta P E=-0.22 \mathrm{eV}$

Hydrogen Bond
$\mathrm{N}-\mathrm{H}$
$\mathrm{O}-\mathrm{C}$ $\longrightarrow \mathrm{N}-\mathrm{H} \mathrm{O}^{-} \mathrm{C}$

The hydrogen bond energy can be estimated by partial charges

$$
\begin{array}{cc}
-0.3 e^{+0.3 e} \quad-0.4 \mathrm{e}+0.4 \mathrm{e} \\
\mathrm{~N}-\mathrm{H} & \mathrm{O}-\mathrm{C} \\
0.1 & 0.2 \quad 0.25 \\
\mathrm{Nm}
\end{array}
$$

bond energy $=$ sum $\frac{k q_{i} q_{j}}{r_{j}}$ (scalar sum)


DNA
$\Delta \mathrm{PE}=\frac{\mathrm{ke}^{2}}{10^{-9}}\left(\frac{(-.3)(-.4)}{.1+.2}+\frac{-.3(.4)}{.1+.2+.25}+\frac{+.3(-.4)}{.2}+\frac{.3(.4)}{.2+.25}\right)=-3.49 \times 10^{-20} \mathrm{~J}$
$\Delta P E=-0.22 \mathrm{eV} \quad$ Weaker than a ionic bond but still significant.

Two charges of $+q$ each are placed at corners of an equilateral triangle, with sides of 10 cm . If the Electric field due to each charge is $100 \mathrm{~V} / \mathrm{m}$ at the A find the potential at A


Two charges of $+q$ each are placed at corners of an equilateral triangle, with sides of 10 cm . If the Electric field due to each charge is $100 \mathrm{~V} / \mathrm{m}$ at the A find the potential at A

V at A due to each charge


$$
\begin{aligned}
& E=\frac{k_{e} q}{r^{2}} \\
& V=\frac{k_{e} q}{r}
\end{aligned}
$$

Two charges of $+q$ each are placed at corners of an equilateral triangle, with sides of 10 cm . If the Electric field due to each charge is $100 \mathrm{~V} / \mathrm{m}$ at the A find the potential at A

V at A due to each charge


$$
\begin{aligned}
& E=\frac{k_{e} q}{r^{2}} \\
& V=\frac{k_{e} q}{r} \\
& \frac{E}{V}=\frac{1}{r} \\
& V=E r=100(0.1)=10 V
\end{aligned}
$$

Two charges of $+q$ each are placed at corners of an equilateral triangle, with sides of 10 cm . If the Electric field due to each charge is $100 \mathrm{~V} / \mathrm{m}$ at the A find the potential at A

V at A due to each charge


$$
\begin{aligned}
& E=\frac{k_{e} q}{r^{2}} \\
& V=\frac{k_{e} q}{r} \\
& \frac{E}{V}=\frac{1}{r} \\
& V=E r=100(0.1)=10 \mathrm{~V}
\end{aligned}
$$

$\mathrm{V}_{\text {total }}=\mathrm{V}_{\mathrm{BA}}+\mathrm{V}_{\mathrm{CA}}=2 \mathrm{~V}=20 \mathrm{~V}$
Potential is a scalar

Two charges of $+q$ each are placed at corners of an equilateral triangle, with sides of 10 cm . The Electric field due to each charge is $100 \mathrm{~V} / \mathrm{m}$ at A.

What is the potential at A?

A. 10 V
B. 100 V
C. 1000 V

Two charges of $+q$ each are placed at corners of an equilateral triangle, with sides of 10 cm . The Electric field due to each charge is $100 \mathrm{~V} / \mathrm{m}$ at A.

What is the potential at A?

A. 10 V
B. 100 V C. 1000 V

$$
\begin{aligned}
& \frac{E}{V}=\frac{1}{r} \\
& V=E r=100(0.1)=10 V
\end{aligned}
$$

Two charges of $+q$ each are placed at corners of an equilateral triangle, with sides of 10 cm . The Electric field due to each charge is $100 \mathrm{~V} / \mathrm{m}$ at A.

What is the potential at A?

$$
\begin{array}{ll}
B & \text { C }
\end{array}
$$

A. 10 V
B. 100 V
C. 1000 V

$$
\begin{aligned}
& \frac{E}{V}=\frac{1}{r} \\
& V=E r=100(0.1)=10 V
\end{aligned}
$$

Potential is a scalar

3 charges of $1 \times 10^{-9} \mathrm{C}$ are placed at the corners of a equilateral triangle Each side of the triangle has a length of 1.0 cm . Find the work needed to bring the charges together from a long distance away.


PE due to Coulomb interaction
How many interactions?
$P E=$

3 charges of $1 \times 10^{-9} \mathrm{C}$ are placed at the corners of a equilateral triangle Each side of the triangle has a length of 1.0 cm . Find the work needed to bring the charges together from a long distance away.


PE due to Coulomb interaction
How many interactions?
$P E=$

3 charges of $1 \times 10^{-9} \mathrm{C}$ are placed at the corners of a equilateral triangle Each side of the triangle has a length of 1.0 cm . Find the work needed to bring the charges together from a long distance away.


PE due to Coulomb interaction
How many interactions?
$\mathrm{PE}=\quad \mathrm{PE}_{12}+\mathrm{PE}_{13}+\mathrm{PE}_{23}$

3 charges of $1 \times 10^{-9} \mathrm{C}$ are placed at the corners of a equilateral triangle Each side of the triangle has a length of 1.0 cm . Find the work needed to bring the charges together from a long distance away.


PE due to Coulomb interaction
How many interactions?

$$
P E=P E_{12}+P E_{13}+P E_{23}
$$

$$
P E=3 \frac{k_{e} q^{2}}{r}
$$

3 charges of $1 \times 10^{-9} \mathrm{C}$ are placed at the corners of a equilateral triangle Each side of the triangle has a length of 1.0 cm . Find the work needed to bring the charges together from a long distance away.


PE due to Coulomb interaction
How many interactions?
$\mathrm{PE}=\quad \mathrm{PE}_{12}+\mathrm{PE}_{13}+\mathrm{PE}_{23}$
$P E=3 \frac{k_{e} q^{2}}{r}$
$P E=3 \frac{9 \times 10^{9}\left(1 \times 10^{-9}\right)^{2}}{(0.01)^{2}}=2.7 \times 10^{-4} \mathrm{~J}$

The following charges are brought together from a large distance away. What is the change in PE? Is the charge distribution stable? (i.e. does it have a negative PE)


The following charges are brought together from a large distance away. What is the change in PE? Is the charge distribution stable? (i.e. does it have a negative PE)

How many interactions?


How many positive?
How many negative?
What is the total change in PE?

The following charges are brought together from a large distance away. What is the change in PE? Is the charge distribution stable? (i.e. does it have a negative PE)

How many interactions?


How many positive?
How many negative?
What is the total change in PE?

The following charges are brought together from a large distance away. What is the change in PE? Is the charge distribution stable? (i.e. does it have a negative PE)

How many interactions?


How many positive?
How many negative?
What is the total change in PE?

The following charges are brought together from a large distance away. What is the change in PE? Is the charge distribution stable? (i.e. does it have a negative PE)

How many interactions?


How many positive?
How many negative?
2
What is the total change in PE?

The following charges are brought together from a large distance away. What is the change in PE? Is the charge distribution stable? (i.e. does it have a negative PE)

How many interactions?


How many positive?
How many negative?
2
What is the total change in PE?

$$
P E=P E_{12}+P E_{13}+P E_{23}
$$

The following charges are brought together from a large distance away. What is the change in PE? Is the charge distribution stable? (i.e. does it have a negative PE)

How many interactions?
How many positive?
How many negative?
2
What is the total change in PE?

$$
\begin{aligned}
& P E=P E_{12}+P E_{13}+P E_{23} \\
& P E=P E_{0}-2 P E_{0}=-P E_{0}=-\frac{k_{e} q^{2}}{a}
\end{aligned}
$$

The following charges are brought together from a large distance away. What is the change in PE? Is the charge distribution stable? (i.e. does it have a negative PE)

How many interactions?


How many positive?
How many negative?
2
What is the total change in PE?

$$
P E=P E_{12}+P E_{13}+P E_{23}
$$

$$
P E=P E_{0}-2 P E_{0}=-P E_{0}=-\frac{k_{e} q^{2}}{a}
$$

STABLE

## Which of the charge distributions is the most stable?

 (has the lowest PE)

Which of the charge distributions is the most stable? (has the lowest PE)


Which of the charge distributions is the most stable? (has the lowest PE)


Which of the charge distributions is the most stable? (has the lowest PE)


Which of the charge distributions is the most stable? (has the lowest PE)


Which of the charge distributions is the most stable? (has the lowest PE)
$P E_{0}=\frac{k_{e} q^{2}}{a}$


$$
\left.\begin{array}{cc}
\mathrm{PE}_{0} & +2
\end{array}\right)-2
$$

Which of the charge distributions is the most stable? (has the lowest PE)


Which of the charge distributions is the most stable? (has the lowest PE)
$P E_{0}=\frac{k_{e} q^{2}}{a}$


$$
\left.\begin{array}{cc}
\mathrm{PE}_{0} & +2
\end{array}\right)-2
$$

Which of the charge distributions is the most stable? (has the lowest PE)


Which of the charge distributions is the most stable? (has the lowest PE)


