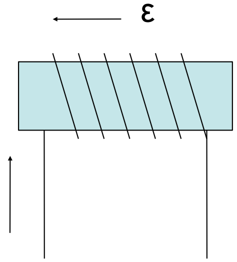
Inductance L is a measure of the self-induced emf



The self-induced emf is

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

but $$\frac{\Delta\Phi_B}{\Delta t} \propto \frac{\Delta I}{\Delta t}$$

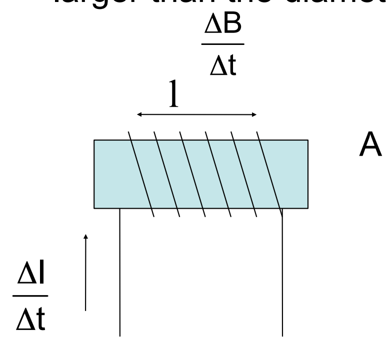
proportionality constant is L

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

L is a property of the coil, Units of L , Henry (H) $\frac{Vs}{A}$

Inductance of a solenoid with N turns and length ℓ ,

wound around an air core (assume the length is much larger than the diameter).



$$\Phi_B = BA = \mu_o \frac{N}{\ell} IA$$

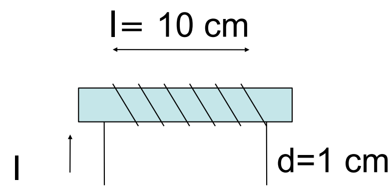
$$\frac{\Delta\Phi_B}{\Delta t} = \mu_o \frac{N}{\ell}\frac{\Delta I}{\Delta t} A$$

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = -\mu_o \frac{N^2}{\ell} A \frac{\Delta I}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

$$L = \mu_o \frac{N^2}{\ell} A$$

inductance proportional to N squared x area/length

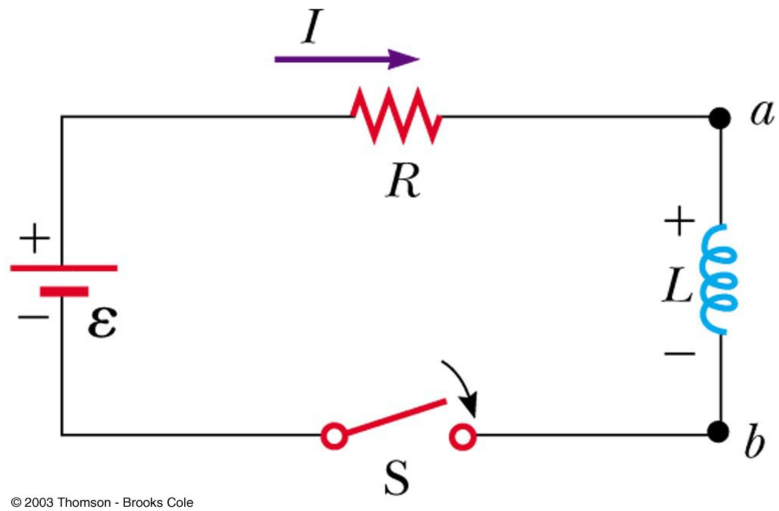
An air wound solenoid of 100 turns has a length of 10 cm and a diameter of 1 cm. Find the inductance of the coil.



$$L = \mu_o \frac{N^2}{\ell} A = \mu_o \frac{N^2}{\ell} \pi \frac{d^2}{4}$$

$$L = \frac{4\pi 10^{-7} (100)^2 \pi (0.01)^2}{0.1(4)} = 1.0x10^{-5} H$$

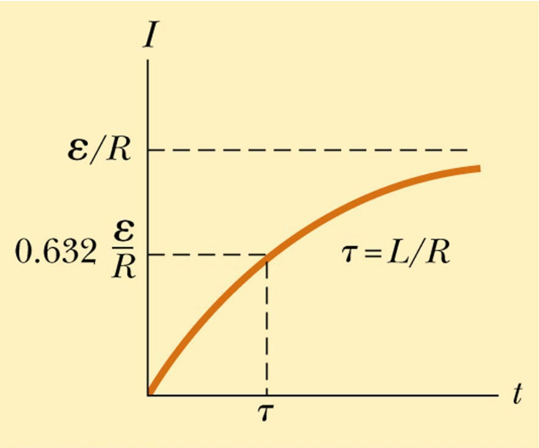
RL circuit



The inductor prevents the rapid buildup of current

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

But at long time does not reduce the current, $\frac{\Delta I}{\Delta t} = 0$ at t=∞



$$I = I_o(1 - e^{-\frac{t}{\tau}})$$

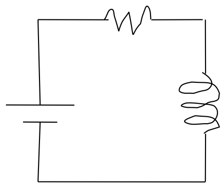
$$\tau = \frac{L}{R}$$

Inductive reactance, X_L

$\Delta V_L = X_L I$

$$I = \frac{\Delta V_L}{X_L}$$

$$X_L = 2\pi fL$$



Dimensional analysis

$$\tau = \frac{L}{R}$$

$$\frac{1}{\tau} = \omega = 2\pi f = \frac{R}{L}$$

$$R = 2\pi fL$$

$$X_L = 2\pi fL$$

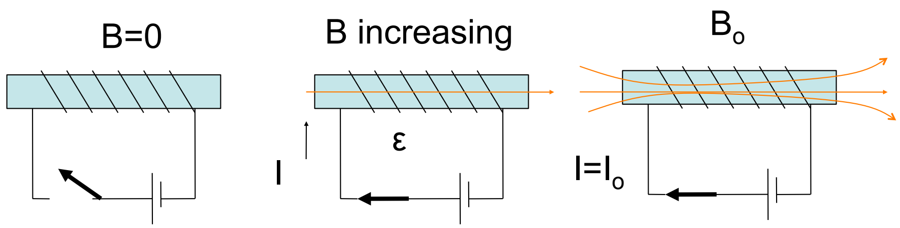
So X_L has units of ohms

Applications of Inductors

Reduce rapid changes of current in circuits

Produce high voltages in automobile ignition.

Energy is stored in a magnetic field of an inductor.



Work is done against ε to produce the B field.

This produces a change in the PE of the inductor

$$PE_L = \frac{1}{2}LI^2$$

This stored PE can be used to do work

21.1 RLC circuit

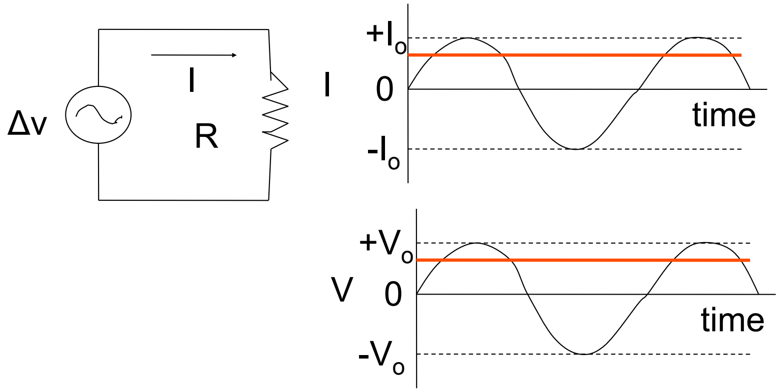
AC circuits
RLC circuit
Resonance

AC Circuits

- Current changes with time
- Current is both positive and negative
- Voltage (V) and Current (I) are not always "in phase" – when the voltage is a max the current may not be a max
- ***<u>Only</u>*** for a resistor are V & I always "in phase" (voltage max occurs when current is max).

AC circuits-Resistor

Household currents are alternating currents AC that vary with time. For circuits only involving resistors the only difference is that the average currents and voltages must be used, I_{rms}, V_{rms}. (rms –root mean square)



$$I_{RMS} = \frac{1}{\sqrt{2}} I_o$$

$$V_{RMS} = \frac{1}{\sqrt{2}} V_o$$

AC circuit with capacitors, inductors and resistor.

Resistors, capacitors and inductors react differently to time dependent voltages.

These components behave differently in an AC circuit.

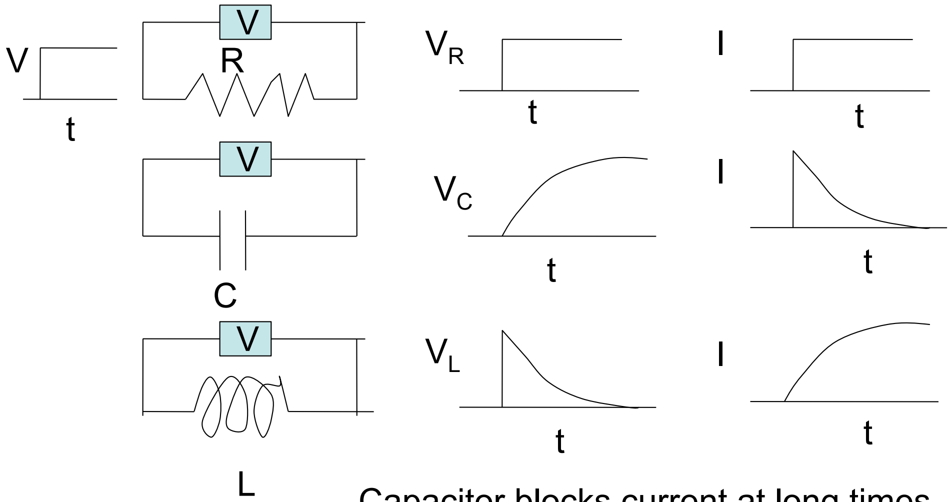
We have seen this already for the capacitor,
so we already know about R & C, just need to examine L

If the average voltages, currents and power are used then the relations for the between current, voltage and power are the same as for DC

$$V_{rms} = I_{rms} R$$

$$P_{rms} = I_{rms} V_{rms} = \frac{V_{rms}^{2}}{R} = I_{rms}^{2} R$$

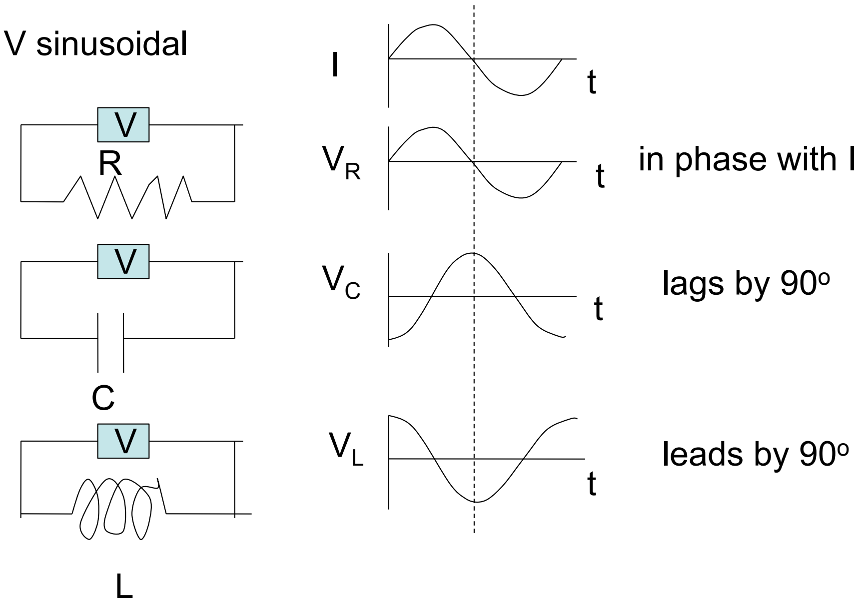
Response to a step voltage: Resistor, Capacitor, and Inductor



Capacitor blocks current at long times
Inductor blocks current at short times

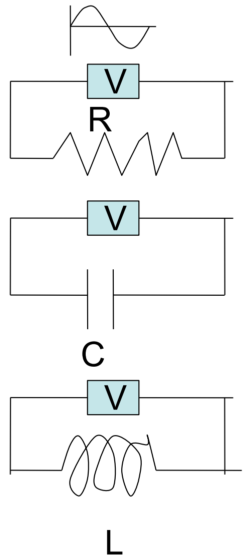
Response to a sinusoidal voltage

V sinusoidal



different phase shift between current and voltage

Response to a sinusoidal voltage



$$V_{rms} = I_{rms} R$$

$$V_{rms} = I_{rms}\left(\frac{1}{\omega C}\right) = I_{rms} X_C$$

$$V_{rms} = I_{rms}(\omega L) = I_{rms} X_L$$

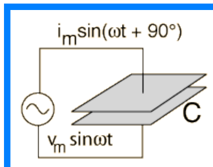
$X_C = \frac{1}{\omega C}$ Capacitive Reactance

$X_L = \omega L$ Inductive Reactance

Capacitive Reactance, X_c

$$\Delta V_C = X_C\, I$$

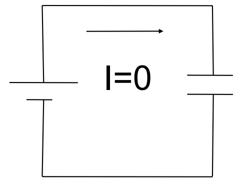
$$X_c = \frac{1}{\omega C}$$



$$X_C = \frac{1}{\omega C}$$

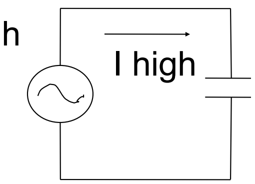
X_C is higher at low frequency. The capacitor block current at long time. because more charge accumulates.

f=0
DC



X_C= infinity

f=high



X_C = low

Capacitive Reactance

Dimensional analysis

$$\tau = RC$$

$$\frac{1}{\tau} = \omega = 2\pi f = \frac{1}{RC}$$

$$R = \frac{1}{2\pi fC}$$

$$X_c = \frac{1}{2\pi fC}$$

So X_c has units of Ohms

A 10 microfarad capacitor is in an ac circuit with a voltage source with RMS voltage of 10 V. a) Find the current for a frequency of 100 Hz. b) Find the current for a frequency of 1000 Hz.

a)

$$\Delta V_C = X_C I$$

$$I = \frac{\Delta V_C}{X_C} = \frac{\Delta V_C(2\pi fC)}{1}$$

$$I = 10(2\pi)(100)(10^{-5}) = 6.3x10^{-2}A$$

b) The frequency is 10 x higher, the current is 10 x higher

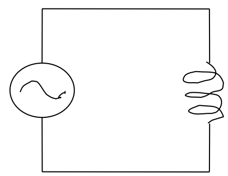
$I=10x6.3x10^{-2}=6.3x10^{-1}A$

Inductive reactance, X_L

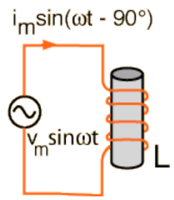
$\Delta V_L = X_L I$

$$I = \frac{\Delta V_L}{X_L}$$

$$X_L = \omega L$$



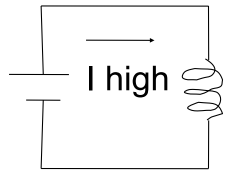
An inductor has higher back emf when ΔI/Δt is greater, i.e. at high frequency. Inductive reactance higher at high frequency.



$$X_L = \omega L$$

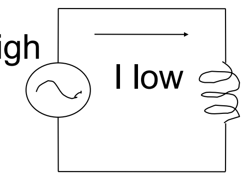
$$X_L = 2\pi f L \qquad I = \frac{\Delta V_L}{X_L}$$

f=0



$X_L = 0$

f high



$X_L =$ high

Inductive reactance is higher at high frequency

A inductor with L= 10^{-5} H is driven by a 10 V ac source.
a) Find the current at f=100 Hz.
b) Find the current at f=1000Hz

a)

$$I_{RMS} = \frac{\Delta V_{L,RMS}}{X_L} = \frac{\Delta V_{L,RMS}}{2\pi fL}$$

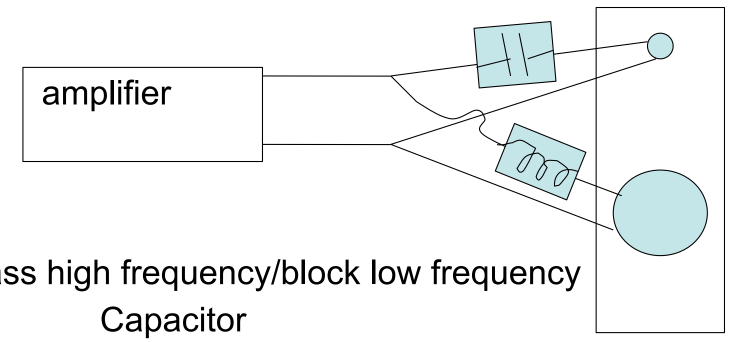
$$I = \frac{10}{2\pi(100)(10^{-5})} = 1.6x10^3 A$$

b)

the frequency is 10x greater
the current is inversely proportional to f
the current is 10x less

$I=1.6x10^3/10=1.6x10^2A$

Application.
High pass and low pass filters.



pass high frequency/block low frequency

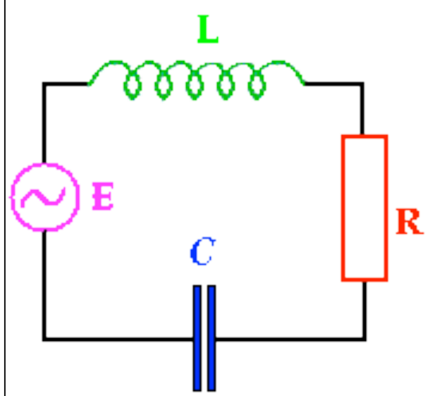
Capacitor

pass low frequency/block high frequency

Inductor

stereo speakers

RLC circuit



Currents and voltages are sinusoidal

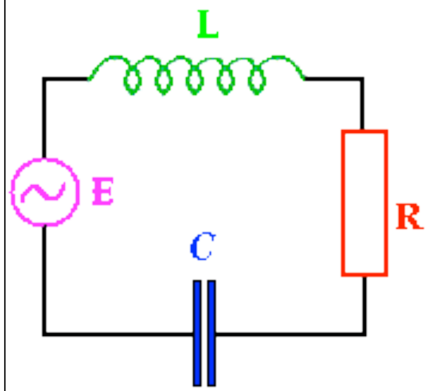
Charge and discharge of capacitor

Energy only dissipated in R

At resonance frequency maximum energy stored in electric and magnetic fields.

This circuit can be used to pick out selected frequencies. e.g. in a radio receiver.

Voltages



Voltage across R ,L , C are sinusoidal

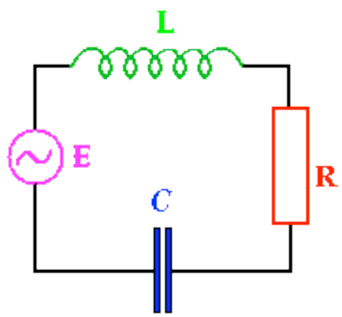
But with different phase relative to the current, and relative to each other

The sum of voltage

$$\Delta V_s = \Delta V_R + \Delta V_L + \Delta V_C$$

But at any time the voltages are not maximum across R, L and C but differ because of phase shifts.

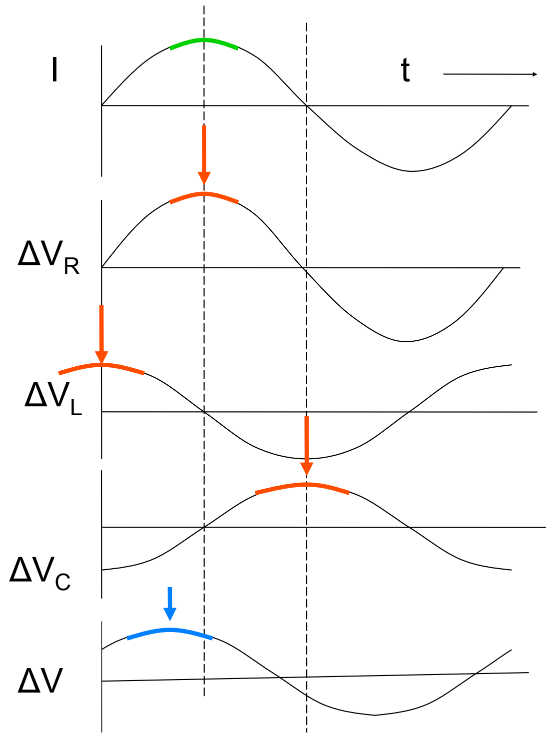
Sum of Voltages



$I = I_{max} \sin \omega t$

$\Delta V = \Delta V_R + \Delta V_L + \Delta V_C$

$\Delta V = \Delta V_{max} \sin(\omega t + \Phi_i)$



Impedance, Z

$$\Delta V = \sqrt{(\Delta V_L - \Delta V_C)^2 + \Delta V_R{}^2}$$

$$\Delta V = \sqrt{(IX_L - IX_C)^2 + I^2R^2}$$

$$\Delta V = I\sqrt{(X_L - X_C)^2 + R^2}$$

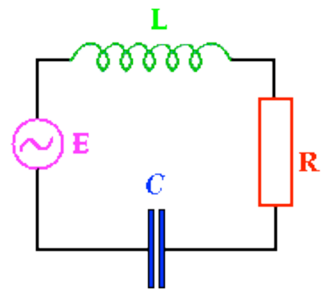
$$\Delta V = IZ$$

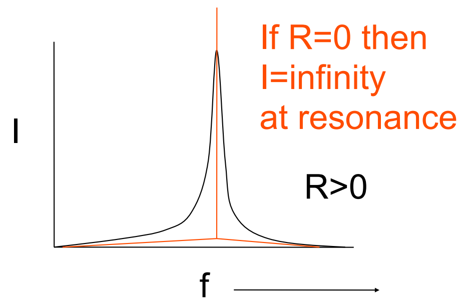
Like Ohm's Law

$$\boxed{Z = \sqrt{(X_L - X_C)^2 + R^2}}$$

L, C and R contribute to Z, Impedance.

Resonance





$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

When $X_L = X_C$
then $X_L - X_C = 0$
Z becomes a minimum
I becomes maximum

resonance frequency

$$X_C = X_L$$

$$\frac{1}{2\pi f_o C} = 2\pi f_o L$$

$$f_o = \frac{1}{2\pi \sqrt{LC}}$$

f

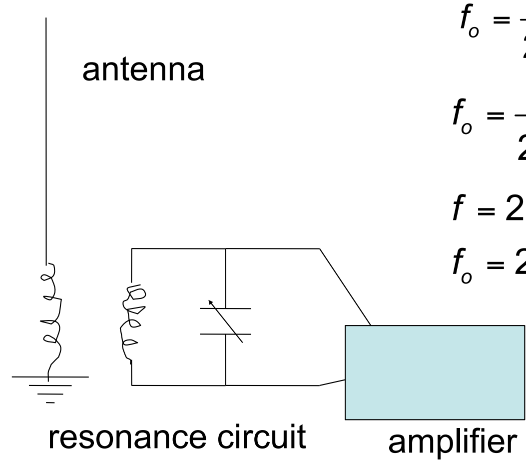
34. A resonance circuit in a radio receiver is tuned to a certain station when the inductor has a value of 0.20 mH and the capacitor has a value of 30 pF.   Find the frequency of the station.

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{0.2x10^{-3}(30x10^{-12})}}$$

$$f = 2.05x10^6 Hz$$

$$f_o = 2.05MHz$$



For LC circuit (R->0) at resonance
Energy oscillates between Electric and Magnetic Fields