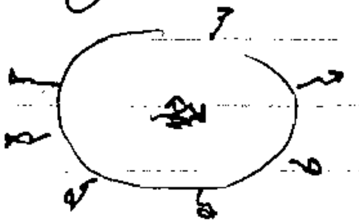
Recap

(1) Last time we discussed fusion reactions in solar core that released energy in core that balances luminosity losses from surface of the sun:



How energy ~~flow~~ in core gets transported to the surface is a topic that will be discussed.

(2) Two chains:

(A) pp chain: in which 4 protons are consumed and one $_2He^4$ nucleus produced along with $Q_{heat} = 26.2$ MeV. Heat in form of photons and kinetic energies of particles. These get deposited into ambient gas.

- Actually 3 chains (ppI, ppII, ppIII)
- Neutrino production in all 3
- Neutrinos escape with no heating

(B) CNO Cycle: unlike pp chains, heavier nuclei involved. $_6C^{12}$ acts like a a catalyst. In this case $Q_{heat} = 25.03$ MeV

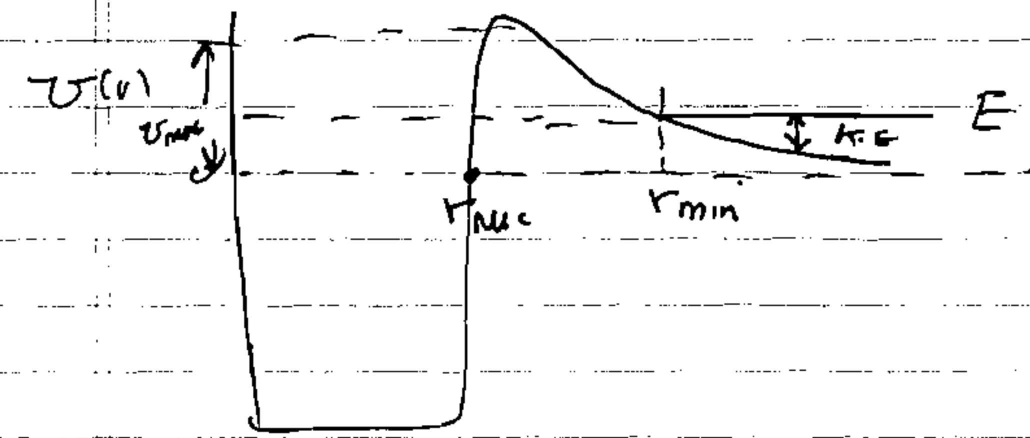
- actually 2 chains
- $_6C^{12}$ get slowly drained into $_7N^{14}$

(C) Reaction Rates:

2 nuclei with reduced mass $\mu_m = \frac{m_1 m_2}{m_1 + m_2}$

$$E = \frac{1}{2}\mu_m v^2 + V(r)$$

Potential Energy Diagram : Barrier Problem



- at $r > r_{nuc}$; Coulomb repulsive potential energy dominates
- At $r < r_{nuc}$; strong interaction mediated by pion exchange yields attractive potential.

Simple case of head-on Collision:

$m_1 \longrightarrow \qquad \longleftarrow m_2$

Classical Description (1) As r decreases $KE = \frac{1}{2}\mu_m V^2$ decreases. At $r = r_{min}$, particle comes to rest, reverses direction and moves to $r > r_{min}$

(2) at $r = r_{min}$, $V = 0 \Rightarrow E = V(r_{min})$

$$E = \frac{Z_1 Z_2 e^2}{r_{min}} = \frac{1}{2}\mu V_\infty^2$$

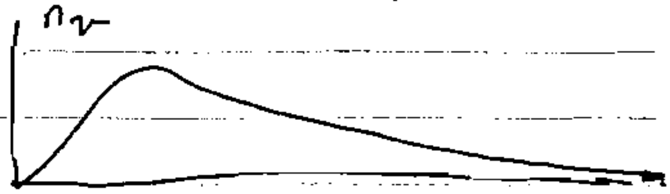
(3) But at $r = r_{nuc} \simeq 10^{-13}\text{cm}$, where strong interaction predominates

$$V_{max}(r_{nuc}) = \frac{Z_1 Z_2 e^2}{r_{nuc}} \simeq \frac{1.4\, Z_1 Z_2}{r_{nuc}/10^{-13}\text{cm}}\ \text{MeV}$$

(4) Dilemma: $= V_{max} >> \frac{1}{2}\mu m V^2 \simeq \frac{3}{2}kT = 1k_e$

Fraction of particles able to leap over V_{max} is small.

For Maxwellian fraction $\simeq \exp(-\frac{V_{max}}{kT})$

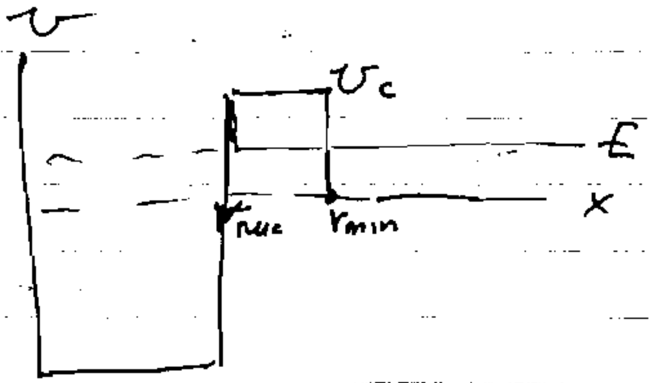


$\simeq \exp(-10^3) \simeq 10^{-434}$ (forget it!)

(5) Quantum Mechanical solution

Distance of closest approach as a classical concept that need not be adhered to in quantum mechanics. Solutions to Schrödinger equation show that wavelike nature of matter admits possibility that particle will penetrate potential Barrier.

Simple analogy:



1-D schrödinger equation

$$-\frac{\hbar^2}{2\mu m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

(a) $r > r_{min}$: $V = 0$, one gets

$$\frac{\hbar^2}{2\mu m}\frac{d^2\psi}{dx^2} + E\psi = 0$$

oscillatory solutions $\quad \frac{d^2\psi}{dx^2} + \left(\frac{2\mu m}{\hbar^2}E\right)\psi = 0$

ingoing and outgoing waves $\Big\}$ $\psi = Ae^{ikx} + Be^{-ikx}$

$$k^2 = \frac{2\mu m}{\hbar^2}E$$

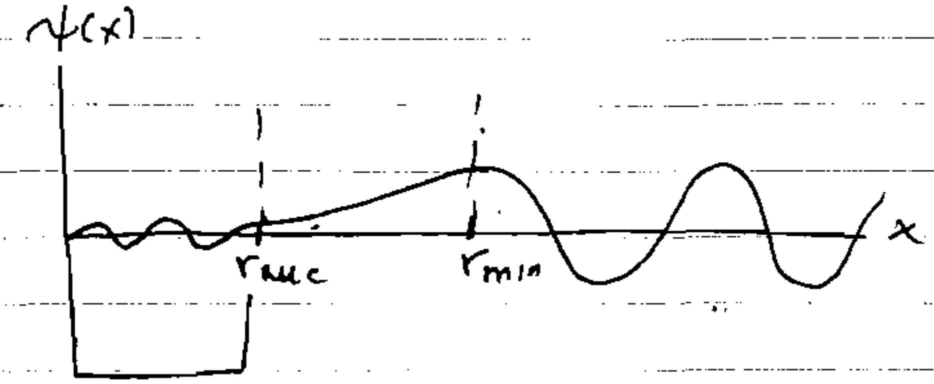
(b) $r_{nuc} < r < r_{min}$

In this range of x $V > E$

$$\frac{\hbar^2}{2\mu m}\frac{d^2\psi}{dx^2} = (V_c - E)\psi \Rightarrow \frac{d^2\psi}{dx^2} - \frac{2\mu m}{\hbar^2}(V_c - E)\psi = 0$$

Here we get $\psi = Ce^{+Kx}$; $K^2 = \frac{2\mu m}{\hbar^2}(V_c - E)$
exponential decay with decreasing x.

Schematically:



where ~~standing~~ oscillating wave solution holds at $r < r_{nuc}$ since $E > V$ here.

Gamow solved this problem and showed that probability for transmission through the Coulomb barrier is given by

$$P = \exp[-2\pi^2 r_{min}/\lambda]$$

{ λ is de Broglie wavelength of reduced mass particle

• Evaluate r_{min}: Recall $E = \frac{1}{2}\mu m V^2 = \frac{Z_1 Z_2 e^2}{r_{min}}$

Therefore $r_{min} = \frac{2 Z_1 Z_2 e^2}{\mu m V^2}$

• Evaluate λ: de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\mu m V}$$

Therefore: $\frac{r_{min}}{\lambda} = r_{min}\cdot\left(\frac{\mu_m V}{\hbar}\right)$

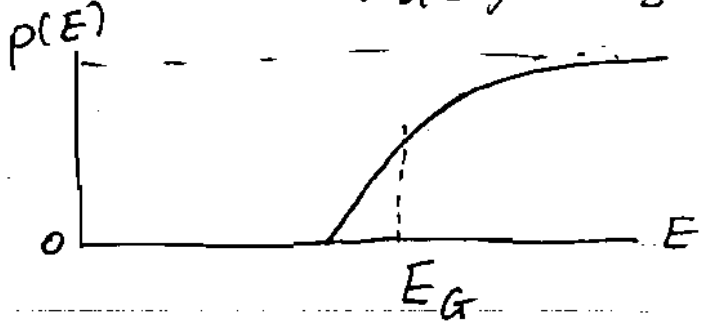
or $\frac{r_{min}}{\lambda} = \frac{2\cdot Z_1 Z_2 e^2}{\mu_m V^2}\times\frac{\mu_m V}{\hbar} = \frac{2 Z_1 Z_2 e^2}{\hbar V}$

Therefore $\boxed{\cancel{P \propto \exp\left[-\frac{Z_1 Z_2 e^2}{\hbar E^{1/2}}\right]}}$

Clear that $P \propto \exp\left[-\frac{Z_1 Z_2}{E^{1/2}}\mu_m^{1/2}\right]$

In fact: $P(E) = \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right]$

where $E_G = \left[\pi\left(\frac{e^2}{\hbar c}\right)Z_1 Z_2\right]^2\cdot 2\mu_m c^2$



Conclude P(E) rises steeply with E, which compensates somewhat for exponential decline in density of particles with increasing energy.

$$n_E(E)\,dE = n_v(v)\,dv \propto v^2 e^{-\frac{\frac{1}{2}mv^2}{kT}}\,dv$$

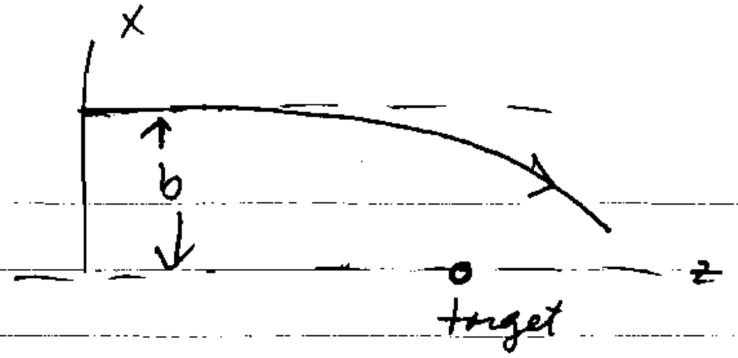
$$n_E(E) = \frac{n_v(v(E))}{\left|\frac{dE}{dv}\right|} \propto \frac{E\cdot e^{-E/kT}}{E^{1/2}}$$

$E = \frac{1}{2}\mu_m v^2$

$\frac{dE}{dv} = \mu_m v = \mu_m\sqrt{\frac{2E}{\mu_m}}$ $\boxed{n_E(E) \propto E^{1/2} e^{-E/kT}}$

Reaction Rates

Cross-section

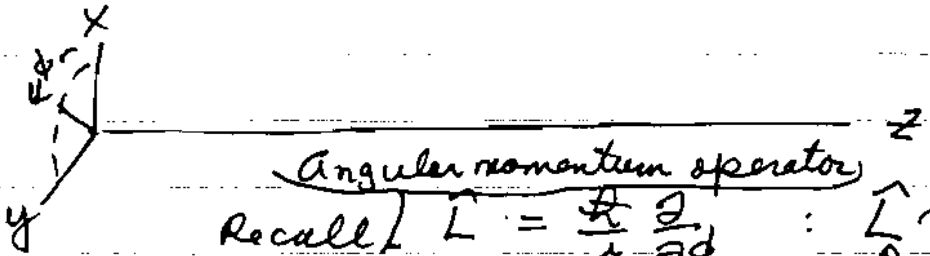


- Let b be the impact parameter, i.e., initial distance along x axis from target. Angular momentum: $L = b \cdot p$; p = momentum.
- In quantum mechanics, $p = \frac{h}{\lambda}$

therefore $\boxed{L = \frac{b \cdot h}{\lambda}}$

Quantization of L

Due to azimuthal symmetry of wave function: $\psi(\phi + 2\pi) = \psi(\phi)$



Angular momentum operator

Recall $\hat{L} = \frac{\hbar}{i}\frac{\partial}{\partial \phi}$: $\hat{L}\psi = L\psi$ (operator, eigenvalue)

Solutions: $\psi \propto e^{i\ell\phi}$

$\therefore \frac{\hbar}{i}\frac{\partial}{\partial \phi}(e^{i\ell\phi}) = L \cdot e^{i\ell\phi}$

$\Rightarrow$ $(\ell\hbar = L)$

But $e^{i\ell\phi} = e^{i\ell(\phi+2\pi)} \Rightarrow e^{i2\ell\pi} = 1 = \cos(2\ell\pi) + i\sin(2\ell\pi)$

$\Rightarrow \ell = 1, 2, \ldots, 3$

$\therefore$ Angular momentum eigenvalue $L = \ell\hbar$ where ℓ is an integer $= 1, 2, \ldots$

So impact parameter b_ℓ is quantized

$\ell \cdot \hbar = \frac{b_\ell h}{\lambda} \Rightarrow \boxed{b_\ell = \frac{\ell\lambda}{2\pi}}$

since $\hbar = \frac{h}{2\pi}$

Cross-section $\sigma_\ell = \pi b_\ell^2 = \frac{\pi \ell^2 \lambda^2}{(2\pi)^2} = \frac{\pi \ell^2}{(2\pi)} \cdot \frac{\hbar^2}{p^2} \propto \frac{1}{E}$

Combining Tunneling probability with angular momentum quantization we have:

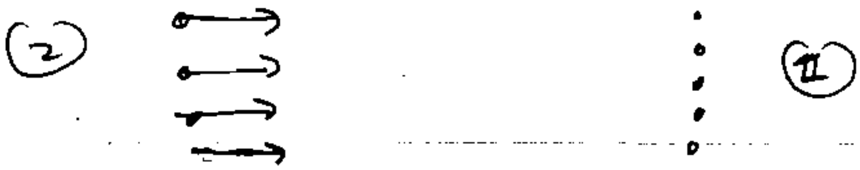
$$\underbrace{S(E)}_{S(E) \propto \text{ constant}} \qquad \sigma(E) = \frac{S(E)}{E} \cdot \exp\left[-\frac{b}{E^{1/2}}\right]$$

not impact parameter

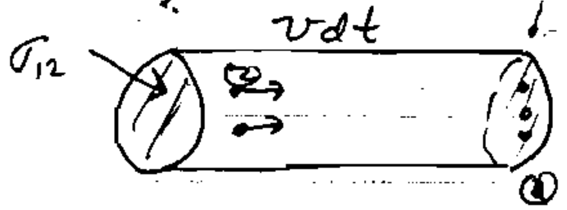
Note: $b \equiv 31.3\, Z_1 Z_2 A^{1/2}\, (\text{keV})^{1/2}$; $A = \frac{A_1 A_2}{A_1 + A_2}$

(From $p(E) \propto \exp\left[-\frac{Z_1 Z_2 A^{1/2}}{E^{1/2}}\right]$)

Reaction Rates: Consider flux of particles (2) incident on particles (1)



Again consider imaginary cross-sectional area σ_{12} and travel length $v \cdot dt$. Then if dN_2 = no. of (2) particles that reach particles (1) in time interval dt, then



$$dN_2 = \underbrace{dn_2(v)}_{\text{density}} \cdot \underbrace{\sigma_{12} \cdot v\, dt}_{\text{volume}}$$

(2) particles in cylinder reach (1)

since σ_{12} is "effective" area per particle ①,

Reaction rate per ① particle: $\frac{dN_2}{dt} = dn_2(v)\sigma_{12}\cdot v$

$$\frac{dN_2}{dt} = \sigma_{12} \cdot \underbrace{v\, dn_2(v)}_{\text{particle flux}} = \text{reaction rate per particle ①}$$

Reaction rate per volume: $= \frac{dN_2}{dt} \times n_1$

$$dr_{12} = \sigma_{12} \cdot v\, dn_2 \times n_1 \quad \left\{ \frac{\text{no. of product particles}}{\text{volume} \cdot \text{time}} \right\}$$

Assuming $dn_2 = n_2 \underbrace{\phi(v)}_{\text{velocity or speed distribution}} dv$

$$dr_{12} = \sigma_{12} \cdot v \cdot n_2 \phi(v)\, dv \cdot n_1$$

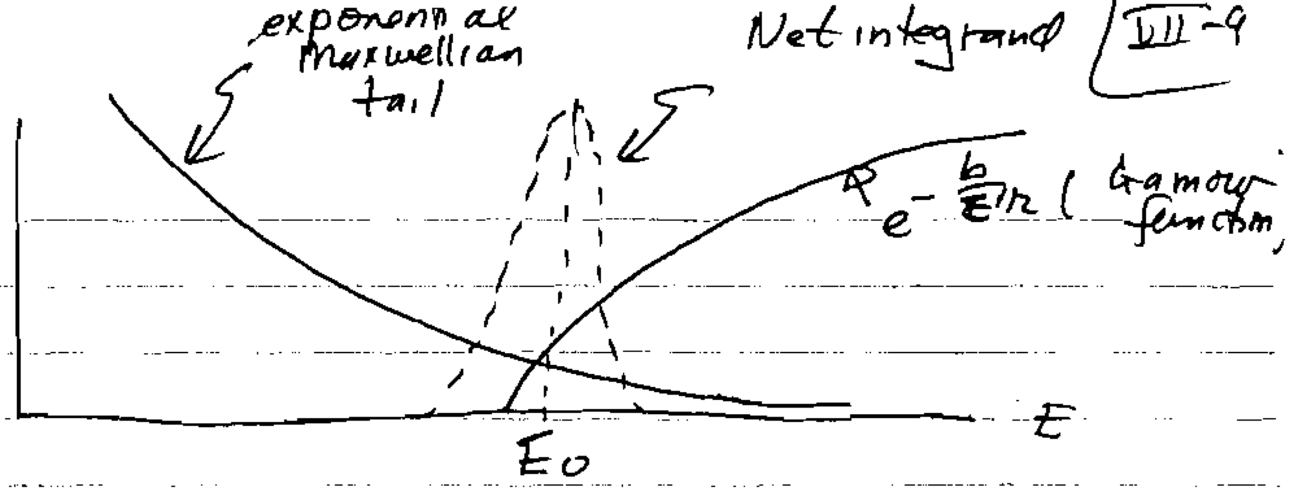
Integrate over v: $r_{12} = n_1 n_2 \int_0^\infty \sigma_{12}\, \phi(v) v\, dv$

Transform to E distribution $\phi(v)dv = n_E(E)dE$

$$\phi(v)dv \propto E^{1/2} e^{-E/kT}$$

$$\Rightarrow r_{12} \propto n_1 n_2 \int \underbrace{\frac{S(E)\exp\left(-\frac{b}{E^{1/2}}\right)}{E}}_{\sigma_{12}} \cdot \underbrace{E^{1/2} e^{-E/kT}}_{n_E(E)} \underbrace{E^{1/2}}_{v(E)} dE$$

$$r_{12} \propto n_1 n_2 \int S(E) \cdot \exp\left[-\frac{b}{E^{1/2}} - \frac{E}{kT}\right] dE$$



Most of reaction rate caused by narrow distribution of particles concentrated around E_0; maximum of integrand occurs at

$$E_0 = \left(\frac{b \cdot kT}{2} \right)^{2/3}$$

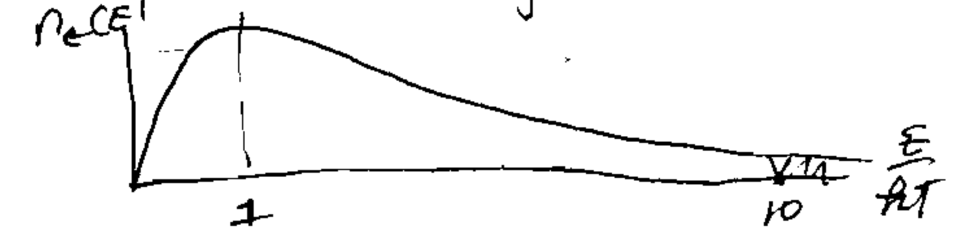
Plugging in $b = 31.3 Z_1 Z_2 A^{1/2} (keV)^{1/2}$ we have

$$E_0 = 1.22 \left(Z_1^2 Z_2^2 A T_6^2 \right)^{1/3} keV \; ; \; T_6 = T/10^6 K$$

average $\langle E \rangle = \frac{3}{2} kT = 0.09\, T_6$ keV

	pp	CNO
E_0 (keV)	10	30
$\langle E \rangle$ (keV)	1	3

Therefore: $E_0 \simeq 10 \langle E \rangle$; Effective energy at which nuclear reactions proceed is ~~10 k~~T. Thus a small fraction of protons in the core of the sun dominate fusion reaction rate.



Recap

Last time, I derived an expression for the reaction rate per unit volume for fusion between m_1 and m_2 particles.

$$r_{12} = n_1 n_2 \int_0^{\infty} \sigma_{12}\, \phi(v)\, v\, dv$$

(a) $$\sigma_{12} = \frac{S(E)\exp\left(-\frac{b}{E^{1/2}}\right)}{E}$$

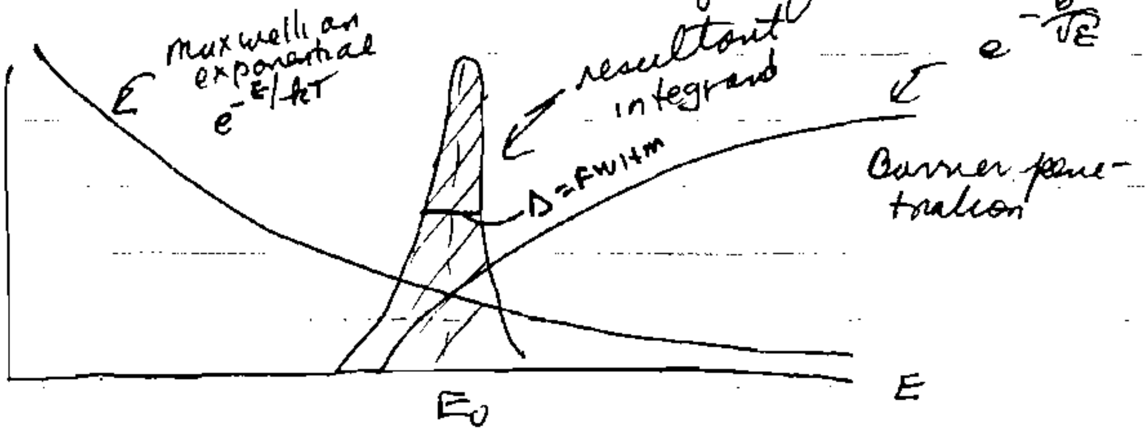
(b) $$\phi(v)\,dv = \Phi(E)\,dE = \frac{2}{\sqrt{\pi}} \cdot \frac{E}{kT} \exp\left(-\frac{E}{kT}\right) \frac{dE}{(kTE)^{1/2}}$$

(c) Let $r_{12} = n_1 n_2 \langle \sigma v \rangle_{12}$ $\quad [\sigma v] = \mathrm{cm}^3/s$

Put it all together:

$$\langle \sigma v \rangle_{12} = \left(\frac{8}{\mu\pi}\right)\left(\frac{S_0}{(kT)^{3/2}}\right)\int_0^{\infty} \exp\left[-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right] dE$$

where I let $S(E) \simeq S_0$ (average of $S(E)$)

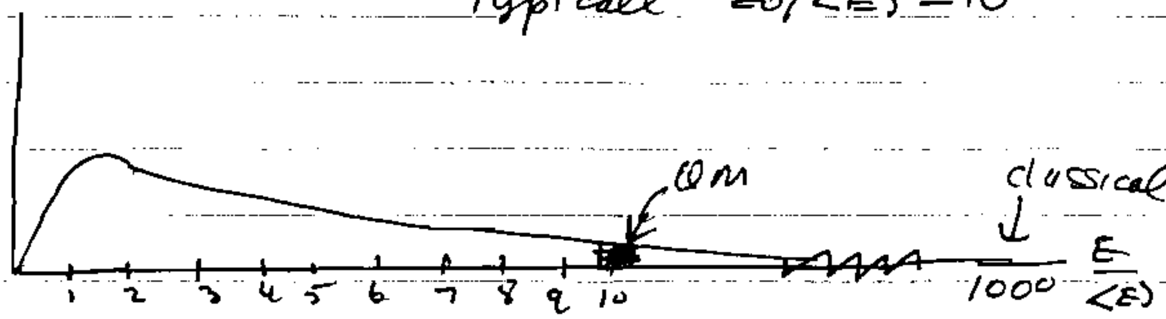


where $E_0 = \left(\frac{b kT}{2}\right)^{3/2}$

and $E_0 = 1.2 \left(Z_1^2 Z_2^2 A T_6^2\right)^{1/3} \text{keV}$ {"effective" energy}

(d) Compare E_0 with $\langle E \rangle$

Typicall $E_0/\langle E \rangle \simeq 10$



Thus, although effective energy is above average, it still is better than classical solution in which $E \geq V_{max}$ and $E \approx 1000 \cdot \langle E \rangle$ is required. Still, fraction of protons causing fusion is about $\sim e^{-\frac{E_0}{kT}} \approx e^{-10} \simeq 10^{-4}$ (still small!)

<u>Computing Energy production Rates</u>

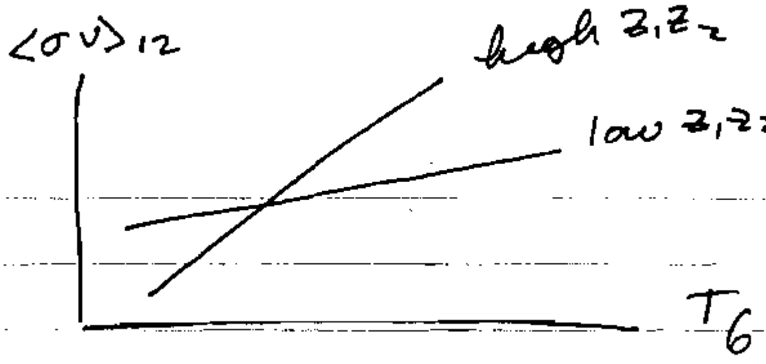
- <u>Rate constant</u>

We can write $\langle \sigma v \rangle_{12} = \frac{4.5 \times 10^{14} \tau^2 e^{-\tau}}{A Z_1 Z_2}$ [cm³/s]

where $\tau = 43 \left(\frac{Z_1^2 Z_2^2 A}{T_6}\right)^{1/3}$

- <u>T-sensitivity</u> : Rates are T-sensitive

- Reaction networks with higher nuclear charges (CNO) are more T-sensitive than networks with lower nuclear charges (pp) [For given T, $\langle \sigma v \rangle$ decreases as $Z_1 \cdot Z_2$ increases]



• <u>Production Rates</u>

Let $\mathcal{E}_0$ be energy released per reaction (Note this is not E_0).

$$\mathcal{E}_0 \cdot r_{12} = \frac{\text{energy}}{\cancel{\text{particle}}\ \text{reaction}} \times \frac{\text{no. } \cancel{\text{of}} \text{ reactions}}{\text{cm}^3 \cdot \text{sec}} = \frac{\text{energy}}{\text{cm}^3 \cdot \text{sec}}$$

Define ϵ by $\rho \epsilon = \mathcal{E}_0 r_{12}$

$$\epsilon = \frac{\mathcal{E}_0 r_{12}}{\rho} = \frac{\text{engery / vol.} \cdot \text{Time}}{\text{mass / vol}}$$

$[\epsilon]$ = energ production rate per unit mass

<u>pp Cycle</u>: Add up all reactions:

$$\rho \epsilon \simeq \frac{2.4 \times 10^4 \rho^2 X^2}{T_9^{2/3}} \exp\left[-3.38 / T_9^{1/3}\right] \frac{\text{ergs}}{\text{cm}^3 \cdot \text{sec}}$$

Fit with power-law: $\epsilon \propto \rho X^2 T^\nu$ ($\nu \simeq 4$)

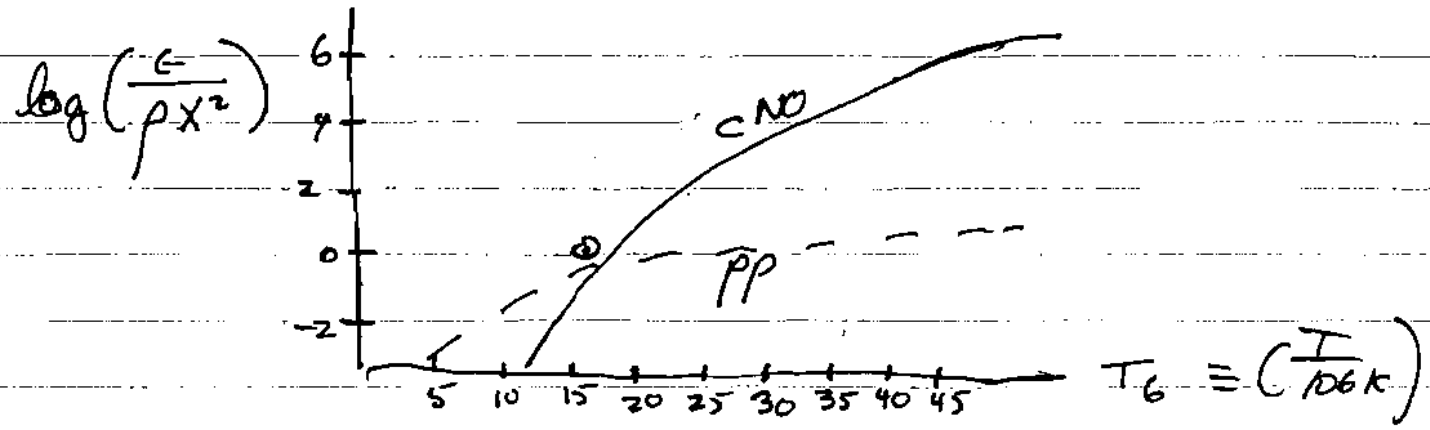
where $T_9 = T/10^9 K$

<u>CNO cycle</u>

$$\rho \epsilon \simeq \frac{9.4 \times 10^{24} \rho^2 X \cdot Z}{T_9^{2/3}} \exp\left[-15.23 / T_9^{1/3}\right]$$

$$\epsilon \propto \rho X \cdot Z T^\nu \ (\nu \simeq 20)$$

Compare Two rates



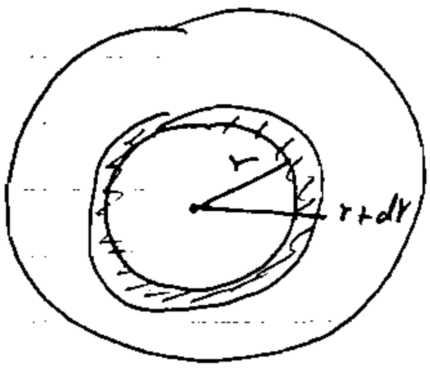
where $Z/X = .02$ is assumed. this shows that stars with $T_c > 2\times10^7 K$, CNO cycle dominates. These are clearly the more massive stars:

Recall $T_c \propto \frac{\mu M}{R}$

and $R \propto M^{.5} \Rightarrow T_c \propto \mu M^{.5}$

This has important implications as we shall see.

Fourth equation of Stellar Structure:



Let dL = luminosity ~~produced~~ in shell with radius $(r, r+dr)$

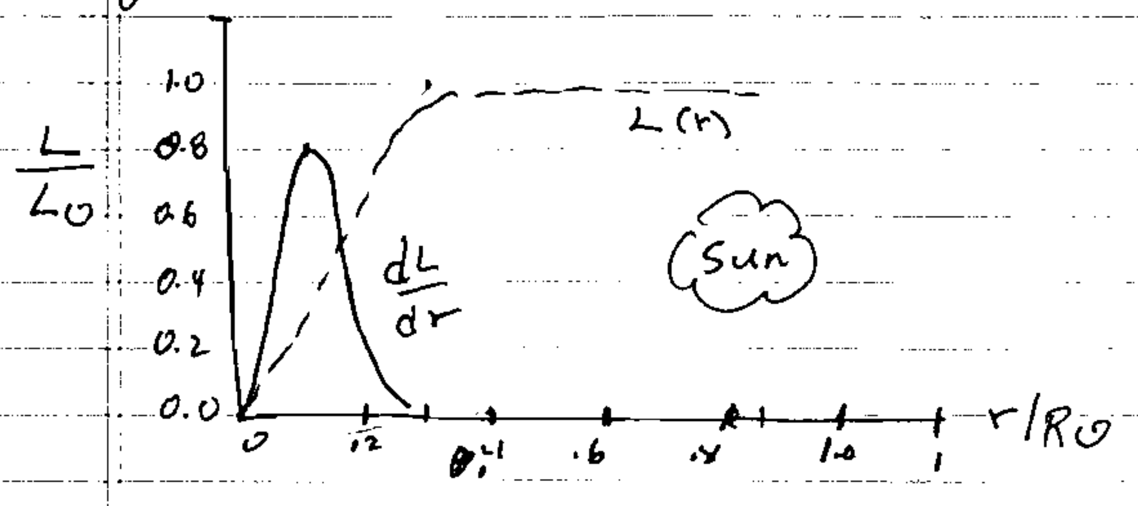
If shell has mass dM

$\therefore$

$$dL = \epsilon \cdot dM = \epsilon \cdot 4\pi\rho r^2 dr$$

$$\therefore \boxed{\frac{dL}{dr} = 4\pi r^2 \rho \epsilon}$$

Note ϵ is strongly peaked in central regions of the star. Thus beyond radii of energy production $dL/dr \simeq 0 \Rightarrow L \simeq \text{const}$.



So, we now have eq. for energy production. Peaked near core of the sun. This gives us 3 equations for stellar structure

$$\frac{dP}{dr} = -\frac{G M(r) \rho(r)}{r^2}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dL}{dr} = 4\pi r^2 \rho(r) \epsilon(r)$$

$$P = \frac{R}{\mu}\rho T + \underbrace{\tfrac{1}{3}aT^4}_{}$$ rad. pressure

$$\underbrace{\kappa = \kappa(\rho, T, X, Z)}_{\text{constitutive relations}}$$

We need a fourth equation. Of course this one tells how energy that is produced in the core reaches the surface.

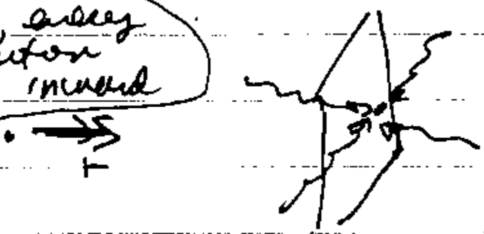
For MS stars there are two primary mechanisms: ① Radiative transport
② Convection

Radiative transport: slow outward diffusion of radiative flux transports energy to surface.

How does this work? The most important or crucial factor is that $|dT/dr| \neq 0$.

(A) Suppose $dT/dr = 0$. Solar interior is isothermal. In that case net outward flux of radiation would vanish, since radiation would be perfect isotropic

on average, every outward photon balanced by inward photon

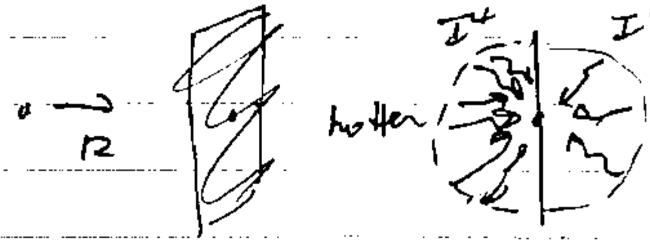


If T is same everywhere
$I_\nu = B_\nu(T)$, $T = \text{const}$
$I_\nu = \text{const}$ (no θ dependence)

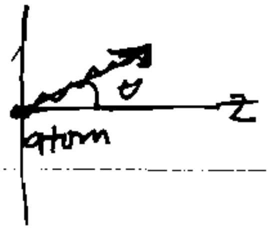
$$F = \int_{4\pi} I \cos d\Omega = 0 \quad \text{since} \quad F = I \int_0^{2\pi}\int_0^{\pi} (\cos\theta \sin\theta)\, d\phi$$

(B) But I is not exactly isotropic. There is a slight asymmetry due to T gradient; due to higher T at smaller r.

Recall, at given point I can only see 1 m.f.p.; i.e., photons travel freely for 1 mfp

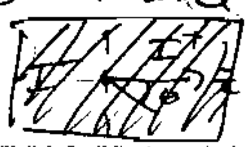


I^+ of outward photons > I^- of inward ones
$I^+ > I^-$



: Photons moving outward have $0 \le \theta \le \pi/2$ and constant intensity I^+

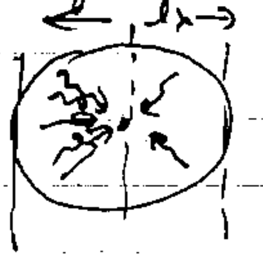
Photons moving inward have $\pi/2 \le \theta \le \pi$ and constant intensity I^-



$$F = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \, (\cos\theta) \, I(\theta) = 2\pi \left\{ I_+ \int_0^{\pi/2} d\theta \sin\theta\cos\theta + I_- \int_{\pi/2}^{\pi} d\theta \sin\theta\cos\theta \right\}$$

$$F = 2\pi\left(\frac{I_+}{2} - \frac{I_-}{2}\right) = \pi(I_+ - I_-)$$

But this is change over ± photon mfp:



$B(R)$ $\quad$ $B(R+\Delta R)$

I_+ $\quad$ I_-

$$I_+ - I_- \simeq B(R) - B(R+\Delta R) = B(R) - \left[B(R) + \frac{dB}{dR} l_\lambda\right]$$

$$I_+ - I_- \simeq -\frac{dB}{dR} l_\lambda$$

$$\therefore \boxed{F(R) = -\pi \frac{dB}{dR} l_\lambda}$$

Luminosity flowing through sphere $A = 4\pi R^2$

$$L(R) = 4\pi R^2 \cdot F(R)$$

$$\cancel{\boxed{L(R) = 4\pi R^2 \left[-\pi \frac{dB}{dR} \ell_\lambda\right]}}$$

Recall for black-body: $B(R) = \dfrac{\sigma T^4}{\pi}$

write in terms of a where $\sigma = \dfrac{ca}{4}$

$$\therefore\ B(R) = \left(\frac{ca}{4\pi}\right) T^4 \quad \begin{cases} a = 7.56\times10^{-15}\ (\text{cgs}) \\ a = 7.56\times10^{-16}\ (\text{MKS}) \end{cases}$$

$$\text{or}\ \frac{dB}{dR} = \frac{ca}{4\pi} \times 4T^3 \frac{dT}{dR} = \frac{ca}{\pi} T^3 \frac{dT}{dR}$$

$$\text{Thus:}\quad F(R) = -\cancel{\pi}\left[\frac{ca}{\cancel{\pi}} T^3 \frac{dT}{dR}\right] \ell_\lambda$$

$$L(R) = \frac{4\pi R^2}{3}\left[-(ca)\, T^3 \frac{dT}{dR}\right] \ell_\lambda$$

more accurate 3-D

$$\boxed{L(R) = -\frac{4\pi R^2 (ca)\, T^3\, dT/dR}{3\cdot \kappa\rho}}$$

where $\ell_\lambda = \dfrac{1}{n\sigma} = \dfrac{1}{\kappa\rho}$

4^{th} eq. of stellar structure:

$$\frac{dT}{dR} = -\frac{3\kappa\rho \cdot L(R)}{4\pi (ca)\, R^2 T^3} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L(R)}{4\pi R^2}$$

Implications

(1) Luminosity of star determined by temperature gradient. dT/dR

(2) But $T \propto GM/R$; dT/dR set by global structure of the star not by L_{nuc}, i.e., ϵ.

(3) L fixed by M, R, κ not L_{nuc}

Example: Order of magnitude:

$$L \propto \frac{R^2 (T^4/R)}{\rho \kappa} = \frac{R T^4}{\kappa \rho}$$

Recall: $T \propto \frac{\mu M}{R}$; $\rho \propto M/R^3$

$$\Rightarrow \boxed{L \propto \frac{R \left(\frac{\mu M}{R}\right)^4}{\kappa M/R^3} \propto \frac{\mu^4 M^3}{\kappa}}$$

Two points here

(A) Voigt-Russel theorem: L determined mainly by mass, molecular weight composition, not by L_{nuc}
(Get $L \propto M^3$ dependence (almost))

(B) For fixed mass $_2He^4$ star will be much more luminous than $_1H^1$

$\mathcal{L} \propto M^3$ dependence if σ is fixed. While this is not rigorously true, it is not a bad approx. indicating that eqs. of stellar structure yield correct mass-luminosity relationship.

Stability of Global Thermodynamic Equilibrium

Let's go back to condition that nuclear energy input balances radiative output from the star:

$$\mathcal{L}_{nuc} = \mathcal{L}_{rad}$$

How accurate is this equation?

(A) Suppose $\mathcal{L}_{nuc} < \mathcal{L}_{rad}$ in the core

(i) U (thermal kinetic energy density) will be radiated away in t_{KH} (as I showed previously)

(ii) $\frac{dE}{dt} = -\mathcal{L}_{rad} + \mathcal{L}_{nuc} < 0$

(iii) Virial theorem tells us that

$$\frac{dE}{dt} = \frac{1}{2}\frac{d\Omega_g}{dt}$$

since $dE/dt < 0$, then $\frac{d\Omega_g}{dt} < 0$

What happens? Core contracts

(iv) But since $E_{KE} = -\frac{1}{2}\Omega_g$

$$\frac{dE_{KE}}{dt} = -\frac{1}{2}\frac{d\Omega_g}{dt} \Rightarrow \frac{dE_{KE}}{dt} > 0$$

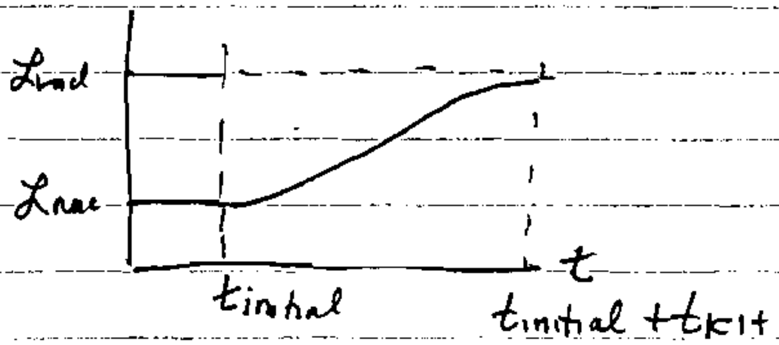
As a result E_{KE} increases, meaning temperature of core rises.

(v) We ~~showed~~ saw that that $L_{nuc} \propto T^{\alpha}$ where $\alpha \gg 1$.
thus core contraction results in an increase in L_{nuc} until $L_{nuc} = L_{rad}$; i.e., until energy input again balances radiative output

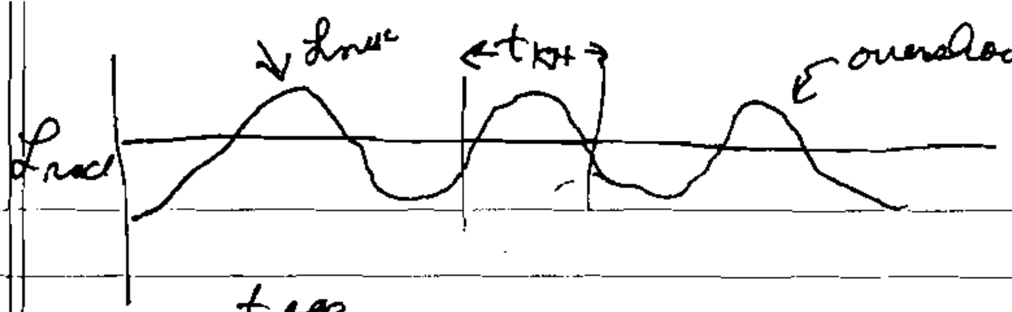
(vi) Since $\frac{dE}{dt} = -L_{rad} + L_{nuc}$

then $\frac{dE}{dt} = 0$ and since $\frac{dE}{dt} = \frac{1}{2}\frac{d\Omega_g}{dt}$,

Contraction of the core is halted.



So MS stars like the sun, i.e. MS stars, adjust their core sizes such that L_{nuc} balances L_{rad}. On time scales $t \gg t_{KH}$, balance is excellent.



$$\langle L_{nuc} \rangle = \frac{\int_0^{t_{age}} L_{nuc}(t)\,dt}{t_{age}} = L_{rad} \text{ where}$$

$t_{age} \gg t_{KH} \simeq 2\times10^7$ years

At same time L_{rad} doesn't vary. Why not? Because it is fixed by dT/dR, which is set by global structure of the star.

Convection