Phys 160

# Stars

① Preliminaries:

Book - F. Shu / "Physical Universe"

Requirements: Graded homework assignments
Final ~~exam~~ paper

Goal: Cover part II of the text; i.e., chps 5-10. Coverage will not be complete nor will it be literal. In some cases I may venture into topics not covered in book: I will assign outside readings.

Reserve books:

Schwarzschild: Structure & Evolution of the Stars

Clayton: Nuclear Astrophysics

## Introduction

① Historical: Stars have played a fundamental role in the way in which we look at the world. Stars have been observed by humans throughout recorded history, and undoubtedly a long time before that.

Ⓐ Time + Periodicity

The diurnal rising and setting of stars due to the spin of the earth gave rise to 2 crucial concepts

(i) Time: The locations of a given star through-out the night were the hour angles of the first clock.

(ii) Periodicity: The fact that the same stars repeated themselves (sun) gave rise to idea that events were reproducable.

Both concepts were crucial to development of modern science.

(B) Navigational guides

Stars were also used as fixed points against which ships were guided, astrological .. etc.

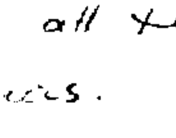
(C) Sun: Although it wasn't realized at the time, the sun is in fact the nearest star. So... one could state that stars are essential for life. We shall see that this doesn't just pertain to sunlight. Rather, I mean that with the exception of light elements such as H, He, Li, D, all the elements are manufactured in stellar interiors.

So we are "star stuff".

Physical Nature:

But the task of this course is to learn something about the nature of stars. In fact little progress on this subject was made until the beginning of the 20th century.

And during the 20th century we have been able to answer fundamental questions about the nature of stars

(a) Energy transport? [Stars radiate EM energy] 

b Energy Source? Do we need a source?

(c) Internal structure? $T(R)$, $\rho(R)$, $\mathcal{L}(R)$?

(d) What are parameters that determine $\mathcal{L}$?

$$\mathcal{L} = \mathcal{L}(M, \mu, t)$$

(e) How do stars evolve? or Do stars evolve?

(f) How do stars die? Endpoints of stellar evolution

(g) How do stars form?.

Slowly, through painstaking research, astrophysicists have answered these questions.

Breakthroughs:

We shall see that it was

(a) Eddington's suggestion that energy is transported by radiation played a major role in understanding

- internal structure
- $\mathcal{L}(M,\mu)$ relation

(b) Nuclear Physics

Bethe, Von Weizacker, Salpeter, Hoyle, Fowler, B², showed convincingly that nuclear fusion was source of energy in the sun and in other stars. Continuous Energy production required to keep sun shining. Production via Nuclear reactions

Energy available from fusion accounts for tremendous age of the sun (4.5 Gyr), and, indirectly, of other stars.

(c) Evolution -

(1) Nuclear fusion in stellar interiors, (2) the virial theorem, and straightforward (3) thermodynamics are all that one needs to explain stellar evolution.

byproduct: element synthesis

d Degeneracy

Work of Chandrasekhar, Oppenheimer, Volkoff, and Zwicky showed that fermions

degeneracy plays crucial role in the endpoints of stellar evolution -
Final configurations such as
white dwarfs
Neutron stars
are determined by laws of gravity and
Fermi-Dirac statistics, a quantum mechanical concept
& General Relativity

Finally, endpoint of very massive stars is a black hole. First pointed out by Oppenheimer! Here, final structure determined by Einstein Field Equations.

These are the major themes I will be discussing this term!

Preliminaries

First, let's get a feeling for some of the basic parameters characterizing the sun

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Luminosity and ...

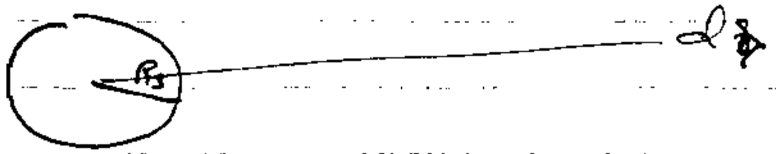
~~That the sun can be regarded as a point source emitting radiation... Since it is a point-like source, it emits spherical waves.~~

## Preliminaries

How do we measure parameters such as luminosity, $L$, mass, $M$, temperature, $T$, ..etc

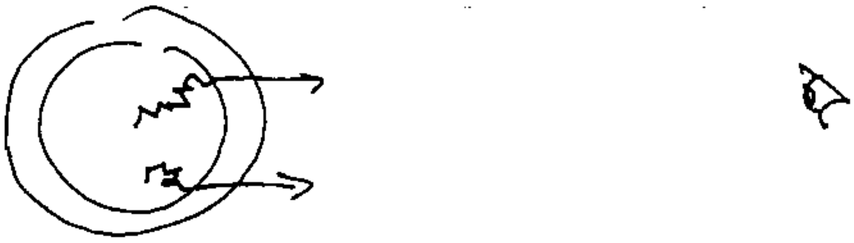
### Luminosity and flux

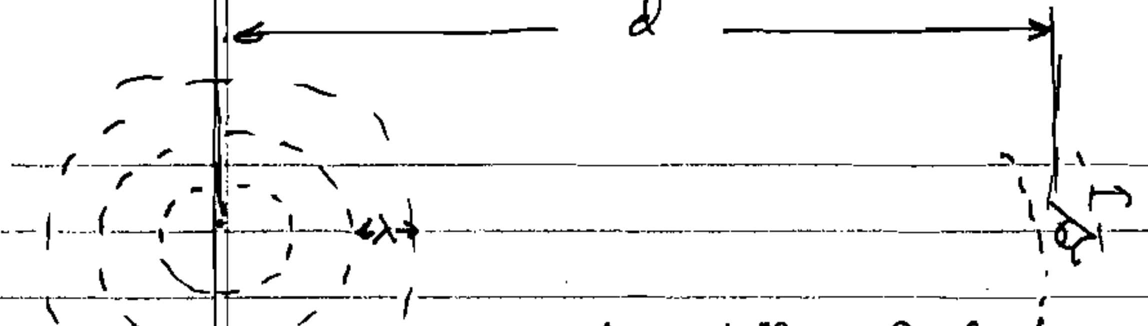
Let sun, or any star, be a sphere




that emits monochromatic radiation. At distance $d \gg R_s$, think of sun as a point-like oscillator emitting spherical waves.

Of course sun does not have a solid surface. As we shall see, light escapes from a thin surface layer, the solar atmosphere or "photosphere"






at astronomical distance $d$ from the star
I observe the star for time interval
$\Delta t = N \cdot p$

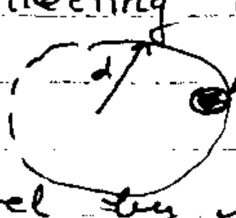

- $p$ = period of wave ($p = \frac{\lambda}{c}$)
- $N$ = integral no. of wavecrests

In time interval $\Delta t$, star radiates energy $\Delta E = \mathcal{L} \Delta t$, where $\mathcal{L}$ is stellar luminosity (here assumed to be at $\lambda$).
of course $d \gg c\,\Delta t$

In absence of interaction between light waves and matter the $\Delta E$ associated with $N$ crests remains invariant. All that happens is that $\Delta E$ gets spread over increasing area of expanding spherical wave.

At distance $d$, energy/area of waves: $\frac{\Delta E}{4\pi d^2}$

Since telescope with collecting area $A$, intercepts fractional area: $A/4\pi d^2$




the EM energy detected by the telescope:

$$\Delta e = \Delta E \,(A/4\pi d^2) = \frac{\mathcal{L}\Delta t}{4\pi d^2} \cdot A$$

Rate of energy detection: $b = \frac{\Delta e}{\Delta t} = \frac{\mathcal{L} A}{4\pi d^2}$

Define Flux: $\boxed{F = \frac{b}{A} = \frac{\mathcal{L}}{4\pi d^2}}$ $\quad [F] = \frac{\text{Energy}}{\text{Time} \cdot \text{area}}$

## Distance

<u>Flux</u>: Determines appearance of the star on the sky. Star is bright for 2 reasons

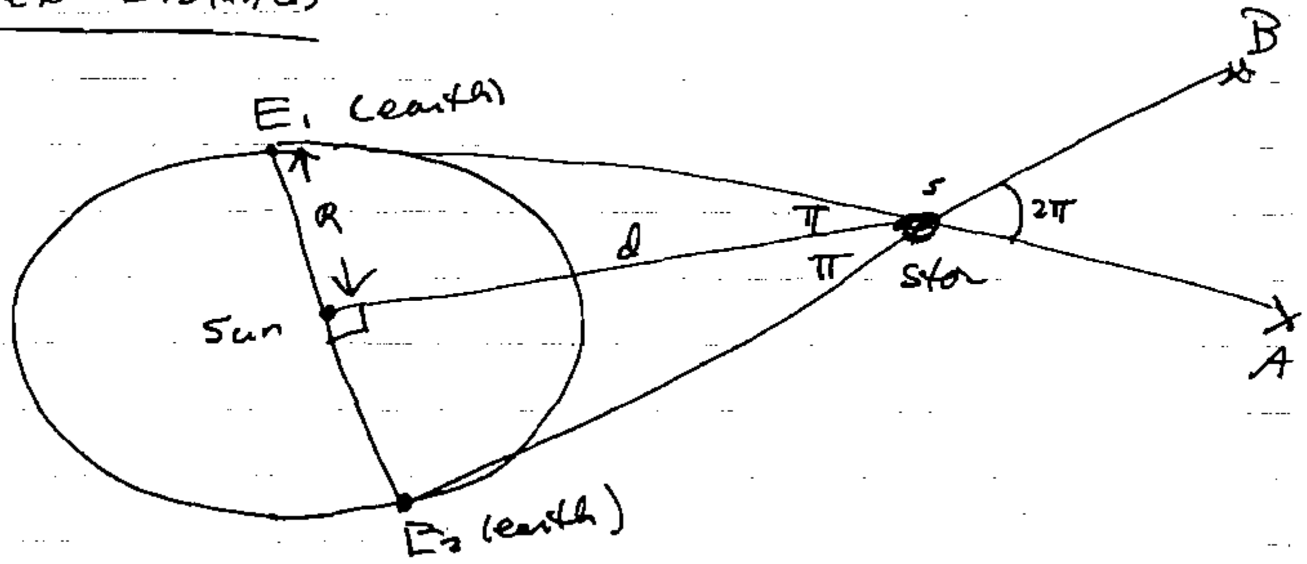
(1) medium $\mathcal{L}$, small $d$

(2) high $\mathcal{L}$, medium $d$

<u>Luminosity</u>: Crucial <u>intrinsic</u> property of a star. To determine $\mathcal{L}$ ~~we~~ need to determined distance, $d$, because

$$\boxed{\mathcal{L} = 4\pi d^2 \cdot F}$$

### Parallax Distances




- Draw line from sun to star
- Draw $\perp$ line in ecliptic plane through sun
- Earth at $E_1$. $E_1 \cdot S$ toward star A
- 6 months later $E_2 S$ toward star B
- $E_1 \cdot S$ and $E_2 \cdot S$ subtend angle given by $2\pi$ ($\pi \neq 3.14159$) $\angle$ at star. Line $d$ bisects angle

Therefore $\tan \pi = \frac{R}{d}$

Because $\pi < 1$ arcsec $\Rightarrow$ $\pi \ll 1$ rad $\Rightarrow$

$$\tan\pi \simeq \pi \simeq \frac{R}{d} \qquad \text{or} \qquad d = \frac{R}{\pi(\text{rad})}$$

number

$$\text{But } \pi(\text{rad}) = \pi(\text{arcsec}) \times \left[\frac{\pi}{180} \times \frac{1}{60} \times \frac{1}{60}\right]$$

$$\pi(\text{rad}) = \frac{\pi(\text{arcsec})}{2\times10^5}$$

therefore

$$\boxed{d = \frac{2\times10^5 \cdot R}{\pi(\text{arcsec})}}$$

Since displacement or parallax angle is largest for nearest stars, and since $\pi$ is never observed to exceed 1 arcsec, implication is

$$\frac{d}{R} \gg 2\times10^5 \quad \text{for all stars}$$

This was astonishing when in 1838 Bessel made 1st parallax measurement ~~in 19th century~~ for 61 Cygni. Stars are more than $10^5$ more distant than sun.

Physical Units: $R = 1.5\times10^{13}$ cm

$$\therefore \quad d = \frac{2\times10^5\,(1.5\times10^{13}\,\text{cm})}{\pi(\text{arcsec})} = \frac{3\times10^{18}\,\text{cm}}{\pi(\text{arcsec})} = \frac{3\times10^{16}\,\text{m}}{\pi(\text{arcsec})}$$

$$d = \frac{1}{\pi(\text{arcsec})}\ \text{pc.}$$

## Other Units

speed of light: $c = 3\times10^{10}$ cm/s

1 yr $\quad \Delta t = 3\times10^{7}$ s

1 lt yr $= c\cdot\Delta t = 3\times10^{10} \times 3\times10^{7}$

$= 9\times10^{17}$ cm $\approx 10^{18}$ cm.

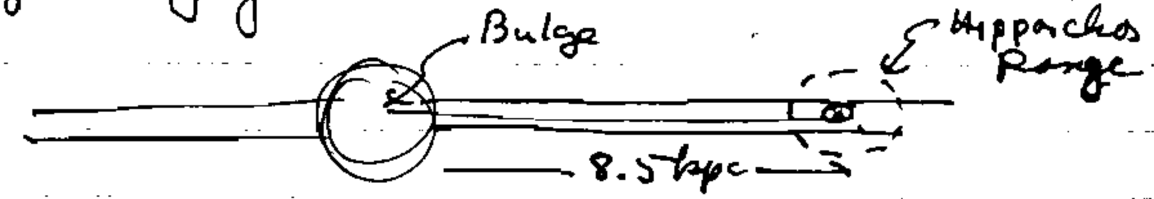
But since 1 pc $= 3\times10^{18}$ cm, we have $\boxed{1\,\text{pc} \simeq 3\,\text{lt yr.}}$

## Parallax Measurements:

(i) Hipparchos satellite: accuracy $\simeq 0.''001$

Since $d\,(\text{pc}) = \dfrac{1}{\pi(\text{arcsec})}$

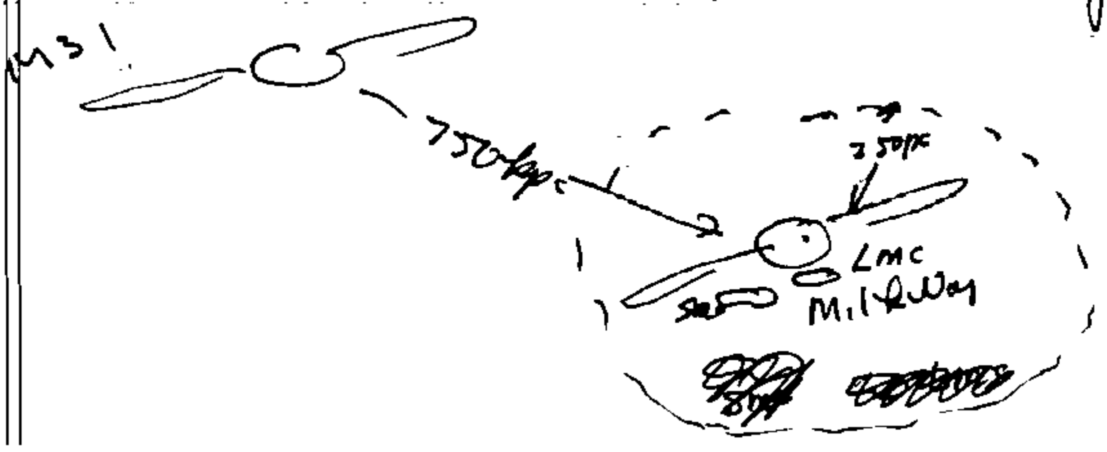
Hipparchos measures stars within $10^3$ pc $= 1$ kpc.
~118,000 stars: well within disk of Milky-Way galaxy




Plans are to ultimately go down to $4\times10^{-6}$ arcsec (SIM)

$$\Rightarrow \quad d \simeq \frac{1}{4\times10^{-6}} = 2.5\times10^{5}\ \text{pc} = 250\ \text{kpc}!$$

Includes distance to LMC, SMC : nearest galaxies.



## Luminosity, flux, magnitudes

How do we determine $\mathcal{L}_\odot$ for sun?

From $F = \frac{\mathcal{L}}{4\pi d^2}$; $\mathcal{L}_\odot = 4\pi R^2 \cdot F_\odot$

Measurements of "solar const": $F_\odot = 1.36\times10^6 \frac{\text{ergs}}{\text{cm}^2\cdot\text{sec}}$

$\therefore \mathcal{L}_\odot = 4\pi(1.5\times10^{13})^2(1.36\times10^6) \simeq 4\times10^{33}$ ergs/s
$= 4\times10^{26}$ J/s $= 4\times10^{26}$ Watts

## Magnitude Scale

Astronomers don't generally work with fluxes. Rather they use magnitude scale (Hipparchos 200-125 B.C.)

(A) Definition

$m = 1$ (~~Brightest~~ Brightest star = 1st magnitude)
$m = 6$ ( Faintest observable w/ eye = 6th mag)

larger algebraic values of magnitudes mean fainter objects.

(B) Connection between flux & magnitude

Pogson (19th century British astronomer) noticed that star with $m=6$ at 10" diameter telescope appeared as bright as a $m=1$ star at 1" telescope.

Brightness $b = F \cdot A$

$$\therefore \frac{b_6}{b_1} = \frac{F_6 \cdot \frac{\pi}{4}(10'')^2}{F_1 \cdot \frac{\pi}{4}(1'')} = 100 \cdot \frac{F_6}{F_1} \simeq 1$$

$$\therefore \frac{F_1}{F_6} = 100 \iff \Delta m = 5$$

Therefore:

| $m_2 - m_1$ | $F_1/F_2$ | $F_1/F_2$ |
|---|---|---|
| 5 | 100 | $(2.51)^5$ |
| 3 | 15.9 | $(2.51)^3$ |
| 2 | 6.3 | $(2.51)^2$ |
| 1 | 2.51 | $(2.51)^1$ |

This implies: $F_1/F_2 = (2.51)^{m_2-m_1} = 10^{.4(m_2-m_1)}$

or $F_2/F_1 = 10^{-.4(m_2-m_1)}$

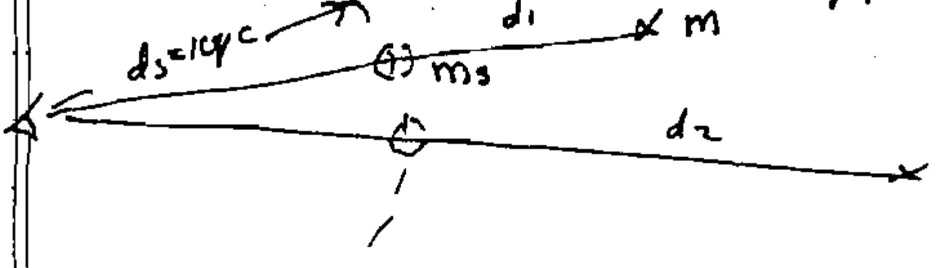
$$\log(F_2/F_1) = -.4(m_2-m_1)$$

$$\text{or } \boxed{m_2 - m_1 = -2.5\log(F_2/F_1)}$$

## Absolute Magnitude

But we want to rank stars by their intrinsic ~~properties~~ luminosities rather than by ~~apparent brigh~~ fluxes which are determined by random position of earth wrt stars!

Idea: Do a thought experiment in which all stars brought into standard distance of $d_s = 10pc$. At same distance, relative fluxes will reflect relative luminosities. Abs. mag. is apparent mag star would have at a distance of $d_s = 10pc$.



Same Star

$$m_s - m = -2.5\log\frac{F_s(d_s)}{F(d)} = -2.5\log\left[\frac{L/4\pi d_s^2}{L/4\pi d^2}\right]$$

$$m_s - m = -2.5\log\left(\frac{d}{d_s}\right)^2 = -5\log\left(d/10pc\right)$$

Standard notation: $M = m$ (absolute magnitude)

$$\boxed{M = m - 5\log(d/10pc)}$$

Interpretation: For an observed apparent magnitude, a larger distance implies more negative $M$, hence more luminous object.

Rewrite: $5\log(d/10pc) = m - M$

$$d/10 = 10^{.2(m-M)}$$

$$\boxed{d = 10^{.2(m-M)+1}}\ pc$$

$m - M$ is distance modulus.

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Example: What is absolute magnitude of the sun?

Apparent mag: $m_\odot = -26.8$ (Sun is very bright)
distance $d = 1\,A.U = \left(1/2\times10^5\right) pc$

$$\therefore\ M_\odot = m_\odot - 5\log\left(\frac{1}{2\times10^5\times10}\right) = m_\odot + 5\log(2\times10^6)$$

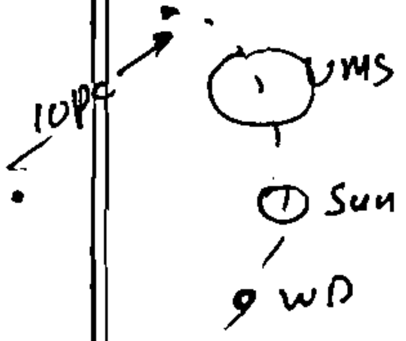
$$\therefore\ M_\odot = -26.8 + 5(.3+6) = -26.8 + 31.5$$

$$\boxed{M_\odot = 4.7}$$ (Sun not very luminous)

## Relative Luminosities

Compare absolute magnitudes with sun: all at $d_s$.

$$M - M_\odot = -2.5 \log \frac{F}{F_\odot} = -2.5 \log \frac{L/4\pi d_s^2}{L_\odot/4\pi d_s^2}$$




$$M - M_\odot = -2.5 \log \left(\frac{L}{L_\odot}\right)$$

### (A) Luminous Stars:

Upper Main Sequence Stars: $\mathcal{L} \approx 10^6 \mathcal{L}_\odot$

$$\therefore \quad M_{UMS} = M_\odot - 2.5 \log 10^6$$

$$M_{UMS} = M_\odot - 15$$

(15 mag. more luminous than Sun)

$$M_{UMS} = 4.7 - 15 = -10.3$$

### (B) Intrinsically faint stars (White Dwarfs)

these stars are 25 mag fainter than UMS stars

$$M_{WD} - M_{UMS} = -2.5 \log \left(\frac{L_{WD}}{L_{UMS}}\right)$$

$$\frac{L_{WD}}{L_{UMS}} = 10^{-.4(M_{WD} - M_{UMS})} = 10^{-.4(25)} = 10^{-10}$$

or $\dfrac{L_{UMS}}{L_{WD}} = 10^{10}$

more Precise $L_{UMS}/L_{WD} = 10^{12}$!

# Black Body Radiation

A crucial physical quantity determining the appearance and physical state of a star is temperature. Specifically, the appearance of a star is largely determined by the temperature at its "surface". If this region is in a state of thermodynamic equilibrium, then the procedure is unambiguous.

In thermodynamic equilibrium (TE) one unique temperature characterizes all degrees of freedom (dof)

Let's take a closer look at TE

(1) Adiabatic, sealed Box

Assume box is sealed with rigid walls. Box contains matter and radiation in TE


(2) In State of TE

(a) rapid interaction between matter, walls, radiation result in $T_{wall} = T_{matter} = T_{radiation} = T$

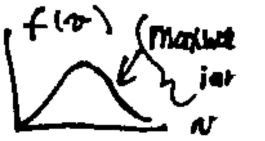
Pressure = const. (no net c.m. motion)

(b) Internal dof coupled to external dof

- $T_{external}$ describing translation motions of free particles ($e^-$, $H^+$, $H^0$, ..) equals $T_{internal}$ describing relative populations of bound stat

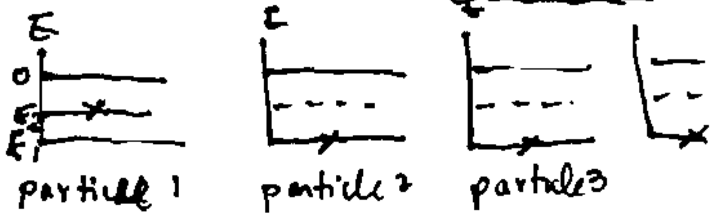
Illustration

- External

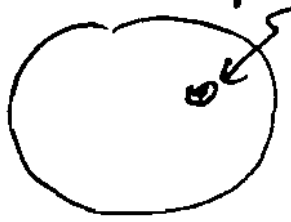




- Internal

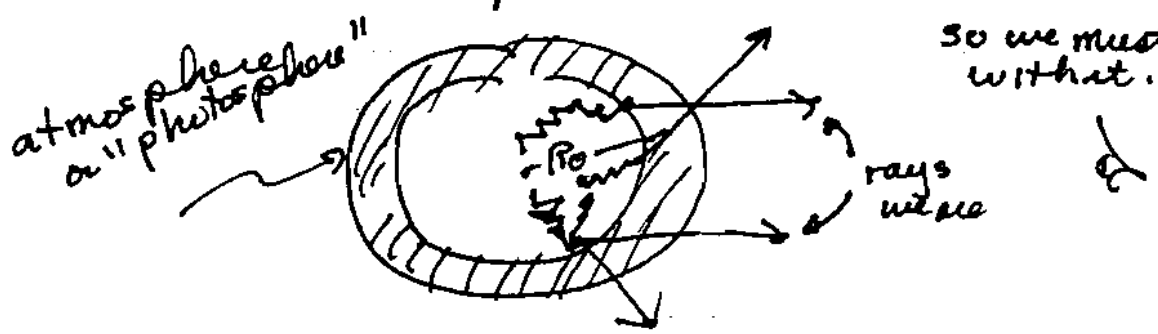


ⓒ We will see this is an excellent approximation for small regions of stellar interior.

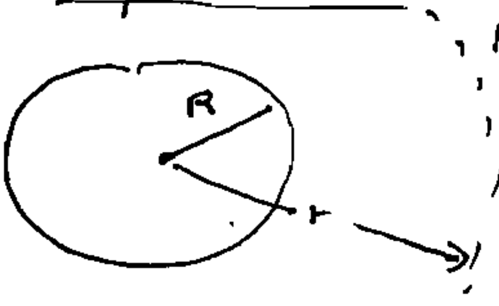


But less accurate for lower densities of stellar atmosphere

Thus, in general $T_{internal} \neq T_{external}$ in atmosphere of sun or any star. But the atmosphere is the outer layer from which radiation escapes: this is what we observe, so we must deal with that.




(3) Equivalent black body or Effective temperature $T_e$:




Recall $F = \frac{L}{4\pi r^2}$

Shrink r down to surface of the Star. We then have –

$$L = 4\pi R^2 \cdot F(R)$$

In the case of black body radiation, I will show that → $\boxed{F(R) = \sigma T^4 \text{. Black Body}}$

But stellar atmospheres are not perfect black bodies. So we define $T_e$ as temperature a black body would have if emitted flux equaled $F(R)$.

That is, $F_{surface} = \sigma T_e^4$

Resulting in $\boxed{L = 4\pi R^2 \cdot \sigma T_e^4}$

where $\sigma = 5.67\times10^{-8}\ W\,m^{-2}K^{-4}$ (mks)
$= 5.67\times10^{-5}$ erg $cm^{-2}K^{-4}$ (cgs)

What is $T_e$ for the sun?

$$T_e = \left(\frac{L_\odot}{4\pi R_\odot^2 \sigma}\right)^{1/4}$$

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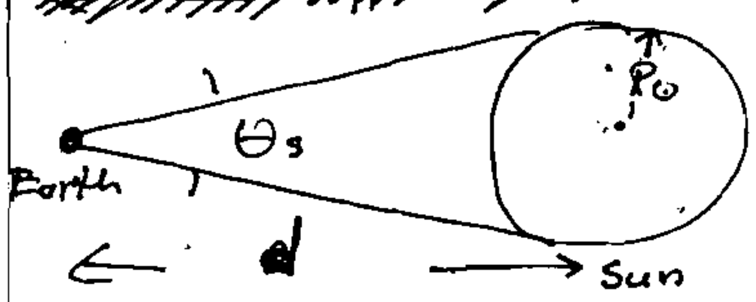
I didn't talk about solar radius:




Angular diameter $\theta_s = \frac{2R_\odot}{d}$

Observations show $\theta_s = 32$ arcmin.

Therefore: $R_\odot = \frac{1}{2}\underbrace{(\theta_s)}_{radian} d$

or $R_\odot = \frac{1}{2}\left(32\times\frac{1}{60}\times\frac{\pi}{180}\right)\times1.5\times10^{13}$ cm

$R_\odot = 7\times10^{10}$ cm or $7\times10^{8}$ m

Earth $R_\oplus = 6.4\times10^{6}$ m $\Rightarrow R_\odot \simeq 10^2 R_\oplus$

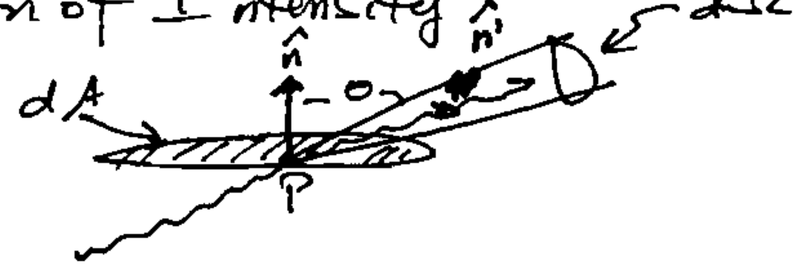
As a result: $T_e = \left( \dfrac{4\times10^{26}\,W}{4\pi\times(7\times10^{8})^2\times 5.67\times10^{-8}} \right)^{1/4}$ (1-17)

$$\boxed{T_e = 5800K}$$

~~Next Lecture~~ Later on I will discuss how Black-body of this temperature has spectral peak at about wavelength $\lambda \approx 5000Å$ or 500nm: Not surprising, from a Darwinian point of view, that ~~this~~ is spectral region where human eye has peak sensitivity!

## Black Body Spectrum:

(1) Definition of Intensity

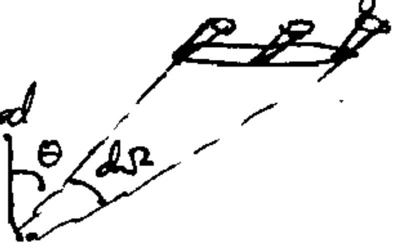



(1) Consider infinitesimal area element $dA$ with unit normal vector $\hat{n}$

(2) Draw line from point P on $dA$ in direction $\hat{n}'$ in which radiation propagates

(3) Consider rays about $\hat{n}'$ confined to solid angle $d\Omega$

(4) Draw family of cones parallel to $\hat{n}'$ at each point on $dA$ with same solid angle $d\Omega$




(5) Envelope of these cones is truncated cone of solid angle $d\Omega$ with apex behind $dA$

(6) In limit $dA \to 0$, $dA$ slides down toward apex and meets $\underset{dt}{}$ in limit $dA \to 0$ which we will consider

(7) Let $dE_\nu$ = radiation energy
with
(a) frequencies in interval $(\nu, \nu + d\nu)$
(b) flows across $dA$ in time interval $dt$
(c) In directions confined to $d\Omega$

In that case $dE_\nu \propto d\nu \, d\Omega \, dt \, \underbrace{dA \cos\theta}_{\text{projected area}}$

Definition of intensity:

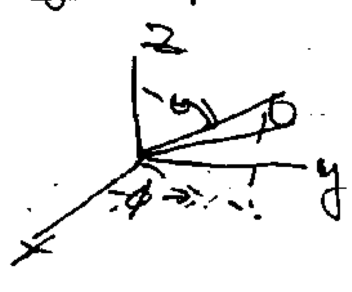
$$dE_\nu = I_\nu \, d\nu \, d\Omega \, dt \, dA \cos\theta$$

$$\boxed{I_\nu(\vec{n}) = \frac{dE_\nu}{d\nu \, d\Omega \, dt \, dA \cos\theta}}$$

$$[I_\nu] = \frac{\text{ergs}}{\text{cm}^2 \cdot \text{sec Hz sr}} \quad \text{or} \quad \frac{\text{Joules}}{\text{m}^2 \text{ sec Hz sr}} = \frac{\text{Watt}}{\text{m}^2 \text{ Hz sr}}$$

Flux: Consider net amount of radiative energy crossing $dA$ in $(t, t+dt)$, $(\nu, \nu + d\nu)$ over all solid angles.

$$\frac{dE_\nu}{dt \, d\nu} = I_\nu(\theta, \phi) \, d\Omega \, dA \cos\theta$$




Integrate over $4\pi$ sterad

$$\frac{dE_\nu}{dt \, d\nu} = dA \int_{4\pi} I_\nu \, d\Omega \cos\theta$$

Flux: $F_\nu \equiv \dfrac{dE_\nu}{dt\,d\nu\,dA} = \displaystyle\int_{4\pi} I_\nu \cos\theta\, d\Omega$

(1)  in space or interior to star we integrate over all directions

$$F_\nu = \int_0^{2\pi}\!\!\int_0^{\pi} I_\nu \cos\theta\,(\sin\theta\, d\theta\, d\phi)$$

isotropic rad. (CMB), $F_\nu \equiv 0$

(2) But at surface of star we ignore incoming radiation so integrate $\theta$ from ~~...~~ $0 \to \pi/2$

(3) In case of Black Body $I_\nu = B_\nu(T)$ (not a function of $\theta$)

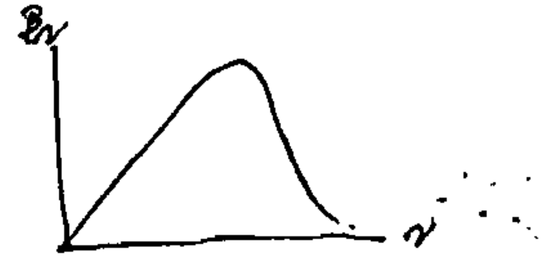
Surface flux $\Big\}$ $\therefore\ F_\nu^{+} = B_\nu(T)\displaystyle\int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta\, \cos\theta \sin\theta$

or $F_\nu^{+} = B_\nu \cdot 2\pi \cdot \frac{1}{2} = \pi B_\nu$

where $B_\nu = \dfrac{2h\nu^3/c^2}{e^{\frac{h\nu}{kT}} - 1}$, $h$ Planck const

$h = 6.626 \times 10^{-27}$ erg·s

$h = 6.626 \times 10^{-34}$ J·s




Bolometric flux that we have considered so far

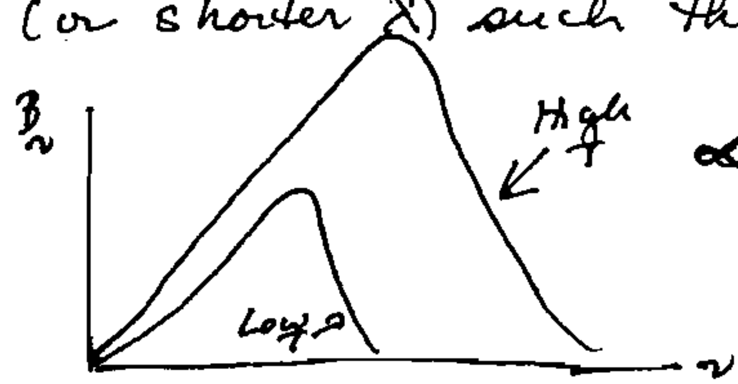
$$F = \int_0^\infty F_\nu \, d\nu = \pi \int_0^\infty B_\nu \, d\nu$$

But $\int_0^\infty B_\nu \, d\nu = \sigma T^4 / \pi$ ($\sigma$ = homework problem)

Therefore $\boxed{F = \sigma T^4}$ for flux emerging from surface of BB

Two Crucial Aspects of BB

(1) An increase of T (a) raises entire BB curve and (b) pushes peak to higher $\nu$ (or shorter $\lambda$) such that area under peak is $\propto T^4$.



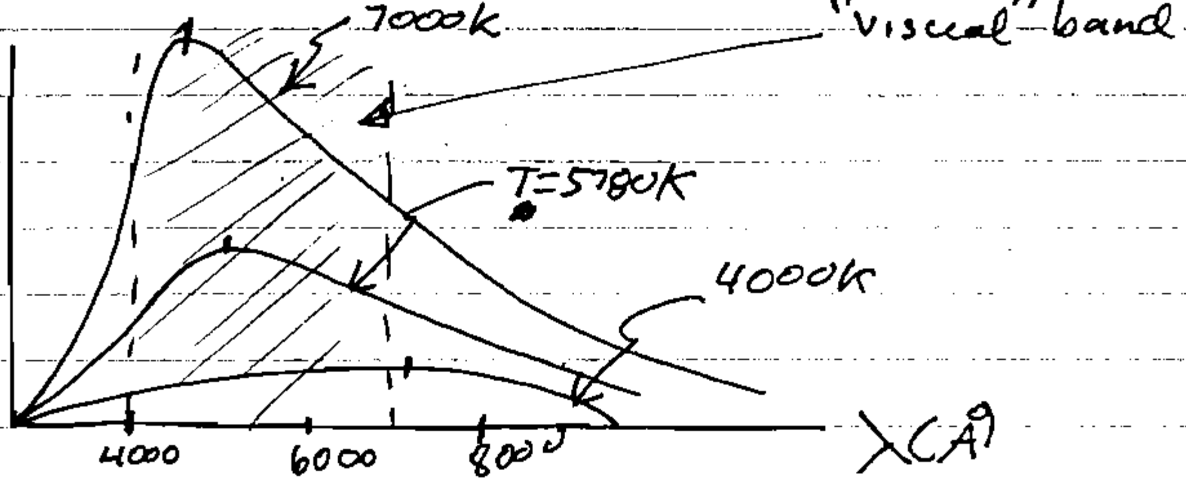

(2) Sometimes convenient to use intensity per unit wavelength interval

$$B_\lambda \, d\lambda = B_\nu \, d\nu \Rightarrow B_\lambda = B_\nu \left| \frac{d\nu}{d\lambda} \right|$$

where $\nu = c/\lambda$ $\therefore \left| \frac{d\nu}{d\lambda} \right| = \frac{c}{\lambda^2}$

In that case $B_\lambda = \dfrac{2h\left(\frac{c}{\lambda}\right)^3 / c^2}{e^{\frac{hc}{\lambda kT}} - 1} \left( \dfrac{c}{\lambda^2} \right)$

or $$\boxed{B_\lambda = \frac{2hc^2/\lambda^5}{e^{\frac{hc}{\lambda k T}} - 1}}$$



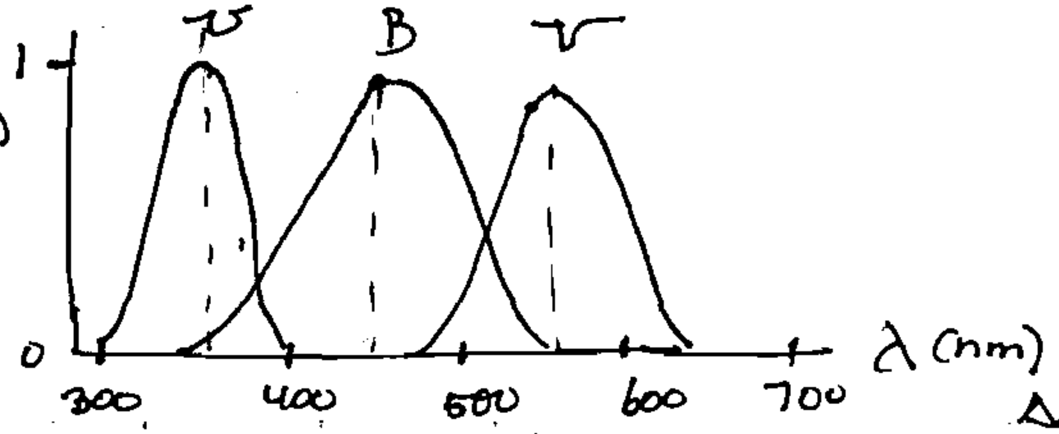

$\lambda_{max} T = 0.0029$ m K (HW Problem)

For sun where $T \simeq 5771$ K we find
$\lambda_{max} = \frac{30 \times 10^{-4}}{6 \times 10^3} \simeq 5 \times 10^{-7}$ m

or $\lambda_{max} \simeq 500$ nm or 5000 Å
just at peak sensitivity of eye.
Cooler stars have longer $\lambda_{max}$ and hotter stars " shorter $\lambda_{max}$.
Thus red appearance of Betelgeuse implies it is a cool star while bluish appearance of Sirius implies a higher surface temperature

<u>Colors</u>.
Detectors work only over a limited range of λ's. In general fluxes are integrated over broad-band filters, just as the eye acts as a visual filter.

In standard UBV system apparent magnitudes are measured through three filters with sensitivity functions that look like this.




| | | | $\Delta\lambda$ (Å) |
|---|---|---|---|
| U | $\lambda_{max}(U) = 365$ nm | (3650 Å) | 680 |
| B | $\lambda_{max}(B) = 440$ nm | (4400 Å) | 980 |
| V | $\lambda_{max}(V) = 550$ nm | (5500 Å) | 890 |

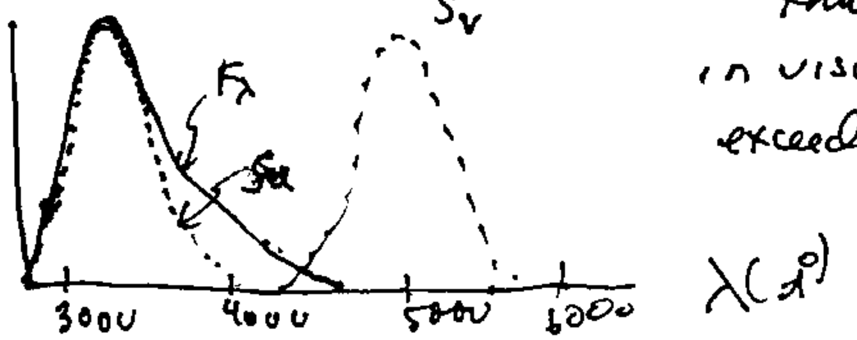
Flux in Visual band:

$$F_V = \int_0^\infty d\lambda\, S_V(\lambda)\, F_\lambda$$


$[F_\lambda] = $ ergs cm$^{-2}$ s$^{-1}$ Å$^{-1}$ $\qquad$ $[F] = $ ergs cm$^{-2}$ s$^{-1}$

$[S_V] = 1$

Effect of filters: Visual filter will miss much of the flux of a hot stars, while U filter will get it. But hot stars are 50× luminous that luminosity in visual band can exceed visual luminos of cool dwarfs.



visual mag. refer to stars with $m_V = 0$.
Such stars have visual fluxes:

$$F_V(0) = 3.2 \times 10^{-6} \ \text{erg/cm}^2\text{sec}$$

$$m_V = -2.5 \log F_V + C_V$$

$$\therefore m_V - 0 = -2.5 \log \frac{F_V}{F_V(0)} = -2.5 \log \frac{F_V}{3.2\times10^{-6}}$$

$$\boxed{m_V = -2.5 \log F_V - 13.74} \quad (C_V = -13.74)$$

Color independent of distance

$$m_B = -2.5 \log F_B + C_B$$
$$m_V = -2.5 \log F_V + C_V$$

$$V - B = m_V - m_B = -2.5 \log \frac{F_V}{F_B} + C_B - C_V$$

independent of distance since

$$\frac{F_V}{F_B} = \frac{L_V/4\pi d^2}{L_B/4\pi d^2} = L_V/L_B.$$

Color is a function only of surface temperature of star.

Large $V-B$ means cool star $\Rightarrow F_V/F_B$ small
Small $V-B$ " hot star $\Rightarrow F_V/F_B$ lg.

## Bolometric Corrections.

In real world we measure fluxes with some filter. But physically we want to know flux integrated over all ~~bands~~ wavelengths; i.e., bolometric flux. To do this we make corrections using a model for $F_\lambda$ or $F_\nu$.

## Bolometric Magnitude:

$$m_{bol} = -2.5\log\left(\int_0^\infty F_\lambda d\lambda\right) + C_{bol}$$

visual mag.: $m_V = -2.5\log\left(\int_0^\infty F_\lambda S_V(\lambda) d\lambda\right) + C_V$

therefore $m_{bol} - V = m_{bol} - m_V$

$$BC = m_{bol} - V = -2.5\log\left(\frac{\int_0^\infty F_\lambda d\lambda}{\int_0^\infty F_\lambda S_V(\lambda) d\lambda}\right) + C_{bol} - C_V$$

(1) Determine $C_{bol}$ from Sun where we know $M_{bol}$ and $\int_0^\infty F_\lambda d\lambda$

(2) $C_V$ is known $\begin{cases} C_V = -13.74 \\ C_B = -12.97 \\ C_U = -13.87 \end{cases}$

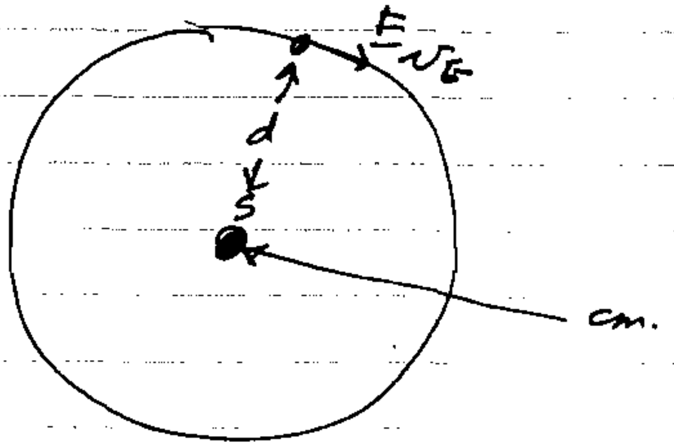
(3) So for a given star all we need to know is ratio of integrals or $\int F_\lambda S_V(\lambda) d\lambda \simeq F_\lambda(\lambda^V_{max}) \Delta\lambda_V$

In principle $BC < 0$ since $M_{bol} < m_V$.

# Mass determination

~~Binary~~

Sun-~~Earth~~ System




Newton's law for circular orbit

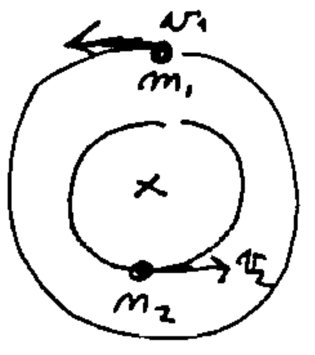
$$m_E \frac{v_E^2}{d} = \frac{G M_S M_E}{d^2}$$

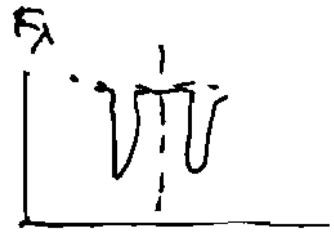
implies $\quad M_S = \frac{v_E^2 \cdot d}{G}$

$$= \frac{(30\times10^5 \,\text{cm/s})^2 (1.5\times10^{13})}{6.7\times10^{-8}}$$

$$M_2 = 2\times10^{33}\,g \quad \text{or} \quad 2\times10^{33}\,kg$$

## Binary Stars

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