2.35 (a) $\sigma_R = \frac{q}{4\pi R^2}$, $\sigma_b = -\frac{q}{4\pi b^2}$, $\sigma_a = \frac{q}{4\pi a^2}$

(b) $V_{ct} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{b} + \frac{1}{a}\right)$

(c) $\sigma_a = 0$ (otherwise, $\vec{E}$ lines go from a to $\infty \Rightarrow \Delta V_{a\to\infty} \neq 0$)

$$V_{ct} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{b}\right)$$

2.36 (a) Guess $\sigma_a = -\frac{q_a}{4\pi a^2}$, $\sigma_b = -\frac{q_b}{4\pi b^2}$,

$$\sigma_R = \frac{q_a + q_b}{4\pi R^2}$$

With such arrangement, the metallic part of the sphere is still an equipotential body. Then by uniqueness theorem, this arrangement is <u>the</u> solution.

(b) For outside, σ_a & q_a cancel, σ_b & q_b cancel,

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q_a + q_b}{r^2}\hat{r} \quad (r > R)$$

(c) For cavity a, σ_b & q_b cancel, σ_R & σ_a have no effect,

$$\Rightarrow E_a = \frac{1}{4\pi\varepsilon_0}\frac{q_a}{r_a^2}\hat{r}_a, \quad r_a < a$$: with q_a at origin.

Similar argument for b,

$$\Rightarrow E_b = \frac{1}{4\pi\varepsilon_0}\frac{q_b}{r_b^2}\hat{r}_b, \quad r_b < b$$: with q_b at origin.

(d). For q_a, σ_a & σ_R exert no force.
σ_b & q_b cancel

$\Rightarrow F_{q_a} = 0$. Similarly, $F_{q_b} = 0$.

(e). (a): σ_R will change. (Polarization)
others don't change due to screening provided by σ_R

(b): will change b/c field lines will now start/end on q_c

(c) & (d): won't change due to screening by σ_R

2.37. $\vec{E}$ field due to one plate is

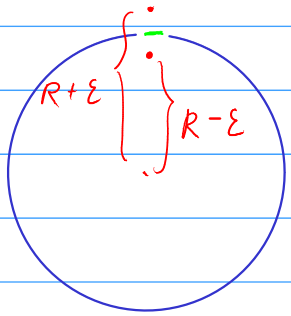
$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}.$$

$\Rightarrow$ Force on one plate due to field of the other is

$$F = \sigma \cdot E \cdot A$$

$$\Rightarrow P = \frac{F}{A} = \sigma E = \frac{Q^2}{2\varepsilon_0 A^2}$$

2-38 NB: force "between" hemispheres is <u>not</u> the force on one hemisphere exerted <u>by the other</u>, but simply the force on one semisphere (part of the force will come from its own charge).



First calculate field <u>on</u> shell.

For $\varepsilon \to 0$,

@ $r = R - \varepsilon$, $E = 0 = E_{blue} - E_{green}$.

@ $r = R + \varepsilon$, $E = \dfrac{Q}{4\pi\varepsilon_0 R^2} = E_{blue} + E_{green}$.

$$\Rightarrow E_{blue} = E_{green} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2}$$

@ $r = R$, the field <u>on</u> the green piece is $E_{blue} = \dfrac{1}{2}\dfrac{Q}{4\pi\varepsilon_0 R^2}$

$\Rightarrow$ E field <u>on shell</u> is

$$E_{shell} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2}\hat{r} \equiv E_0 \hat{r}$$

$\Rightarrow$ force on hemisphere is

$$F = \sigma \int (\vec{E} \cdot \hat{z})\, dA$$

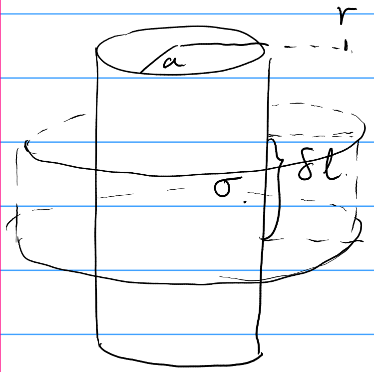
$$= \sigma E_0 \int_0^{\frac{\pi}{2}} \underbrace{\cos\theta \cdot 2\pi R \sin\theta \cdot R\, d\theta}_{= 2\pi R^2 \sin\theta\, d\sin\theta}$$

$$= \sigma E_0 \cdot \pi R^2$$

$$= \frac{Q}{4\pi R^2} \cdot \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2} \cdot \pi R^2$$

$$= \frac{Q^2}{32\pi\varepsilon_0 R^2}$$

2.39



$$\underbrace{\sigma \cdot 2\pi a \,\delta l}_{\delta q} = \epsilon_0 E(r) \cdot 2\pi r \,\delta l.$$

$$\Rightarrow \quad E(r) = \frac{a\sigma}{\epsilon_0 r}$$

$$\Rightarrow |V_{a\to b}| = \left| \int_a^b E \cdot dr \right|$$

$$= \frac{a\sigma}{\epsilon_0} \log \frac{b}{a}$$

$$\Rightarrow \quad C = \frac{Q}{V\delta l} = \frac{\sigma \cdot 2\pi a \cdot \delta l}{\frac{a\sigma}{\epsilon_0} \log \frac{b}{a}\, \delta l} = \frac{2\pi \epsilon_0}{\log b/a}$$

2.40. (a) $P = \frac{\epsilon_0}{2} E^2 \Rightarrow \quad W = P \cdot A \cdot \varepsilon = \frac{1}{2} \varepsilon_0 E^2 \cdot A \cdot \varepsilon.$

(b) $w = \frac{1}{2} \epsilon_0 E^2 \Rightarrow E_{cost} = w \cdot Vol_{cost} = w \cdot A \cdot \varepsilon$

$$= \frac{1}{2} \varepsilon_0 E^2 A \varepsilon$$