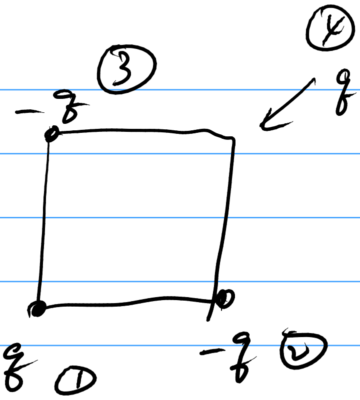
2.31 (a) $U = \frac{1}{4\pi\varepsilon_0}\cdot\left(\underset{1}{\frac{q^2}{\sqrt{2}a}} - \underset{2\,\&\,3}{\frac{2q^2}{a}}\right)$



$$= -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(2-\frac{1}{\sqrt{2}}\right)$$

(b) $U = \frac{1}{4\pi\varepsilon_0}\sum_{i<k}\frac{q_i q_k}{r_{ik}}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\cdot\left(\underset{12}{-1}\ \overset{13}{-1} + \underset{14}{\frac{1}{\sqrt{2}}} + \underset{23}{\frac{1}{\sqrt{2}}}\ \overset{24}{-1}\ \underset{34}{-1}\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(4-\sqrt{2}\right)$$

2.32 (a). $V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{2R}\left(3-\frac{r^2}{R^2}\right)$; $\rho = \frac{3q}{4\pi R^3}$

$$\Rightarrow U = \frac{1}{2}\int_0^R \rho\, V(r)\, 4\pi r^2\, dr = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3q^2}{5R}$$

(b). $E(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cdot\frac{r^3}{R^3} & r<R \\ \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} & r>R \end{cases}$

$$W = \frac{\varepsilon_0}{2}\int_0^\infty E^2\cdot 4\pi r^2\, dr$$

$$= \int_0^R + \int_R^\infty = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{10R}+\frac{q^2}{2R}\right) = \frac{1}{4\pi\varepsilon_0}\frac{3q^2}{5R}$$

(C). Assume $a > R$.

$$W = \frac{\varepsilon_0}{2}\left[\underbrace{\int_0^a E^2 \cdot 4\pi r^2 dr}_{I} + \underbrace{E(a)\cdot V(a)\cdot 4\pi a^2}_{\frac{1}{(4\pi\varepsilon_0)^2}\frac{q}{a^2}\cdot\frac{q}{a}\cdot 4\pi a^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0^2}\frac{q^2}{a}$$

$$I = \int_0^R + \int_R^a = \frac{q^2}{4\pi\varepsilon_0^2}\left(\frac{1}{5R} - \frac{1}{r}\Big|_R^a\right)$$

$$= \frac{1}{4\pi\varepsilon_0^2}\left(\frac{1}{5R} + \frac{1}{R} - \frac{1}{a}\right)$$

$$\Rightarrow W = \frac{1}{4\pi\varepsilon_0}\frac{3q^2}{5R^2}$$

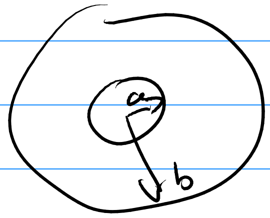
2.33 $V(r) = \frac{q}{4\pi\varepsilon_0}\frac{r^2}{R^3}$; $dq(r) = 4\pi r^2 dr \cdot \underbrace{\frac{3q}{4\pi R^3}}_{\rho}$

$$\Rightarrow W = \int_{r=0}^{R} V(r)\, dq(r)$$

$$= \frac{q}{4\pi\varepsilon_0 R^3}\cdot\frac{3q}{R^3}\int_0^R r^4 dr$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3q^2}{5R}$$

2.34. (a). $\vec{E}$ only between shells.



$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}, \quad r \in [a, b]$$

$$\Rightarrow W = \frac{\varepsilon_0}{2}\frac{q^2}{(4\pi\varepsilon_0)^2}\int_a^b \frac{1}{r^4}\cdot 4\pi r^2\, dr$$

$$= \frac{q^2}{8\pi\varepsilon_0}\cdot\left(\frac{1}{a} - \frac{1}{b}\right)$$

(b). Inner shell: Outer shell:

$$E_1 = \begin{cases} 0, & r < a \\ \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} & r > a \end{cases} \qquad E_2 = \begin{cases} 0, & r < b \\ \frac{-1}{4\pi\varepsilon_0}\frac{q}{r^2} & r > b \end{cases}$$

$$E_1 \cdot E_2 = \begin{cases} 0, & r < a \\ 0, & r \in (a, b) \\ -\frac{1}{(4\pi\varepsilon_0)^2}\frac{q^2}{r^4} & r > b \end{cases}$$

$$\Rightarrow W_{12} \equiv \epsilon_0 \int E_1 \cdot E_2\, d\tau = \varepsilon_0 \int_b^\infty -\frac{1}{(4\pi\varepsilon_0)^2}\frac{q^2}{r^4}\cdot 4\pi r^2\, dr$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{b}.$$

Ex 2.8: $W_1 = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{a}, \quad W_2 = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{b}$

$$\Rightarrow W = W_1 + W_2 + W_{12} = \frac{1}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$