Lecture 21

The Expanding Universe

Outline of Lecture 21

- Hubble's Law:
  - Apparent velocity of recession of distant galaxy is proportional to its distance from us.
  - Value of proportionality constant, Hubble's constant, is equal (today) to 22 km/s per Mly.
- Cosmological Principle:

  To an observer in any typical galaxy, the universe appears at any instant of time to be
  - Homogeneous (same at all points)
  - Isotropic (same in all directions)
- Newtonian Cosmology:
  - Bound model: mass density is greater than critical value
  - Critical model: mass density is equal to critical value
  - Unbound model: mass density is less than critical value

Redshifts of Galaxies

- In 1912 Vesto Slipher (1875-1969) discovers that spiral “nebulae” generally have lines in their spectra with wavelengths λ that are typically redshifted from their rest values λ_0 .
- Nomenclature:

$$1+z=\frac{\lambda}{\lambda_0} \Rightarrow z=\frac{\lambda-\lambda_0}{\lambda_0} \equiv \frac{\Delta\lambda}{\lambda_0}.$$

- Normal interpretation:

  non-relativistic Doppler shift at recessional velocity v: $z = \mathrm{v}/c$.

- Example: Suppose have Hα and H β

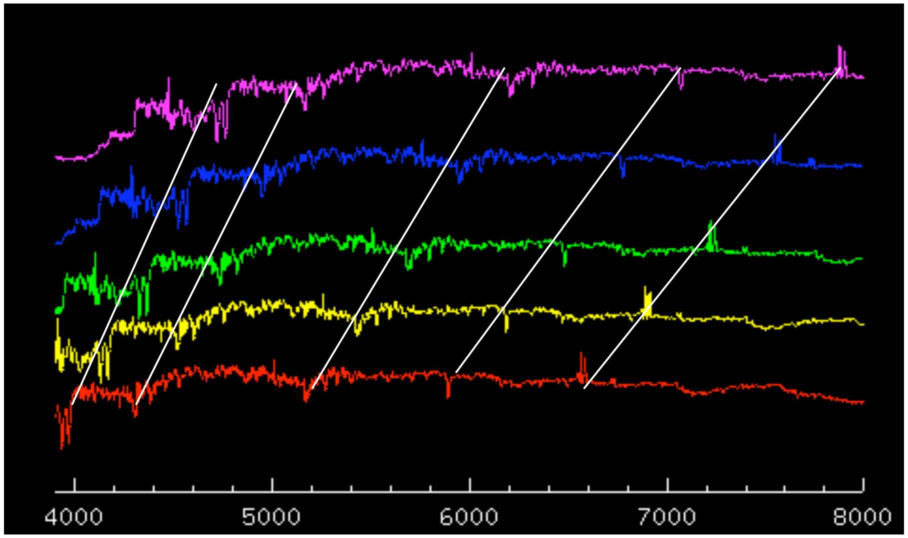
  In galaxy: $\lambda = 787.6 \text{ and } 583.3 \text{ nm}$

  In laboratory: $\lambda_0 = 656.3 \text{ and } 496.1 \text{ nm}$

$$\Delta\lambda = 131.3 \text{ and } 97.2 \text{ nm}$$

$$z = \Delta\lambda / \lambda_0 = 0.20 \quad \text{and } 0.20$$

- Checks; thus, $\mathrm{v} = 0.20c = 60{,}000 \text{ km s}^{-1}$.



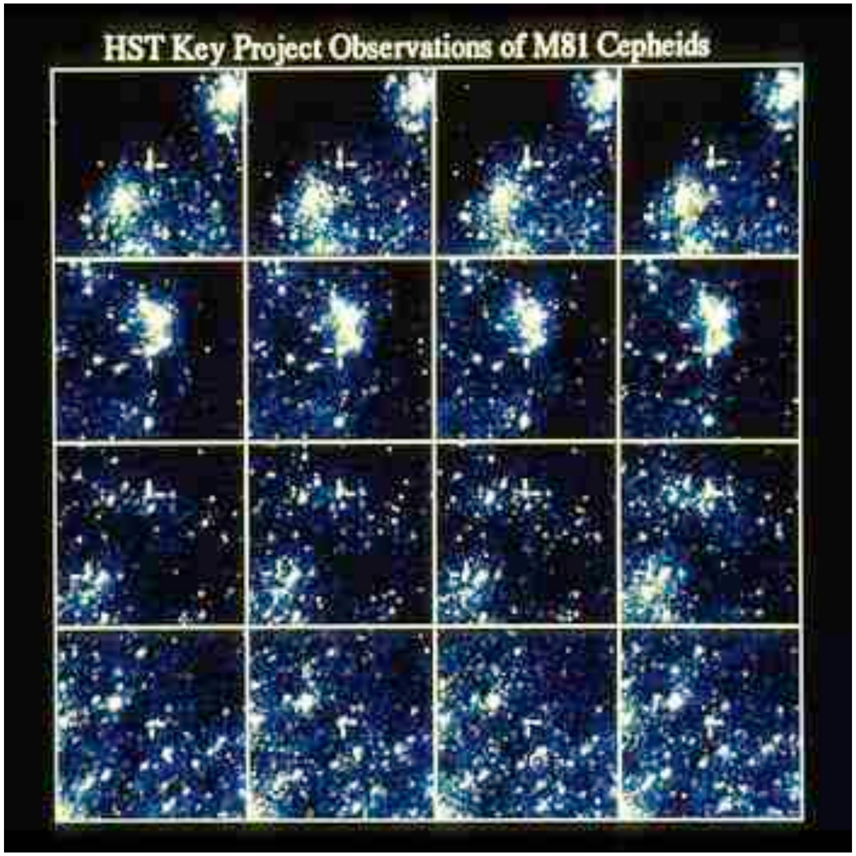
Spectra (λ in Angstroms) of galaxies at redshifts (bottom to top) 0.00, 0.05, 0.10, 0.15, and 0.20. White lines trace progression of particular feature. Note that apparent slope increases for increasing rest wavelength, reflecting that ratio $\lambda / \lambda_0 = 1 + z$ is staying the same for each galaxy.

Sloan Digital Sky Survey

Distances of Galaxies

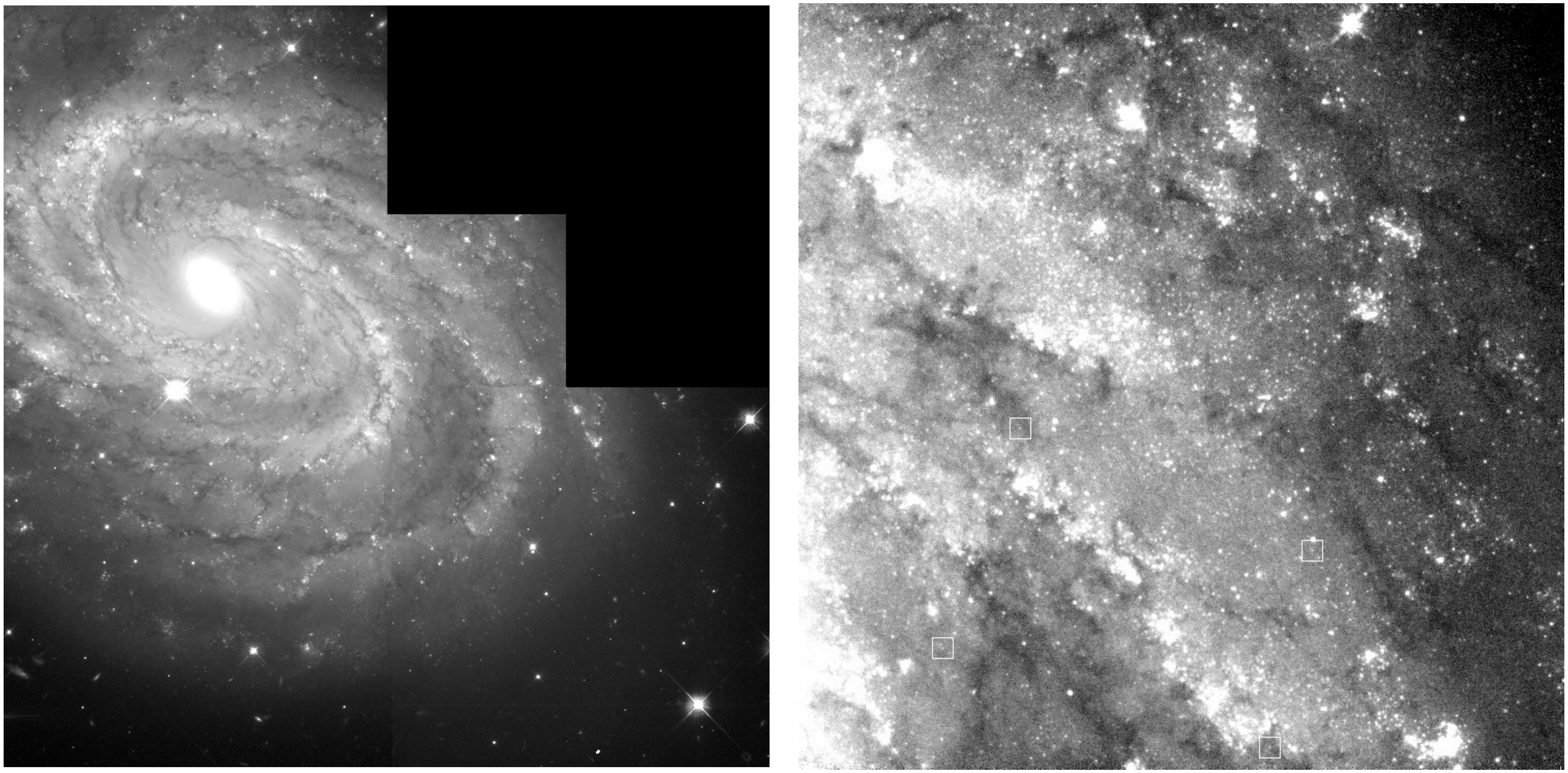
Cepheids in M81 Yield Its Distance as 12 Mly

Photo Credit: R. Gendler



NASA/HST

At 108 Mly, NGC 4693 is as Far as the Cepheid Method Will Work



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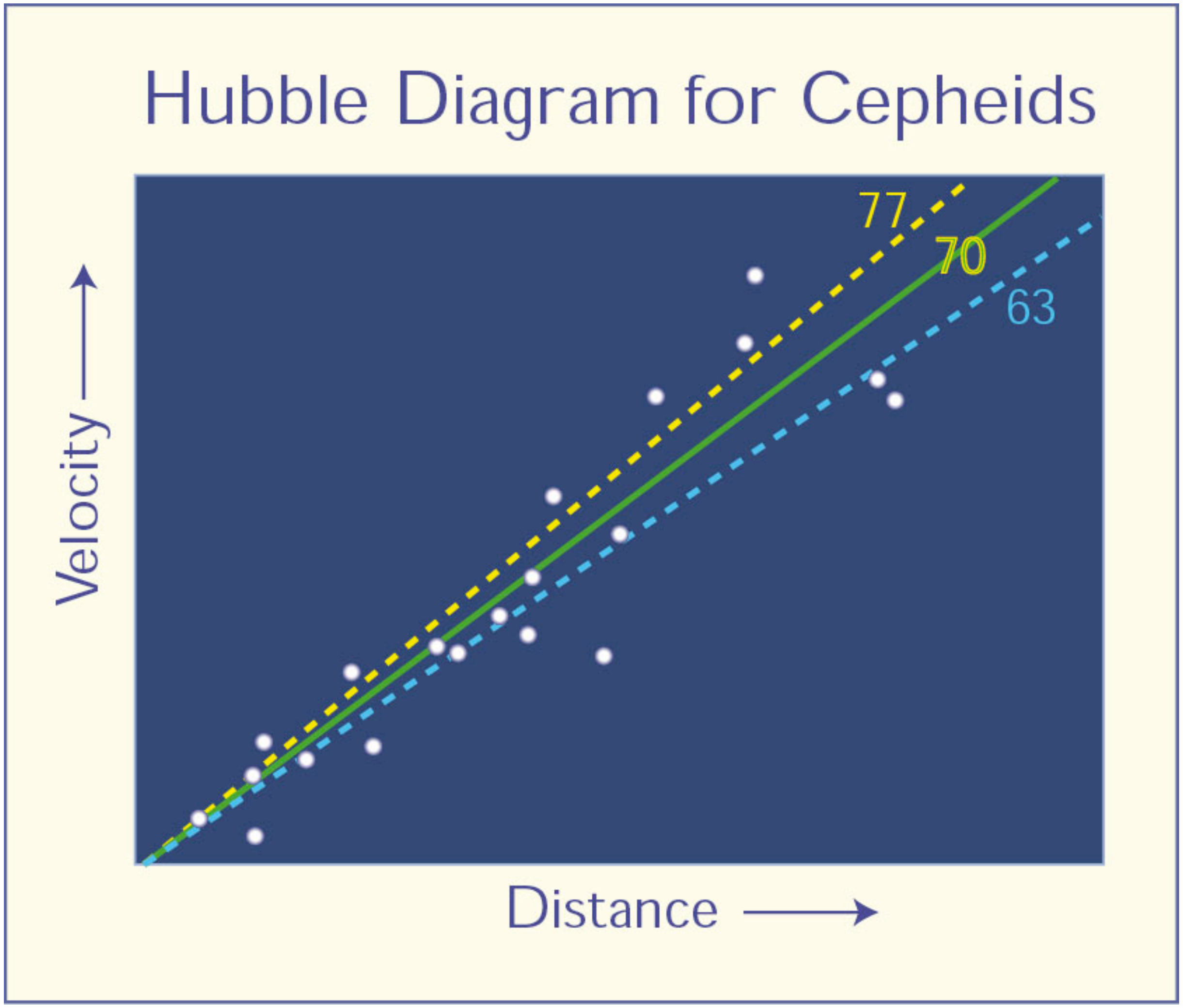
Hubble's Law: *v* = *Hr* (discovered in 1929)



$$H_0 = 70 \text{ km/s per Mpc}$$
$$= 22 \text{ km/s per Mly}$$

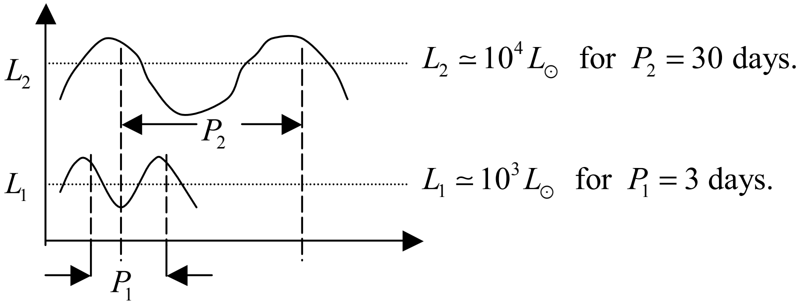
Mpc = 3.26 Mly

Historically, determination of *H* involved many mistakes and controversies. Next two slides illustrate how systematic errors were eventually overcome.

Value of Hubble's Constant from Distance of Virgo Cluster Needs to Account for Local Infall

- Method of "standard candles": $f = \dfrac{L}{4\pi r^2}$,

  where *L* = luminosity of spherically symmetric or transparent source and *r* = distance. If we can calibrate *L*, a measurement of *f* (radiant energy falling per s on a unit area of detector) then yields *r*.
- Example: Period-Luminosity of Cepheid variable:



$L_2 \simeq 10^4 L_\odot$ for $P_2 = 30$ days.

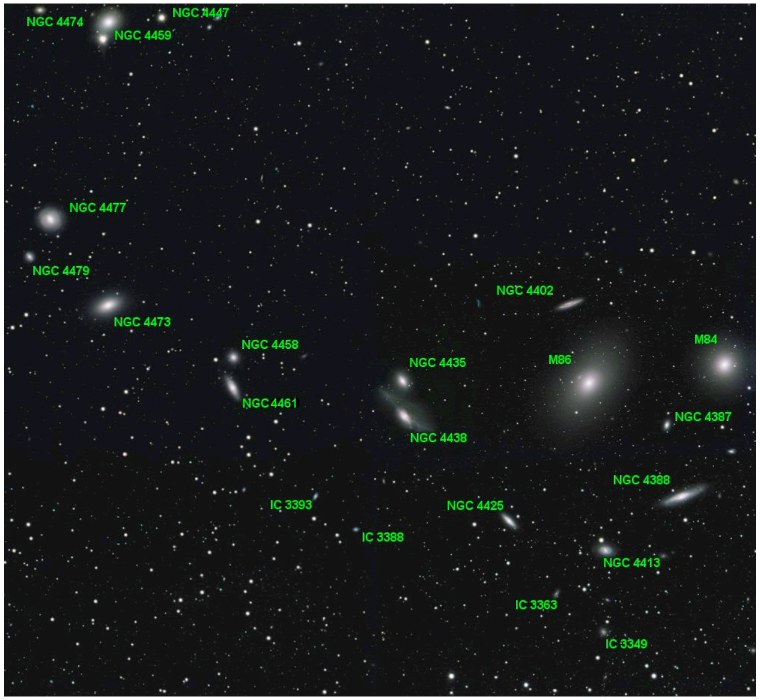
$L_1 \simeq 10^3 L_\odot$ for $P_1 = 3$ days.

- For a Cepheid with P = 30 days in Virgo cluster galaxy, measure $f \approx 1\times10^{-18}$ watts m^{-2} (detectable by HST in a few hours). With

  $$L = 10^4 L_\odot = 4\times10^{30} \text{ watts},$$

  we get for the *distance to the Virgo cluster*:

  $$r = \left(\frac{L}{4\pi f}\right)^{1/2} \approx 6\times10^{23}\text{ m} \approx 60\text{ Mly}.$$
- Average recession velocity of Virgo galaxies: v = 1000 km/s. However, cannot derive accurate *H* from this combination v/*r*.



Virgo cluster at a distance of ~ 60 Mly is the nearest large collection of some 1000 spirals and ellipticals. Since our Galaxy is moving toward Virgo at ~ 300 km/s with respect to the Hubble flow (as deduced from Hubble's constant; see next slide), it will probably become a member of this cluster in the distant future.

Expansion of the Universe

- The farther away in distance r is a galaxy from us, the faster its speed of recession v. The relationship between v and r is linear:

$$v = Hr.$$

- The distance of the Virgo cluster, 60 Mly, is not great enough for the cosmic expansion to completely overcome deviation of velocities of galaxies due to local gravitational excesses or deficits (hundreds of km/s).
- The relative apparent brightnesses of comparable galaxies in Coma and Virgo suggest that the Coma cluster is 5.5 times farther away, i.e., at a distance of r = 330 Mly.
- The mean velocity of recession of galaxies in Coma is

  v = 7000 km/s, large enough to make random velocities of hundreds of km/s negligible. From this data on v and r for Coma, we derive

$$H = v/r = 22 \text{ km/s per Mly.}$$

The Coma cluster contains about 2000 galaxies and, at a distance of 330 Mly, is the nearest rich galaxy cluster to us.

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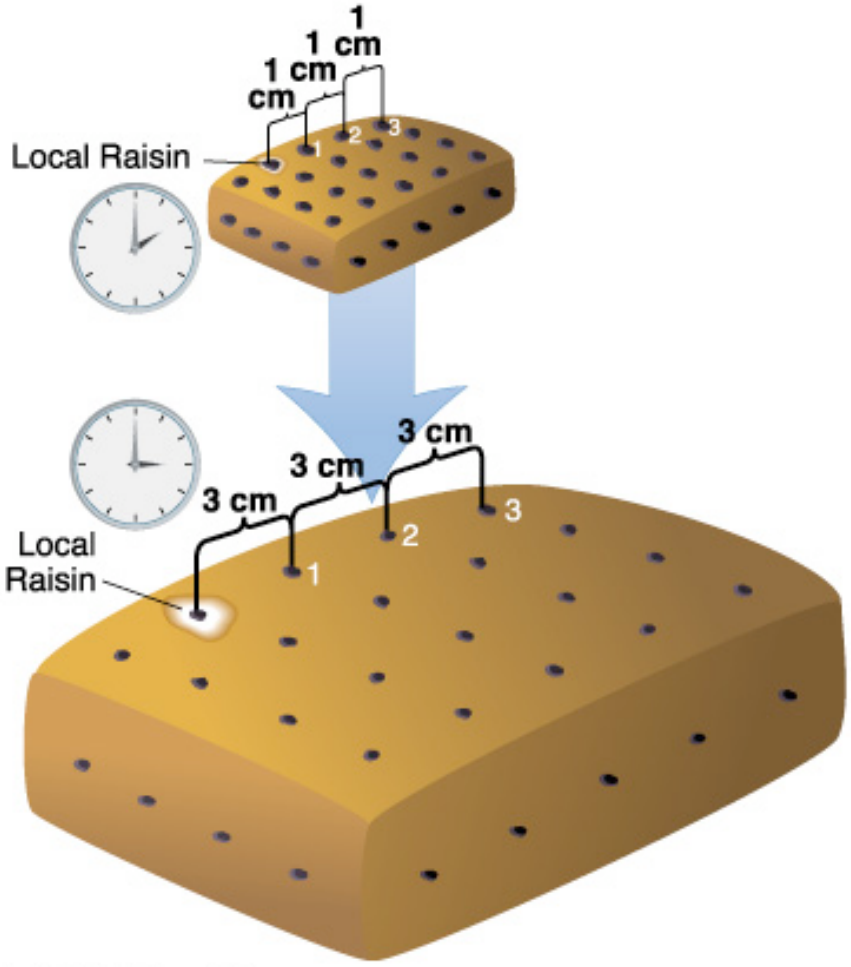
Age of the Universe

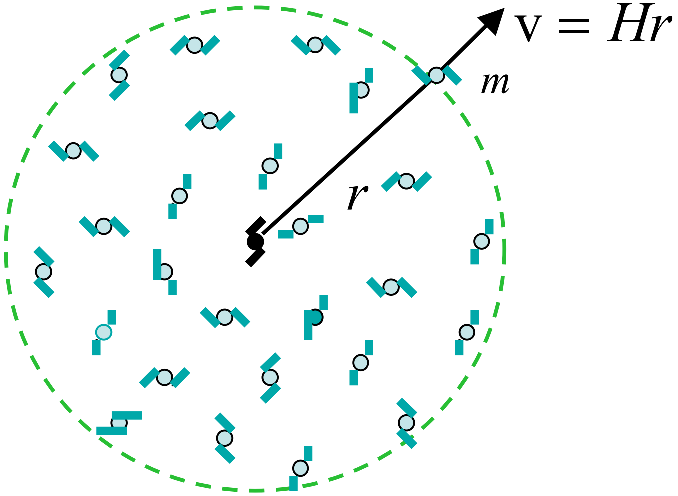
- Hubble's law (1929): $v = Hr$, where v = recessional velocity of galaxy, r = its distance from us, H = Hubble's "constant."
- Best determination: H = 22 km/s per Mly (1 Mly $\approx 10^{19}$ km).
- Naive interpretation: explosion from single point, with us at center and with no subsequent acceleration or deceleration. If t = time passed between moment of big bang and now, then $r = vt$ is distance of any object traveling at speed v, which is Hubble's law if we identify $t = 1/H$. (Note: H is generally a function of cosmic time t. Number 22 km/s per Mly is its current value H_0.)
- Thus, naive interpretation gives current age of universe as t = 1 Mly divided by 22 km/s = 14 billion years.
- Problems with naïve interpretation:
  - Need for us to be at center of explosion?
  - What happens at large r when Hubble's law, $v = Hr$ predicts $v > c$, the speed of light?
  - Deceleration produced by gravitation of all of the matter contained in galaxies?
- Adequate resolution of these difficulties requires Einstein's theory of general relativity combined with cosmological principle (Lecture 22).
- Cosmological principle (an assumption): On average, universe is homogeneous and isotropic when viewed at any instant by an observer in a typical galaxy.

Cosmological Principle as Illustrated by Rising Raisin Cake



Newtonian Cosmology

- Birkhoff's rule: For a homogeneous density distribution ρ(*t*), the effect of gravitation on a galaxy *m* at a small distance *r* from an observer in any other galaxy can be computed by Newtonian rules considering only the mass *M* within radius *r*.



$$M = \rho \frac{4\pi r^3}{3} = const$$

$$E = \frac{1}{2} m \mathrm{v}^2 - \frac{GMm}{r} = const$$

$$= \frac{1}{2} m r^2 \left(H^2 - \frac{8\pi G}{3} \rho \right)$$

where $\mathrm{v} \equiv \frac{dr}{dt} = Hr.$

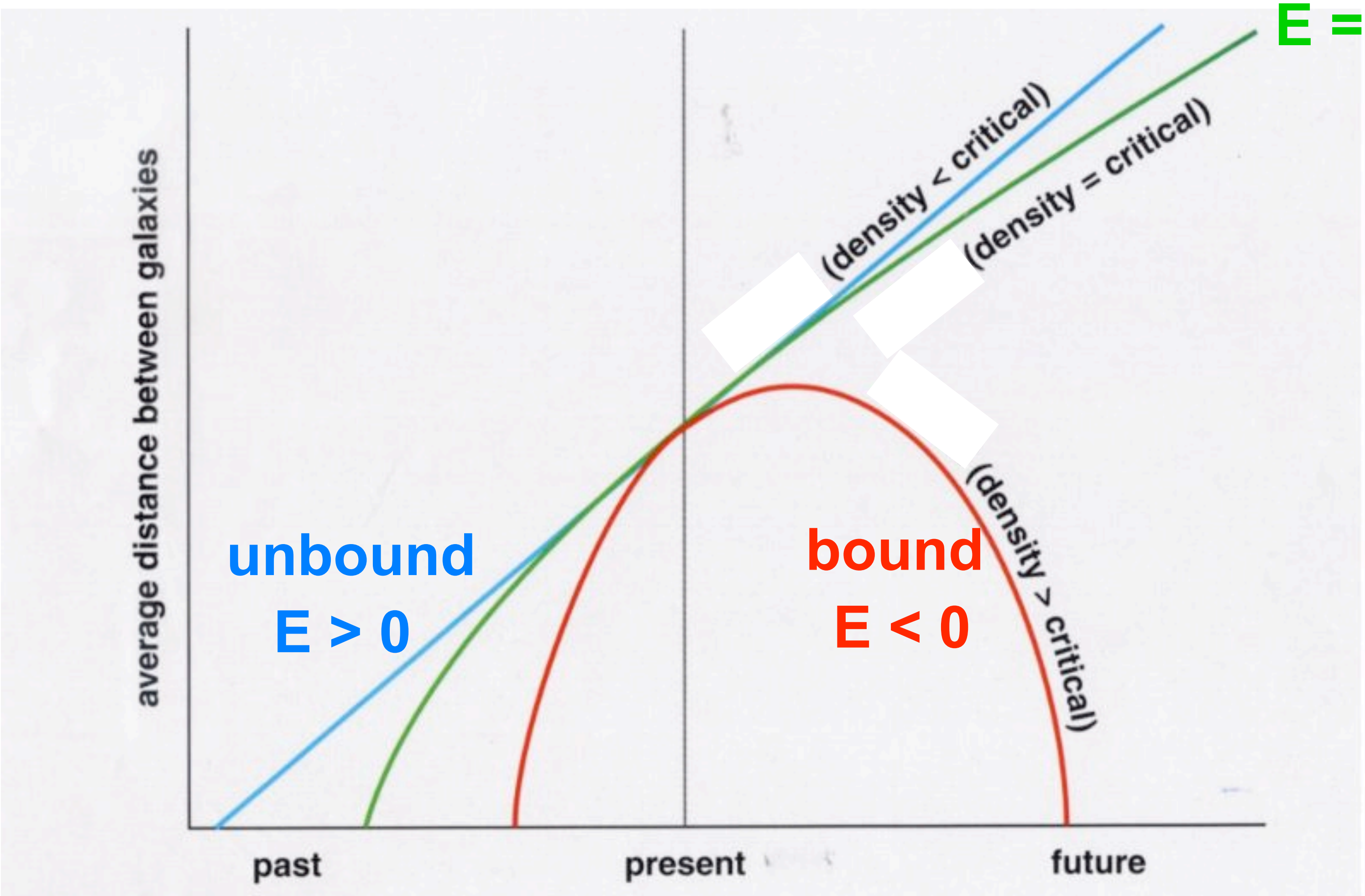
$>$ unbound

- Three cases: $E = 0$ critical → critical density $\rho_c = \frac{3H^2}{8\pi G}.$

$<$ bound

- Numerically, for *H* = 22 km/s per Mly, $\rho_c = 9 \times 10^{-27}\ \mathrm{kg\ m^{-3}}.$

Conventional Cosmological Models & Age of the Universe



Comparison of Actual Density to Critical Density

- The critical density is almost 29 orders of magnitude smaller than the density of liquid water, yet it is about 20-25 times larger than the density of luminous matter in galaxies if this matter were to be spread out smoothly and uniformly throughout space.
- For the universe to be critical, there has to be a lot of unseen matter or energy (dark matter and dark energy).
- Empirical evidence for dark matter concentrated in the halos of galaxies appears in the rotation curves of spiral galaxies. Their outer parts rotate at speeds that would cause them to fly apart unless there is much more gravitational binding mass than exists in the form of luminous matter observed as stars and gas clouds (Lectures 19 and 20). But dark matter is probably only 5 or 6 times as much as ordinary matter, not 20-25 times larger.
- Is there also unseen dark energy, spread smoothly through the universe, and not clustered where matter (ordinary and dark) clusters? (Answer in a future Lecture.)

Difficulties with “Newtonian Cosmology”

- Arbitrary nature of Birkhoff’s rule (in the context of Newtonian theory) to account for (local) effects of gravitation.
- Globally, cannot avoid issue of galaxies separating from each other at arbitrarily high speeds, a result forbidden by the theory of special relativity.
- E = 0 (critical case) is preferred; otherwise, would need to explain from where energy for expansion came? But then the age of the universe turns out to be $(2/3)H^{-1}$, or 9 billion years, in conflict with the ages of the oldest stars in the Milky Way Galaxy (about 13 billion years for globular clusters) as computed from stellar evolution theory (Lecture 17). How can the universe be younger than the stars inside it?
- These issues will be attacked next time when we deal with relativistic cosmology.

Derivation of Age of Critical Universe (extra material)

$E = \frac{m}{2}\left(\frac{dr}{dt}\right)^2 - \frac{GMm}{r} = 0$ for a critical universe,

with $M = \rho \frac{4\pi}{3} r^3 =$ constant. Thus,

$r^{1/2} dr = +\sqrt{2GM}\ dt$ for expanding universe where r increases when t does. Solution with big-bang initial condition $r = 0$ at $t = 0$ reads

$$\frac{2}{3} r^{3/2} = \sqrt{2GM}\ t \Rightarrow r = \left(\frac{9}{2} GM\right)^{1/3} t^{2/3}.$$

This solution yields following relation between H and t:

$H \equiv \frac{1}{r}\frac{dr}{dt} = \frac{2}{3t}$. Thus, $\boxed{t = \frac{2}{3} H^{-1}.}$