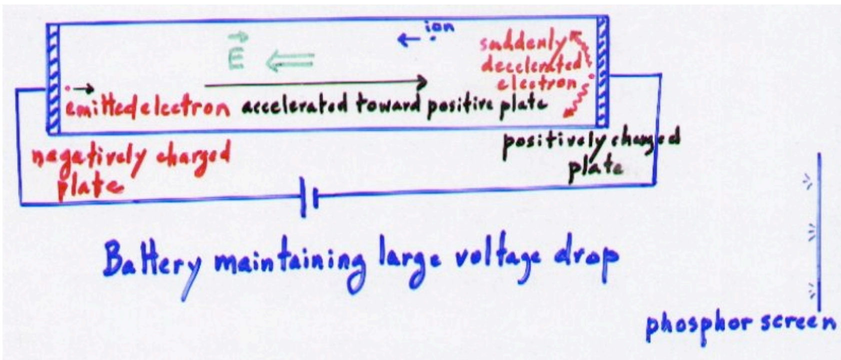
Lecture 11

The Quantum Revolution

Outline of Lecture 11

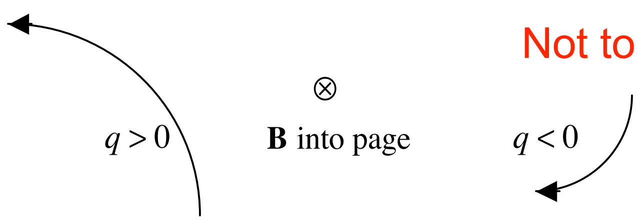
- Structure of Atoms
  - J. J. Thomson finds that electrons and protons comprise the negatively and positively charged compnents of atoms.
  - Marie Curie classifes three types of energetic particle emissions from radioactive elements.
  - Ernest Rutherford uses Curie's alpha particles to show that protons are confined to a small nucleus while electrons circulate the outside of an atom. Classically, such a model is unstable mechanically and radiatively. The classical mechanics that suffices to explain macroscopic objects breaks down for microscopic atoms and molecules.
- The quantum revolution
  - To explain blackbody radiation, Max Planck finds that it necessary to give radiating, oscillating charges quantized energy levels characterized by a new fundamental constant of nature: $h = 6.63 \times 10^{-34}$ joule s, $\hbar \equiv h / 2\pi$.
  - To explain the photoelectric effect, Albert Einstein finds that light must come in quantized packets of energy, now called photons. Light is both a wave and a particle.
  - To explain the mechanical stability of the hydrogen atom and its pattern of spectral lines, Niels Bohr applies the quantum hypothesis of Planck and Einstein to the allowed orbits of an electron and the transitions between them.
- Some astronomical applications of spectroscopy

Discovery of the Electron



- Reprise: Roentgen's discovers X-rays using a plasma tube.
- Thomson superimposes a magnetic field ***B*** going into the page and finds negative charges (electrons) to be bent in much smaller, tighter circles than positive charges (ions).

Not to scale!



- If $q = -e$ and m_e are charge and mass of electron and v is its speed, the radius of curvature r is given by

$$r = \frac{m_e c v}{eB}.$$

- Knowing the values of r, B, and v allows J. J. Thomson (1856-1940) to calculate the ratio of the electron's charge to mass $-e/m_e$.
- Later, Millikan in his famous oil-drop experiment determines the value of e (the same value as we stated for Faraday's law of electroplating when Avogadro's number is known). The mass of the electron then turns out to have the value:

$$m_e = 9.11 \times 10^{-31} \text{ kg}.$$

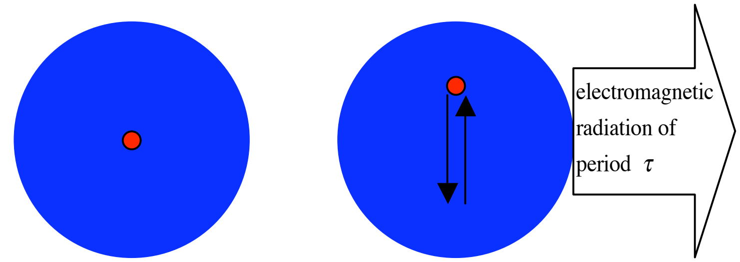
Note: same technique applied to positive ions provide basis for mass-spectrometer.

Inference for Proton and Model of Thomson Atom

- Thomson knew that the electrolysis of water produced positive (hydrogen) ions that had a charge to mass that he estimated was ~ 1700 (now known to be 1836) times less than that of the electron.
- Thus, it was natural to suppose that the hydrogen atom was composed of an electron and a positive charge, today called a proton, whose charge and mass were given by $q = +e$ and $m_p = 1836\, m_e = 1.67 \times 10^{-27}$ kg.
- This is consistent with 1 gm of hydrogen being one mole or N_A atoms:

$$1 \text{ gm}/6.02 \times 10^{23} = 1.67 \times 10^{-27} \text{ kg}$$

$$= m_\text{p} + m_\text{e}$$

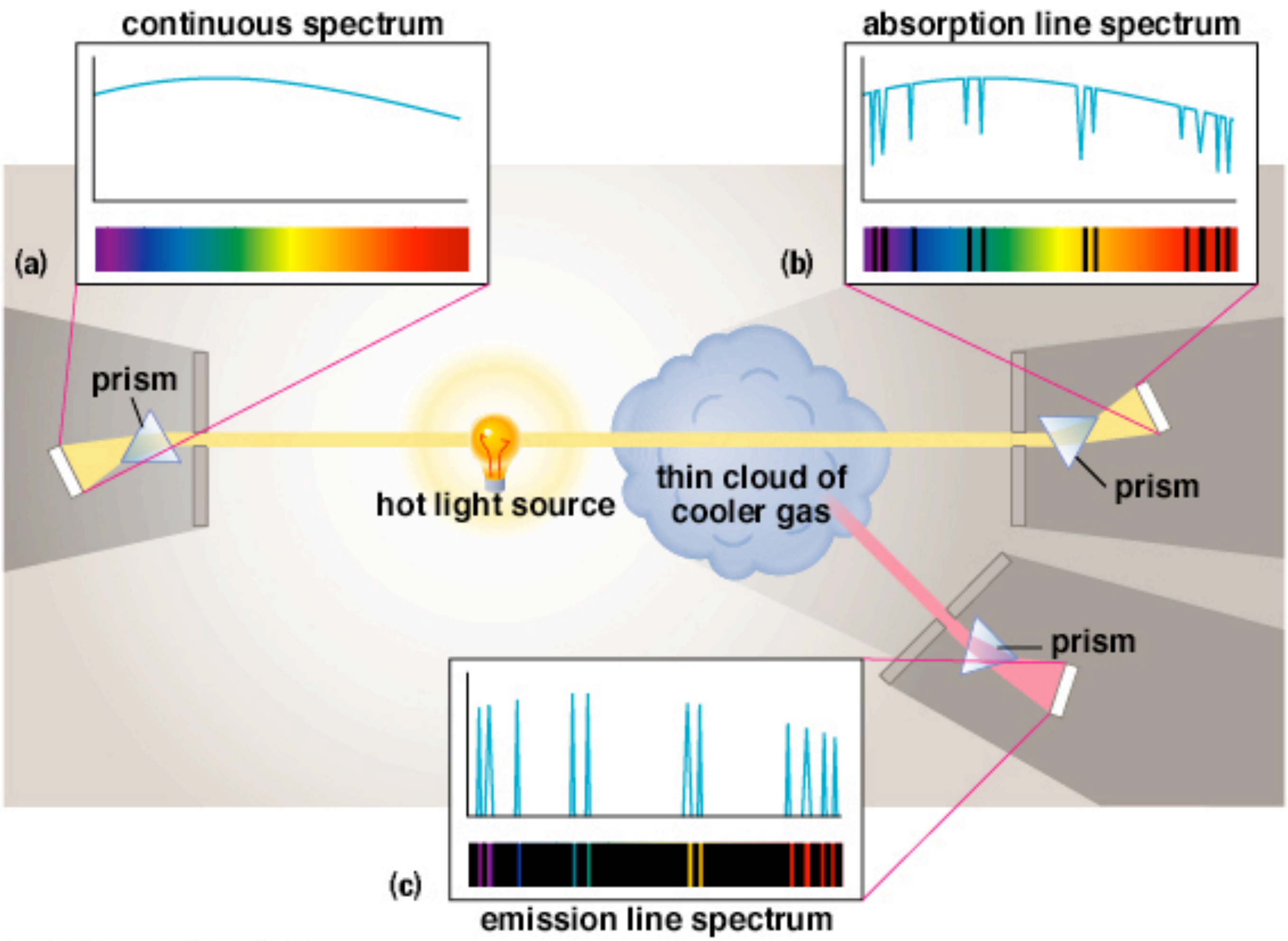


Thomson model of (hydrogen) atom: Small negative electron embedded in large mass of positive charge that contains most of the mass of the atom.

Electron is usually stationary in interior of atom, but if something (which could be a passing light wave) causes it to vibrate, then the oscillating electron can emit, absorb, or scatter radiation.

At first glance, above description appears compatible with Kirchhoff's empirical laws of radiation.

Kirchhoff's Laws of Radiation



Spectrum of Atomic Hydrogen

- Janne Rydberg (1854-1914) finds hydrogen line spectrum empirically satisfies the intriguing integer relationships:

$$\frac{1}{\lambda} = \frac{1}{\lambda_R}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right),$$

where $n_1 = 1, 2, 3, \ldots$ and $n_2 = n_1+1, n_1+2, n_1+3, \ldots$ with

$$\lambda_R = 91.1 \text{ nm}.$$

- For example, with $n_1 = 2$ and $n_2 = 3$, we have Balmer alpha ($H\alpha$) line at $\lambda = 656$ nm; with $n_1 = 2$ and $n_2 = 4$, we have $H\beta$ line at $\lambda = 486$ nm; etc.

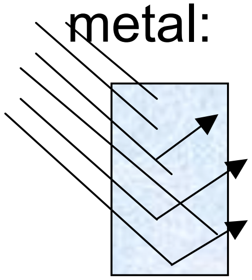
Emission spectrum

Absorption spectrum

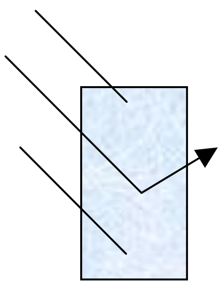
Optical (Balmer) lines of atomic hydrogen

Photoelectric Effect

- Shine light of short enough wavelength on a piece of metal:

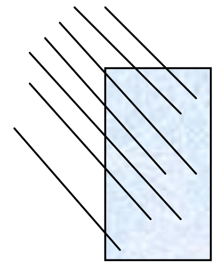


With high light intensity, get many ejected electrons.



With low light intensity, get few ejected electrons.

- Shine light of long wavelength on same piece of metal:



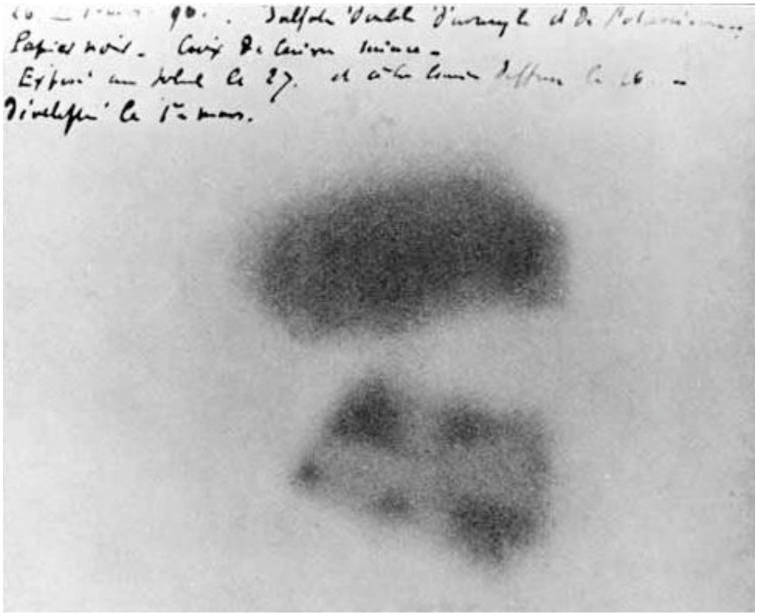
No matter how high is the light intensity, get no ejected electrons.

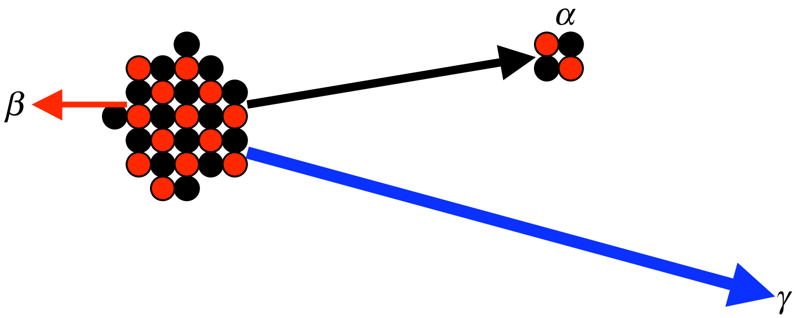
- Conclusion: When λ is too large, light does not have enough energy, in some sense, to eject electrons from metal, no matter how intense is the beam of light.

Einstein in 1905 proposes that light of wavelength λ consists of individual packets of energy, with each photon carrying energy: $E = hc/\lambda$ where h is the constant that Planck introduced (from the quantization of the energy of matter oscillators) in the theory of blackbody radiation.

Discovery of Radioactivity

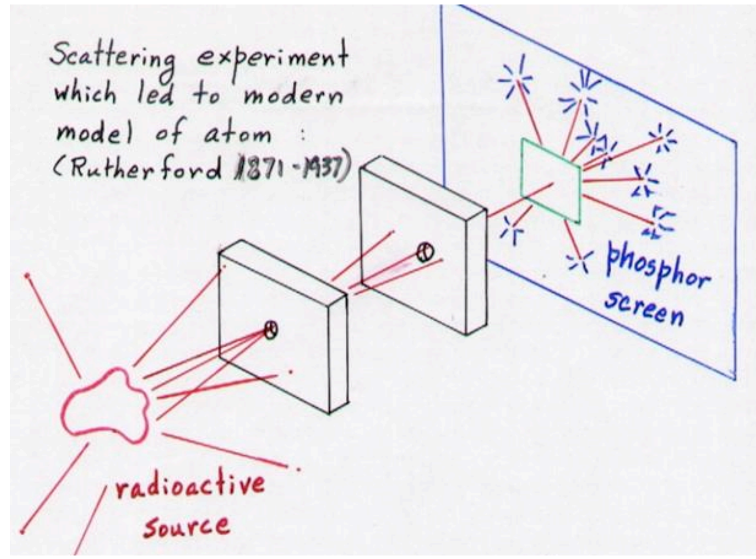
- Henri Becquerel (1852-1908) thinks, incorrectly, that X-rays might have something to do with fluorescence of uranium salts. Story of fogging of photographic plates.
- Marie Curie (1867-1934) classifies three basic kinds of emission from radioactive elements: positive α (helium nuclei), negative β (electrons), and neutral γ (high-energy photons).
- Only person to have won Nobel Prizes both in Physics (awarded before her PhD!) and Chemistry. Only person whose husband, daughter, and son-in-law also won Nobel Prizes.
- She and her husband Pierre deliberately never applied for any patents because they felt scientific knowledge should be free for the betterment of humanity.
- During Wolrd War I, she trained nurses to use X-ray machines to find bullets in wounded soldiers.

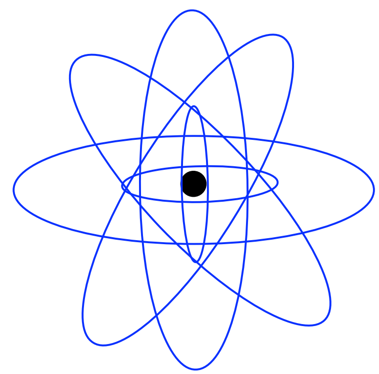




Rutherford Scattering Experiment

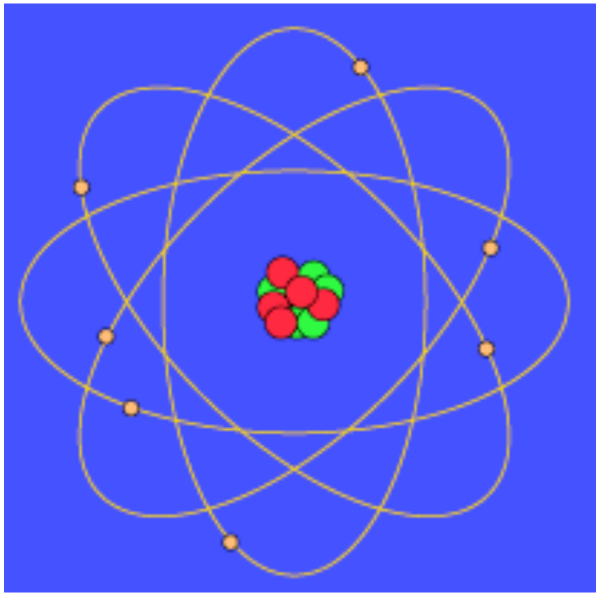
- Use positively charged alpha particles from radioactive source.
- Scatter alpha particles against thin foil of gold.
- Find many large-angle scatterings, which is completely unexpected on basis of Thomson model of atom where positive charge is distributed over entire (large) volume of an atom. As surprising as if “one fires a machine gun at tissue paper and find bullets bouncing back at oneself.”
- Positive repelling charges (protons) must reside in small, hard kernel (nucleus) of atom, which has most of mass, with compensating negative charges (electrons) circulating the outside, which has most of volume.

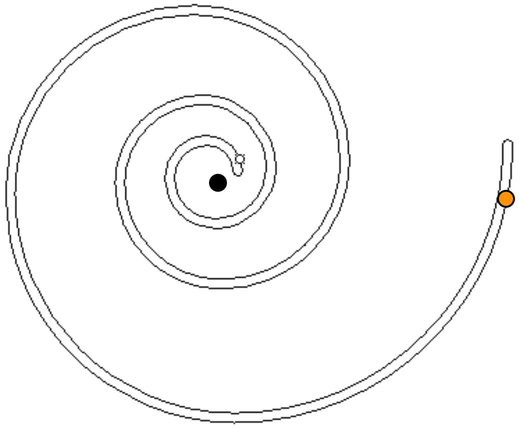




Problems with Classical Model of Atom

- Mechanical stability of multi-electron atoms: Electrons repel each other almost as strongly as they are attracted by nucleus. System is as unstable classically as if Jupiter and Saturn were to repel each other as strongly as they are attracted to the Sun.
- Radiative stability of even a single-electron atom or ion, for example, the hydrogen atom. If the electron orbits the nucleus, it is subject to acceleration, which will cause it to radiate, lose energy, and spiral into the nucleus on a timescale ~ 10^{-11} s.
- These difficulties are clearly at odds with concept of atoms as (almost) "indestructible and uncuttable" elementary units of all matter.





Bohr Atom

- Niels Bohr (1885-1962) hypothesizes that circular electron orbits in an atom can only have quantized units of angular momentum:

$$m_{\mathrm{e}} \mathrm{v} r = n\hbar, \text{ with } n = 1, 2, 3, \ldots$$

- Otherwise, the laws of mechanics and electromagnetism are obeyed as in the classical regime. In particular, for an electron in a circular orbit under the Coulomb attraction of a much heavier proton,

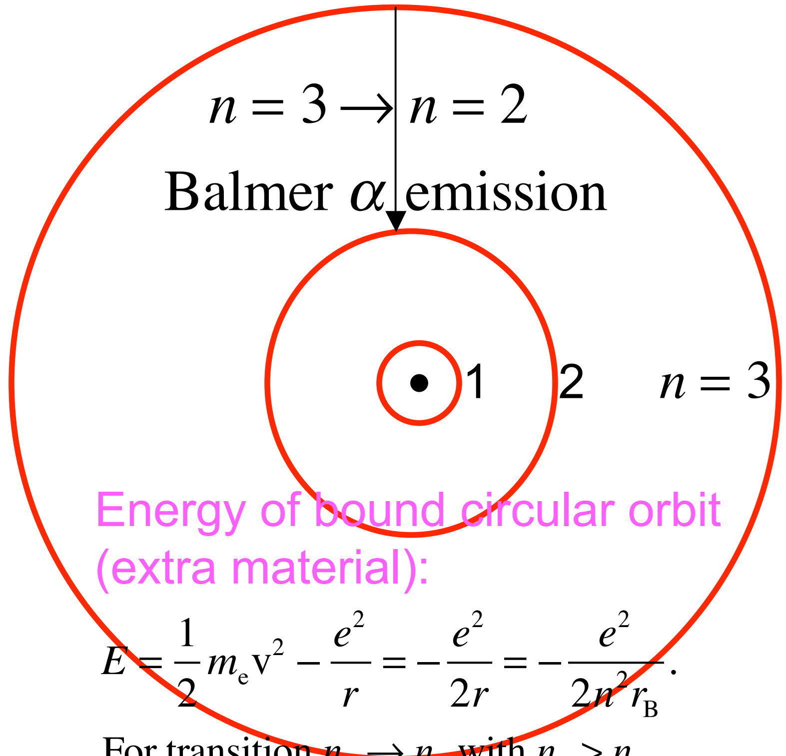
$$F = m_{\mathrm{e}} a,$$

with $a = \mathrm{v}^2 / r$ and $F = e^2 / r^2$.

- Collecting results, we get the radii of allowed orbits as

$$r = n^2 r_{\mathrm{B}} \text{ where } r_{\mathrm{B}} \equiv m_{\mathrm{e}} \hbar^2 / e^2 = 0.053 \text{ nm},$$

$$\text{with } n = 1, 2, 3, \ldots$$



Energy of bound circular orbit (extra material):

$$E = \frac{1}{2} m_{\mathrm{e}} \mathrm{v}^2 - \frac{e^2}{r} = -\frac{e^2}{2r} = -\frac{e^2}{2n^2 r_{\mathrm{B}}}.$$

For transition $n_2 \rightarrow n_1$ with $n_2 > n_1$,

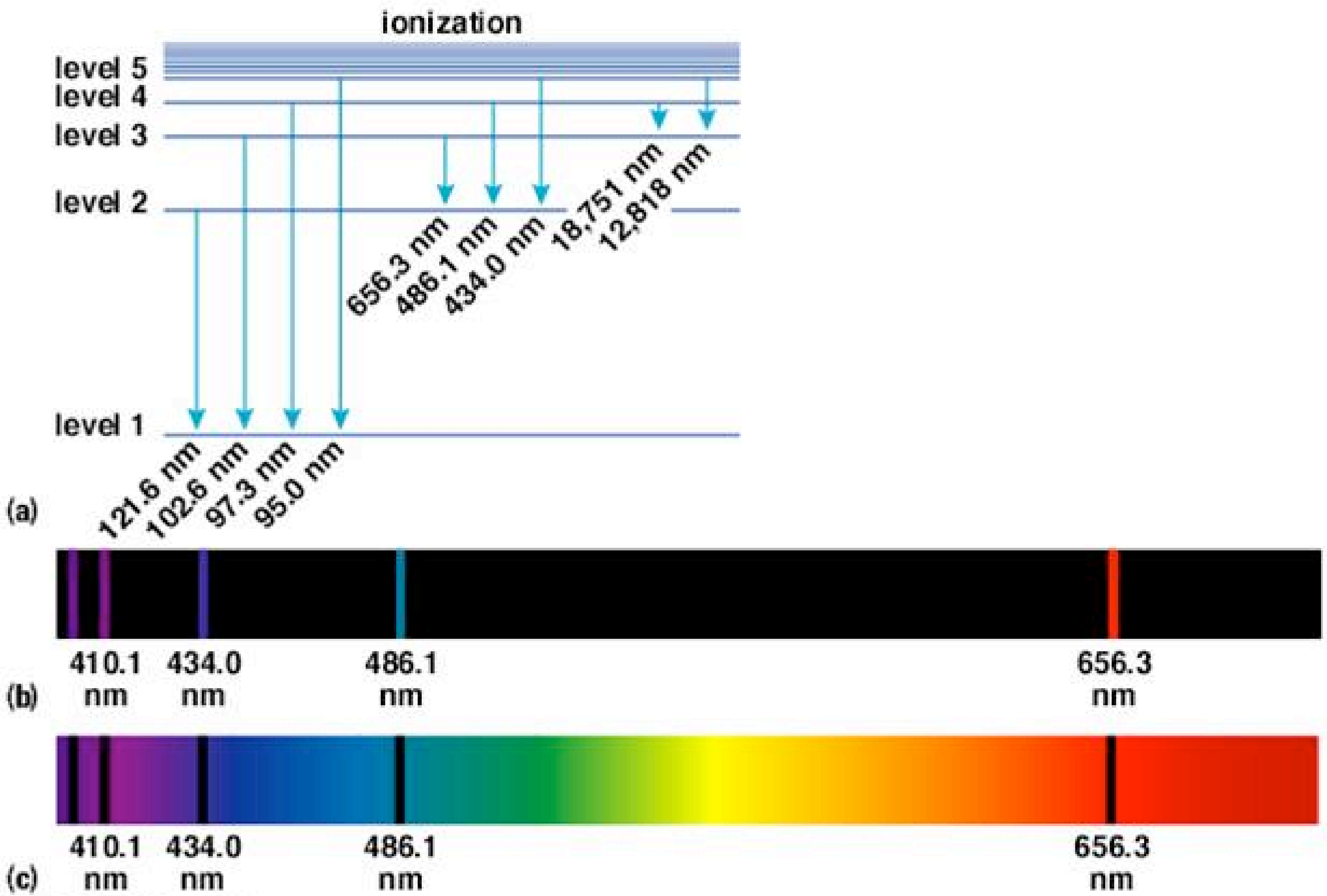
$$\Delta E = \frac{e^2}{2r_{\mathrm{B}}} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = \frac{hc}{\lambda},$$

where λ is wavelength of emitted photon,

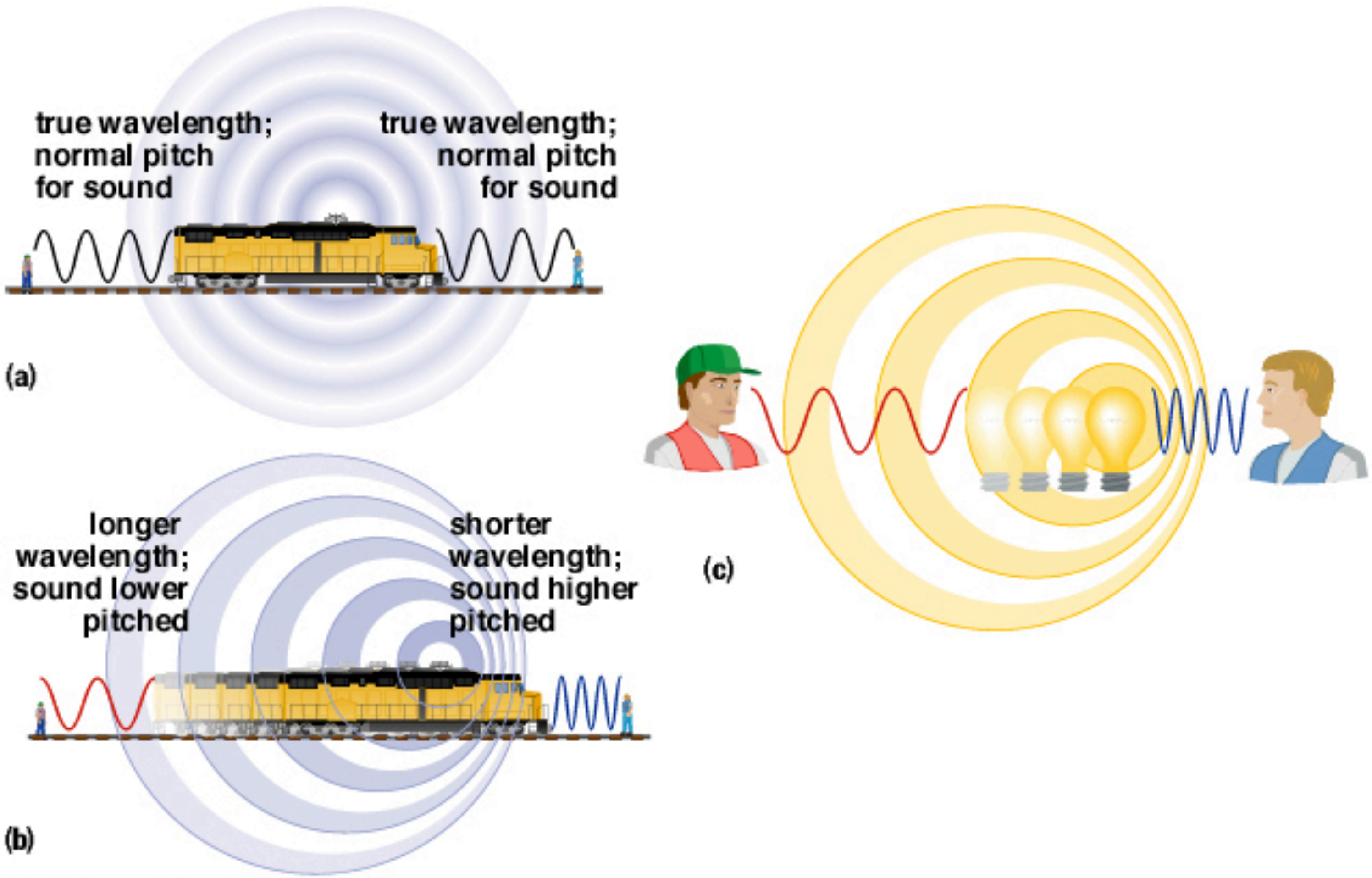
i.e., $$\frac{1}{\lambda} = \frac{1}{\lambda_{\mathrm{R}}} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ with}$$

$$\lambda_{\mathrm{R}} \equiv 2r_{\mathrm{B}} \left(\frac{hc}{e^2} \right) = 4\pi \left(\frac{\hbar c}{e^2} \right) r_{\mathrm{B}} = 91.1 \text{ nm}.$$

Energy Diagram for Hydrogen Line Transitions



Important Application of Discrete Spectral Feature: Doppler Shift (Sound)



Doppler Shift (Spectral Lines)

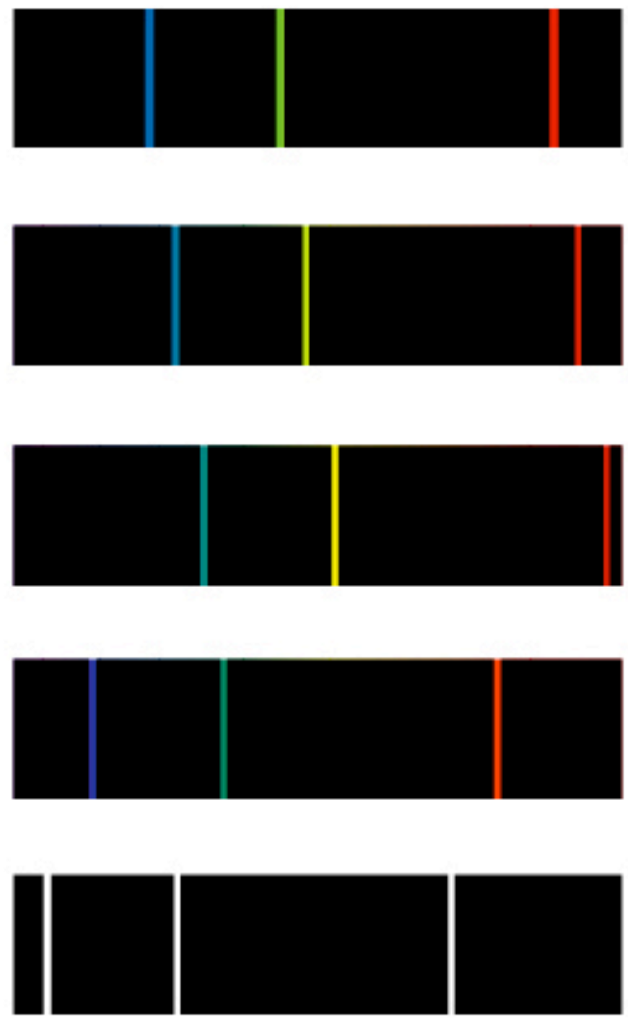
Laboratory spectrum
Lines at rest wavelengths.

Object 1
Lines redshifted: Object is moving away from us.

Object 2
Greater redshift: Object is moving away faster than Object 1.

Object 3
Lines blueshifted: Object is moving toward us.

Object 4
Greater blueshift: Object is moving toward us faster than Object 3.



Some Astronomical Uses of Spectroscopy

- Each element has its own characteristic pattern of spectral lines.
- The relative depth of absorption lines or height of emission lines depends on the amount of absorbing or emitting material along the line of sight.
- A displacement of the wavelength from its normal position in the laboratory (Doppler shift) yields the component velocity along the line of sight (redshift = away, blueshift = toward).
- A splitting of the spectral line (Zeeman effect) can indicate the strength of the magnetic field in which the atoms or molecules are embedded.
- Thus, spectroscopy can yield information on
  - Chemical composition
  - Amount of material along the line of sight
  - Motion along the line of sight
  - Level of magnetization

Kinematic Information Possible Even if Object is Spatially Unresolved (extra material)