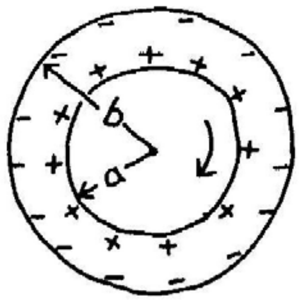
6.18



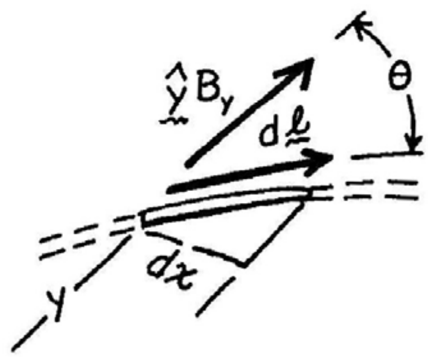
capacitance of coaxial cylinders, length L: $C = \dfrac{L}{2 \ln \frac{b}{a}}$. Charge per unit length on inner cylinder (assumed positive) $= \phi C/L$ $= \phi/2 \ln \frac{b}{a} = 50/2 \ln(4/3) = 87$ esu/cm

If inner cylinder rotates at 30 rev/sec it is a solenoidal surface current of density $\mathcal{J} = 30 \times 87$ $= 2610$ esu cm^{-1} sec^{-1}. Inside that cylinder $B = 4\pi \mathcal{J}/c$ $= 1.09 \times 10^{-6}$ gauss (into paper), $r < a$; $B = 0$, $r > a$.

If both cylinders rotate at 30 rev/sec, $B = 1.09 \times 10^{-6}$ gauss (out of paper) $a < r < b$ and $B = 0$, $r < a$, $r > b$.

6.21 If the conduction electrons are forced closer to the axis there will be uncompensated negative charge near the axis. This will cause a radial electric field E_r pushing outward on the electrons, preventing further constriction when $E_r = (v/c)B$. The field B within the conducting rod is $2\pi r J/c$, where the conduction current density J is nev, n being the number density and v the mean drift velocity of the conduction electrons. Suppose the electron cloud at radius r is squeezed inward by a small distance Δr. The cylinder of radius r will now contain, per unit length, an excess of negative charge in amount $(ne)(2\pi r \Delta r)$ causing an electric field $E_r = 4\pi ne \Delta r$. In equilibrium, then, $4\pi ne \Delta r = (v/c)B = 2\pi r ne (v/c)^2$, or $\Delta r/r = \frac{1}{2}(v/c)^2$. In solid conductors we always find $v/c \ll 1$. In metallic conduction v/c is seldom much greater than 10^{-10}, and $\Delta r/r \approx 10^{-20}$ is too small to detect. In highly ionized gases however, the "Pinch effect", as it is called, can be not only detectable but important.

$\boxed{6.22}$ $d\mathbf{f} = \dfrac{I\, d\boldsymbol{\ell} \times \mathbf{B}}{c}$



The z-component of $\mathbf{B}$ produces a force in the plane of the coil, which contributes nothing to the torque and can therefore be ignored in this calculation.

$d\boldsymbol{\ell} \times \hat{\mathbf{y}} B_y = \hat{\mathbf{z}}\, d\ell\, B_y \sin\theta$ But $d\ell \sin\theta = dx$

Hence $dF_z = \frac{I}{c} B_y\, dx$ The torque about the x-axis is $y \cdot dF_z = \frac{I B_y}{c}\, y\, dx = dN$. We have to integrate this around the loop to find the total torque.

$\int_{\text{loop}} y\, dx = $ area of loop $\equiv a$ (See fig. 11.4, p. 406)

Hence $N = \dfrac{B_y I a}{c}$. The torque vector $\mathbf{N}$ is in the $\hat{\mathbf{x}}$ direction, and if we define $\mathbf{m} \equiv \frac{I\mathbf{a}}{c}$ as in the Figure, our result can be written in the more general form:
$\mathbf{N} = \mathbf{m} \times \mathbf{B}$.

If $\mathbf{B}$ is the same at all points on the loop, the net force on the loop is zero, for

$\int d\mathbf{F} = \int \frac{I}{c}\, d\boldsymbol{\ell} \times \mathbf{B} = -\frac{I}{c}\, \mathbf{B} \times \int d\boldsymbol{\ell}$ and $\int d\boldsymbol{\ell}$ over the whole loop is zero.

6.24 current $= e \cdot \dfrac{v}{2\pi r}$ $\qquad B = \dfrac{2\pi I}{cr} = \dfrac{\beta e}{r^2}$

$$= \frac{.01 \times 4.8 \times 10^{-10}}{10^{-16}} = 4.8 \times 10^{4} \text{ gauss}$$

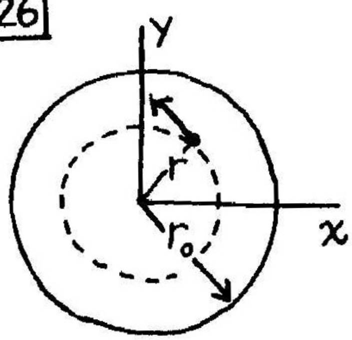
6.25 $B_x = 0 \quad B_y = 0 \quad B_z = B_0$ with $\underset{\sim}{B} = \nabla \times \underset{\sim}{A}$

$$\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0 \qquad \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = 0 \qquad \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = B_0$$

One obvious choice is: $A_x = \dfrac{-B_0 y}{2}$, $A_y = \dfrac{B_0 x}{2}$, $A_z = 0$.

Equally obvious is: $A_y = B_0 x$ $\quad A_x = A_z = 0$. To make others, add any vector function with zero curl.

6.26



Current inside $r = I r^2/r_0^2$

$$B = \frac{2 I r^2}{c r_0^2 r} = \frac{2 I r}{c r_0^2}$$

$$B_x = \frac{-y}{r} B = \frac{-2 y I}{c r_0^2} \qquad B_y = \frac{2 x I}{c r_0^2}$$

If $\underset{\sim}{A} = A_0 \hat{\underset{\sim}{z}}\,(x^2 + y^2)$, $\nabla \times \underset{\sim}{A} = \hat{\underset{\sim}{x}} \dfrac{\partial A_z}{\partial y} - \hat{\underset{\sim}{y}} \dfrac{\partial A_z}{\partial x}$

$$= 2y A_0 \hat{\underset{\sim}{x}} - 2x A_0 \hat{\underset{\sim}{y}} \qquad A_0 = -\frac{I}{c r_0^2}$$

6.35

$$\text{Resistance of ribbon} = \frac{1.6 \text{ ohm-cm} \times 0.5 \text{ cm}}{.001 \text{ cm}^2} = 800 \text{ ohms}$$

$V = 1$ volt $I = 1.25$ milliamp.

$J = 1.25$ amp/cm^2 $= 1.25 \times 10$ amp/m^2

Let's use S.I.: $v = J/ne$; $n = 2 \times 10^{21}$ per m^3

$e = 1.6 \times 10^{-19}$ coulomb $v = \dfrac{1.25 \times 10^4}{2 \times 10^{21} \times 1.6 \times 10^{-19}} = 39$ m/sec

$B = 0.1$ tesla $E_t = vB = 3.9$ volt/m

Across the 0.2 cm width of the ribbon the Hall voltage = 7.8 millivolts.

6.36 The relation between current density J and charge carrier velocity v is $J = nqv$, with v in m/sec, J in amp/m^2, n in m^{-3} and q in coulomb. Force on charge carrier is $q(\mathbf{E}_t + \mathbf{v} \times \mathbf{B})$ which is zero if

$$\mathbf{E}_t = \frac{-\mathbf{J} \times \mathbf{B}}{nq}$$

6.37 The resistance of the winding of the small solenoid will be 10 times that of the large coil. (The wire is 1/10 as long, with 1/100 the cross-sectional area.) If we apply the same voltage, 120 volts, we'll get 1/10 the current. That is just what will be needed to produce a magnetic field equal to that in the large coil, because the small coil has 10 times as many turns per unit length. The power is down by the factor 10, but the small coil has only 1/100 the surface area. It will be much harder to keep it cool.

$\boxed{6.38}$ The grain in Problem 2.22 had a radius of 3×10^{-7} m and was charged to a potential V of 0.15 volt. Its charge $q = 4\pi\epsilon_0 r V = 0.5 \times 10^{-17}$ coulomb. Moving through a magnetic field B the grain experiences a transverse force qvB. If its path is a circle of radius R, around which it moves with angular speed $\omega = v/R$, setting $mR\omega^2 = BqR\omega$ gives us the usual "cyclotron" relation:

$$\omega = Bq/m$$

Given $B = 3 \times 10^{-10}$ tesla, $m = 10^{-16}$ kg, $q = 0.5 \times 10^{-17}$ coulomb, we find $\omega = 1.5 \times 10^{-11}$ sec. The period of one revolution is $2\pi/\omega$ or 4×10^{11} sec, about 1300 years.