PHYSICS 4C | FINAL EXAM | FALL QUARTER 2006
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Formulas:

$F = k\frac{Q_1Q_2}{r^2}$ Coulomb's law ; $k = \frac{1}{4\pi\varepsilon_0} = 8.99\times10^9 \mathrm{N\cdot m^2/C^2}$; $\varepsilon_0 = 8.85\times10^{-12} C^2/Nm^2$

Electric field due to charge Q at distance r: $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{kQ}{r^2}\hat{r}$

Dipole field (p=Ql), along/perpendicular to dipole axis: $E = \frac{1}{2\pi\varepsilon_0}\frac{p}{x^3}$ / $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{y^3}$

Energy of and torque on dipole in external E-field: $U = -\vec{p}\cdot\vec{E}$, $\vec{\tau} = \vec{p}\times\vec{E}$

Linear, surface, volume charge density : $dq = \lambda\, ds$, $dq = \sigma\, dA$, $dq = \rho\, dV$

Electric field of infinite: line of charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$; sheet of charge: $E = \frac{\sigma}{2\varepsilon_0}$

Gauss law: $\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$; Φ = electric flux $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

$U_b - U_a = -W_{ba} = -\int_a^b \vec{F}\cdot \vec{dl} = -\int_a^b q\vec{E}\cdot\vec{dl} = q(V_b - V_a)$ V=J/C=Volts

$V = \frac{kQ}{r}$; $V = \int \frac{kdq}{r}$; $V = \frac{kp\cos\theta}{r^2}$ (dipole) ; $E_l = -\frac{\partial V}{\partial l}$; $\vec{E} = -\vec{\nabla}V$; $U_{12} = k\frac{Q_1Q_2}{r_{12}}$

Capacitors: $Q = CV$; with dielectric: $C \to KC$; $C = \frac{\varepsilon_0 A}{d}$; Energy : $U = \frac{Q^2}{2C}$

Capacitors in parallel: $C_{eq} = C_1 + C_2$; in series : $C_{eq}^{-1} = C_1^{-1} + C_2^{-1}$

$I = \frac{dQ}{dt} = \int \vec{J}\cdot d\vec{A}$; $\vec{J} = \sigma\vec{E}$; $\vec{E} = \rho\vec{J}$; $R = \rho\frac{\ell}{A}$; $V = IR$; $P = I^2R = VI = V^2/R$

Elementary charge: $e = 1.6\times10^{-19} C$; $R_{eq} = R_1 + R_2$ (series) ; $R_{eq}^{-1} = R_1^{-1} + R_2^{-1}$ (parallel)

Charging capacitor: $Q(t) = C\varepsilon(1 - e^{-t/RC})$; Discharging capacitor: $Q(t) = Q_0 e^{-t/RC}$

Force on moving charge: $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$; force on wire: $d\vec{F} = Id\vec{\ell}\times\vec{B}$

Circular motion: $a = \frac{v^2}{r}$; radius $r = \frac{mv}{qB}$; $\omega = \frac{qB}{m}$; $\omega = 2\pi f = 2\pi/T$

Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$; $\mu_0 = 4\pi\times10^{-7}\frac{N}{A^2}$; Ampere's law: $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$

solenoid: $B = \mu_0 nI$ $\vec{\nabla}\cdot\vec{B} = 0$; $\vec{\nabla}\times\vec{B} = \mu_0\vec{j}$

Long wire: $B = \frac{\mu_0 I}{2\pi r}$; loop, along axis: $B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}$; dipole: $\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{x^3}$

Magnetic dipole: $\vec{\mu} = I\vec{A}$; torque: $\vec{\tau} = \vec{\mu}\times\vec{B}$; energy: $U = -\vec{\mu}\cdot\vec{B}$

Faraday law: $\varepsilon = -\frac{d\Phi_B}{dt} = \oint \vec{E}\cdot d\vec{l}$; $\Phi_B = \int \vec{B}\cdot d\vec{A}$ magnetic flux

Local laws: $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ (Faraday) ; $\nabla\times\vec{B} = \mu_0\vec{J}$ (Ampere) ; Stokes theorem: $\oint \vec{X}\cdot d\vec{l} = \int(\vec{\nabla}\times\vec{X})\cdot d\vec{A}$

Self-inductance: $L = \frac{N\Phi_B}{I}$; $\varepsilon_L = -L\frac{dI}{dt}$; $\frac{L}{l} = \mu_0 n^2 A$ for solenoid ; energy $U = \frac{1}{2}LI^2$

Mutual inductance: $M_{21} = \frac{N_2\Phi_{21}}{I_1}$; $\varepsilon_2 = -M_{21}\frac{dI_1}{dt}$; $M_{21} = M_{12} = M$

energy density: magnetic $u = \frac{1}{2}\frac{B^2}{\mu_0}$: electric $u = \frac{1}{2}\varepsilon_0 E^2$

LR circuit: $I = \frac{V_0}{R}(1-e^{-t/\tau_L})$ (rise) ; $I = I_0 e^{-t/\tau_L}$ (decay) ; $\tau_L = L/R$

LC oscillations: $Q(t) = Q_0\cos(\omega_0 t + \phi)$; $I(t) = -\omega_0 Q_0 \sin(\omega_0 t + \phi)$; $\omega_0 = \frac{1}{\sqrt{LC}}$

LRC circuit: $Q(t) = Q_0 e^{-\frac{R}{2L}t}\cos(\omega_0' t + \phi)$; $\omega_0' = \sqrt{\omega_0^2 - \frac{R^2}{4L^2}}$

AC circuit, LRC: $V = V_0\sin(\omega t + \phi)$; $I = I_0\sin(\omega t)$; $I_0 = V_0/Z$; $\tan\phi = \frac{X_L - X_C}{R}$; $\bar{P} = I_{rms}V_{rms}\cos\phi$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$; Voltage ampl.: $V_R = IR,\ V_L = IX_L,\ V_C = IX_C;\ X_L = \omega L,\ X_C = \frac{1}{\omega C}$; $\cos\phi = \frac{R}{Z}$

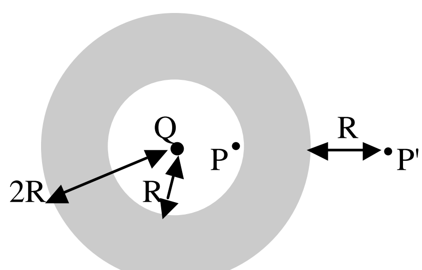
Ampere - Maxwell law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$; displacement current $I_d = \varepsilon_0\frac{d\phi_E}{dt}$; $\phi_E = \int \vec{E}\cdot d\vec{A}$

Local A - M law: $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$; Gauss: $\oint \vec{B}\cdot d\vec{A} = 0$; $\vec{\nabla}\cdot\vec{B} = 0$

Electromag. waves: $\vec{E} = E_0\sin(kx - \omega t)\hat{j}$; $\vec{B} = B_0\sin(kx - \omega t)\hat{k}$; $\frac{E_0}{B_0} = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times 10^8 m/s$

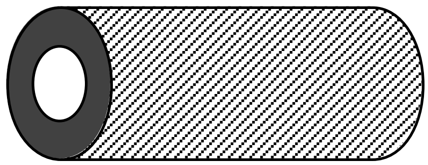
Poynting vector: $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$; radiation pressure: $P = \frac{\bar{S}}{c}$

Problem 1 (10 pts)



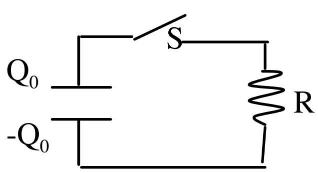
The point charge Q is at the center of a metal shell of inner radius R and outer radius 2R. The point P is inside the cavity very close to the inner surface of the metal shell. The point P' is at distance 3R from the center of the shell. The metal shell has zero net charge.
(a) Find an expression for the surface charge densities on the inner and outer surfaces of the metal shell, in terms of Q and R.
(b) When the metal shell is not there, the electric field at point P due to the charge Q is 3N/C. What is the electric field at point P when the metal shell is there, in N/C? Justify your answer clearly.
(c) What is the electric field at point P' (i) with and (ii) without the metal shell in place, in N/C? Justify your answer.
(d) Assume the charge Q is moved from the center of the shell to another point inside the cavity. (i) Will the electric field at point P change? (ii) Will the inner surface charge density at any point on the inner surface change? If yes, describe how; (iii) Same for the outer surface charge density; (iv) Will the electric field at point P' change? Justify your answers.

Problem 2 (10 pts+3 extra credit)

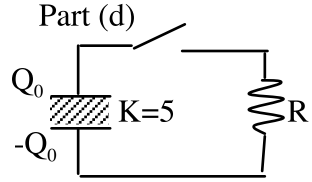


The cylindrical shell in the figure has inner radius R and outer radius 2R, and uniform volume charge density ρ, and length much much longer than 2R.
(a) Find expressions for the electric field at distance r from the axis, far from the ends of the cylinder, for (i) $r<R$, (ii) $R < r < 2R$, (iii) $2R < r$.
(b) Make a qualitative plot of electric field versus r for $0<r<4R$, indicating on the horizontal axis the values r=0, r=R and r=2R.
(c) Defining the electric potential to be zero at the axis of the cylinder, find an expression for the electric potential in the regions $0 < r < R$ and $R < r < 2R$. Where the potential is non-zero, is it always positive, always negative, or both, depending on the value of r?
(d) To bring a charge q from r=4R to r=3R requires work W=5J. How much work (in J) is required to bring the charge from r=3R to r=2R?

Problem 3 (10 pts+4 pts extra credit)



Part (d)



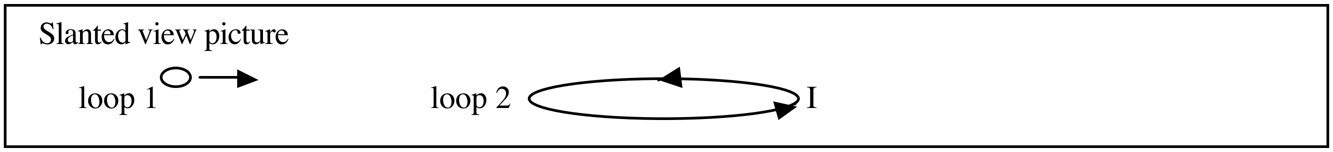
The capacitor in the figure has C=1F and is initially charged with Q_0=1C. The resistor has R=100Ω. At t=0 the switch S is closed.
(a) Find the current through R immediately after the switch is closed, $I(t=0^+)$, in A.
(b) Find the energy dissipated in the resistor during the first second after the switch is closed, in J, accurate to 5 decimal places at least. Compare your answer with $[I(t=0^+)]^2$Rx1s, and explain why it is similar or very different.
(c) Find the total energy dissipated in the resistor after a long time has passed, in J.
(d) Suppose before you close S you insert between the plates of the capacitor a dielectric of dielectric constant K=5. Find the answer for point (c) again.
(e) If the answers to (c) and (d) were different, explain how to account for the difference in energy. Where did the extra energy go, or where did the extra energy come from?

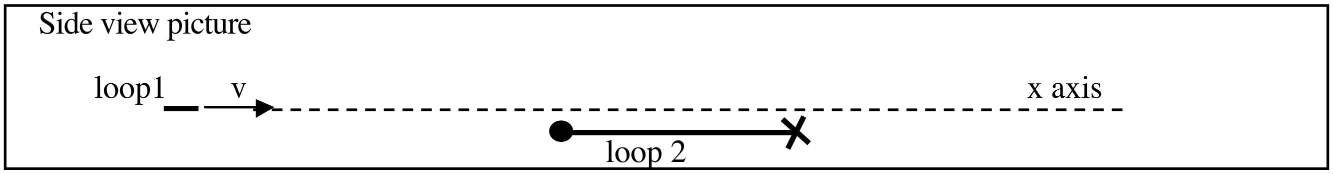
Problem 4 (10 pts)
The electric field in a region of space that extends 2m from the origin in all directions is given by

$$\vec{E}(x,y,z) = (\frac{y}{y_0}\hat{i} - \frac{x}{x_0}\hat{j})V/m \qquad \text{with } x_0=y_0=1\text{m}.$$

(a) Explain why this cannot be an electrostatic field (i.e. a field that is produced by static electric charges).
(b) Assuming the magnetic field at point (0,0,0) is 0 at time t=0, find its magnitude at time t=1s, in T.
(c) A square loop of wire of resistance R=2Ω lies on the xy plane with vertices at points (0,0), (1m,0), (1m,1m), (0,1m). Find the current that circulates through that loop, in A.
(d) Looked at from a point on the +z axis, does the current flow clockwise or counterclockwise?

Problem 5 (10 pts)





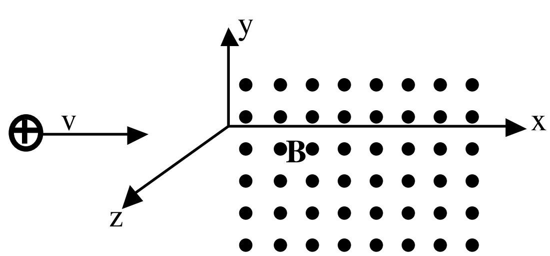
Circular loops 1 and 2 are on a plane perpendicular to the paper (side view picture). Loop 1 moves at uniform speed v along a line (dashed line) (x axis) that goes over loop 2 very close to it , such that the center of loop 1 flies right over the center of loop 2. The radius of loop 2 is much larger than that of loop 1. A steady current I circulates in loop 2, going into the paper on the right side and out of the paper on the left side.

(a) Make a qualitative plot of the flux of magnetic field through loop 1 as function of x=position of the center of loop 1. Put the center of loop 2 at x=0, and indicate on the x-axis of the graph the positions of the edges of loop 2. Use the convention positive flux = flux pointing up, negative flux= flux pointing down.

(b) Make a qualitative plot of the current induced in loop 1 as a function of x, again indicating the position of the edges and center of loop 2. Use the convention: positive current = current in the same sense as in loop 2, negative current = current in the opposite sense as in loop 2. Correlate the points where the current is zero with features of the plot in (a).

<u>Hint:</u> the magnetic field very close to a curved wire looks very similar to that of a straight wire.

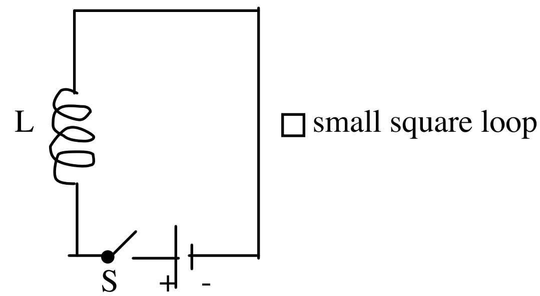
Problem 6 (10 pts)



A proton (mass=$1.67\text{x}10^{-27}$kg, charge=-electron charge) moving in the x direction with speed v=10,000 m/s enters a region starting at x=0 where there is an electric field and a uniform magnetic field. The magnetic field points in the +z direction and has magnitude 0.5G (1G=10^{-4}T). The x-component of the E-field is uniform. The proton continues to move in a straight line in the x direction and comes to a stop at x=1m.

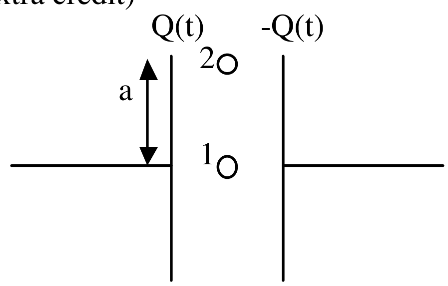
(a) Find the electric field component in the z direction (with its sign) for all x.
(b) Find the electric field component in the y direction (with its sign) at x=0.
(c) Find the electric field component in the x direction (with its sign).
(d) Find the angle that the electric field makes with the x-axis at x=0, in degrees.

Problem 7 (10 pts)



The circuit on the figure, including the solenoid L, is made with Cu wire of cross section $1mm^2$. The total length of wire used in the circuit, including the wire in the solenoid, is 10m. The inductance of the solenoid is L=100mH. The resistivity of Cu is 1.68×10^{-8} Ωm. The battery has the polarity shown and no internal resistance. There is a small square loop of wire next to the long wire in the circuit. Assume the small square loop has negligible self-inductance and neglect any effect of the small loop on the circuit.
At time t=0 the switch S is closed. **1second after S is closed, the current in the small square loop is measured to be 1μA.**
(a) Does the current in the small loop flow clockwise or counterclockwise? Explain.
(b) Find (in μA) (i) the current in the small square loop 2 seconds after the switch S is closed, and (ii) immediately after the switch S is closed. Explain all steps and show all intermediate results clearly.

Problem 8 (10 pts+3 extra credit)



The capacitor in the figure has round plates of radius a=10cm. The charge in the capacitor plates is changing with time according to the equation

$Q(t) = Q_0\left(\frac{t}{\tau}\right)^m$ with Q_0=1C, τ=1ms, and the exponent m=2. There are two small wire loops in the gap between the capacitor plates, labeled 1 and 2 in the figure. They are both in the plane of the paper, loop 1 is centered right on the line that goes through the center of the capacitor plates, and loop 2 is vertically above it at distance 10cm, i.e. right at the edge of the capacitor plates. They have resistance R_1=5Ω and R_2=10Ω respectively, and both have area $1cm^2$. At time t=1s:
(a) Will there be current flowing in loop 1? In loop 2? Explain why qualitatively, without doing any calculation.
(b) Calculate the currents flowing in loop 1 and loop 2, in μA, if any, at time t=1s. Show all steps in the calculation, name all laws that you use, and show all intermediate results.
(c) Assume the exponent in the equation for Q(t) is m=1 instead of m=2, and answer question (a) above again.