1-5 This is a case of dilation. $T=\gamma T'$ in this problem with the proper time $T'=T_0$

$$T=\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2}T_0 \Rightarrow \frac{v}{c}=\left[1-\left(\frac{T_0}{T}\right)^2\right]^{1/2};$$

in this case $T=2T_0$, $v=\left\{1-\left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2}=\left[1-\left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v=0.866c$.

1-6 This is a case of length contraction. $L=\frac{L'}{\gamma}$ in this problem the proper length $L'=L_0$,

$L=\left[1-\frac{v^2}{c^2}\right]^{-1/2}L_0 \Rightarrow v=c\left[1-\left(\frac{L}{L_0}\right)^2\right]^{1/2}$; in this case $L=\frac{L_0}{2}$, $v=\left\{1-\left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2}=\left[1-\left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v=0.866c$.

1-7 The problem is solved by using time dilation. This is also a case of $v \ll c$ so the binomial expansion is used $\Delta t=\gamma\Delta t' \cong \left[1+\frac{v^2}{2c^2}\right]\Delta t'$, $\Delta t-\Delta t'=\frac{v^2\Delta t'}{2c^2}$; $v=\left[\frac{2c^2(\Delta t-\Delta t')}{\Delta t'}\right]^{1/2}$;
$\Delta t=(24\ \text{h/day})(3\,600\ \text{s/h})=86\,400\ \text{s}$; $\Delta t=\Delta t'-1=86\,399\ \text{s}$;

$$v=\left[\frac{2(86\,400\ \text{s}-86\,399\ \text{s})}{86\,399\ \text{s}}\right]^{1/2}=0.004\,8c=1.44\times10^6\ \text{m/s}.$$

1-8 $L=\frac{L'}{\gamma}$

$$\frac{1}{\gamma}=\frac{L}{L'}=\left[1-\frac{v^2}{c^2}\right]^{1/2}$$

$$v=c\left[1-\left(\frac{L}{L'}\right)^2\right]^{1/2}=c\left[1-\left(\frac{75}{100}\right)^2\right]^{1/2}=0.661c$$

1-9 $L_{\text{earth}}=\frac{L'}{\gamma}$

$L_{\text{earth}}=L'\left[1-\frac{v^2}{c^2}\right]^{1/2}$, L', the proper length so $L_{\text{earth}}=L=L\left[1-(0.9)^2\right]^{1/2}=0.436L$.

1-10 (a) $\tau=\gamma\tau'$ where $\beta=\frac{v}{c}$ and

$$\gamma='\left(1-\beta^2\right)^{-1/2}=\tau'\left(1-\frac{v^2}{c^2}\right)^{-1/2}=\left(2.6\times10^{-8}\ \text{s}\right)\left[1-(0.95)^2\right]^{-1/2}=8.33\times10^{-8}\ \text{s}$$

(b) $d=v\tau=(0.95)\left(3\times10^8\right)\left(8.33\times10^8\ \text{s}\right)=24\ \text{m}$

1-11 $\Delta t = \gamma \Delta t'$

$$\Delta t = \Delta t'\left(1-\frac{v^2}{c^2}\right)^{-1/2} \cong \left(1+\frac{v^2}{2c^2}\right)\Delta t' \cong \left[1+\frac{\left(4.0\times10^2\ \text{m/s}\right)^2}{2\left(3.0\times10^8\ \text{m/s}\right)^2}\right](3\,600\ \text{s})$$

$$\cong \left(1+8.89\times10^{-13}\right)(3\,600\ \text{s}) = \left(3\,600+3.2\times10^{-9}\right)\ \text{s}$$

$\Delta t - \Delta t' \cong 3.2\ \text{ns}$. (Moving clocks run slower.)

1-12 (a) 70 beats/min or $\Delta t' = \frac{1}{70}$ min

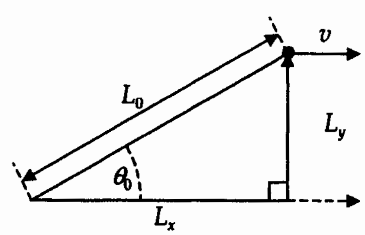
(b) $\Delta t = \gamma \Delta t' = \left[1-(0.9)^2\right]^{-1/2}\left(\frac{1}{70}\right)\text{min} = 0.032\,8\ \text{min/beat}$ or the number of beats per minute $\approx 30.5 \approx 31$.

1-13 (a) $\tau = \gamma\tau' = \left[1-(0.95)^2\right]^{-1/2}(2.2\ \mu\text{s}) = 7.05\ \mu\text{s}$

(b) $\Delta t' = \frac{d}{0.95c} = \frac{3\times10^3\ \text{m}}{0.95c} = 1.05\times10^{-5}\ \text{s}$, therefore,

$$N = N_0 \exp\left(-\frac{\Delta t}{\tau}\right) = \left(5\times10^4\ \text{muons}\right)\exp(-1.487) \approx 1.128\times10^4\ \text{muons}.$$

1-14 (a) Only the x-component of L_0 contacts.



$$L_{x'} = L_0\cos\theta_0 \Rightarrow \frac{L_x[L_0\cos\theta_0]}{\gamma}$$

$$L_{y'} = L_0\sin\theta_0 \Rightarrow L_y = L_0\sin\theta_0$$

$$L = \left[(L_x)^2+\left(L_y\right)^2\right]^{1/2} = \left[\left(\frac{L_0\cos\theta_0}{\gamma}\right)^2+(L_0\sin\theta_0)^2\right]^{1/2}$$

$$= L_0\left[\cos^2\theta_0\left(1-\frac{v^2}{c^2}\right)+\sin^2\theta_0\right]^{1/2} = L_0\left[1-\frac{v^2}{c^2}\cos^2\theta_0\right]^{1/2}$$

(b) As seen by the stationary observer, $\tan\theta = \frac{L_y}{L_x} = \frac{L_0\sin\theta_0}{L_0\cos\theta_0/\gamma} = \gamma\tan\theta_0$.

1-15 (a) For a receding source we replace v by $-v$ in Equation 1.15 and obtain:

$$f_{ob}=\left\{\frac{[c-v]^{1/2}}{[c+v]^{1/2}}\right\}f_{source}=\left\{\frac{[1-v/c]^{1/2}}{[1+v/c]^{1/2}}\right\}f_{source}\cong\left(1-\frac{v}{2c}\right)\left(1-\frac{v}{2c}\right)f_{source}$$
$$\cong\left(1-\frac{v}{c}+\frac{v^2}{4c^2}\right)f_{source}\cong\left(1-\frac{v}{c}\right)f_{source}$$

where we have used the binomial expansion and have neglected terms of second and higher order in $\frac{v}{c}$. Thus, $\frac{\Delta f}{f_{source}}=\frac{f_{ob}-f_{source}}{f_{source}}=-\frac{v}{c}$.

(b) From the relations $f=\frac{c}{\lambda}$, $\frac{df}{d\lambda}=-\frac{c}{\lambda^2}$ we find $\frac{df}{f}=-\frac{c/\lambda^2}{c/\lambda}d\lambda$, or $\frac{\Delta\lambda}{\lambda}=-\frac{\Delta f}{f}=\frac{v}{c}$.

(c) Assuming $v \ll c$, $\frac{v}{c}\cong\frac{\Delta\lambda}{\lambda}$, or $v\cong\left(\frac{\Delta\lambda}{\lambda}\right)c=\left(\frac{20\text{ nm}}{397\text{ nm}}\right)c=0.050c=1.5\times10^7\text{ m/s}$.

1-16 For an observer approaching a light source, $\lambda_{ob}=\left[\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right]\lambda_{source}$. Setting $\beta=\frac{v}{c}$ and after some algebra we find,

$$\beta=\frac{\lambda_{source}^2-\lambda_{obs}^2}{\lambda_{source}^2+\lambda_{obs}^2}=\frac{(650\text{ nm})^2-(550\text{ nm})^2}{(650\text{ nm})^2+(550\text{ nm})^2}=0.166$$
$$v=0.166c=(4.98\times10^7\text{ m/s})(2.237\text{ mi/h})(\text{m/s})^{-1}=1.11\times10^8\text{ mi/h}.$$

1-17 (a) Galaxy A is approaching and as a consequence it exhibits blue shifted radiation. From Example 1.6, $\beta=\frac{v}{c}=\frac{\lambda_{source}^2-\lambda_{obs}^2}{\lambda_{source}^2+\lambda_{obs}^2}$ so that $\beta=\frac{(550\text{ nm})^2-(450\text{ nm})^2}{(550\text{ nm})^2+(450\text{ nm})^2}=0.198$. Galaxy A is approaching at $v=0.198c$.

(b) For a red shift, B is receding. $\beta=\frac{v}{c}=\frac{\lambda_{source}^2-\lambda_{obs}^2}{\lambda_{source}^2+\lambda_{obs}^2}$ so that

$\beta=\frac{(700\text{ nm})^2-(550\text{ nm})^2}{(700\text{ nm})^2+(550\text{ nm})^2}=0.237$. Galaxy B is receding at $v=0.237c$.

1-18 (a) Let f_c be the frequency as seen by the car. Thus, $f_c=f_{source}\sqrt{\frac{c+v}{c-v}}$ and, if f is the frequency of the reflected wave, $f=f_c\sqrt{\frac{c+v}{c-v}}$. Combining these equations gives $f=f_{source}\frac{(c+v)}{(c-v)}$.

(b) Using the above result, $f(c-v)=f_{source}(c+v)$, which gives

$$(f-f_{source})c=(f+f_{source})v\approx 2f_{source}v.$$

The beat frequency is then $f_{beat}=f-f_{source}=\frac{2f_{source}v}{c}=\frac{2v}{\lambda}$.

(c) $$f_{\text{beat}} = \frac{2(30.0\ \text{m/s})(10.0\times10^9\ \text{Hz})}{3.00\times10^8\ \text{m/s}} = \frac{2(30.0\ \text{m/s})}{0.0300\ \text{m}} = 2\,000\ \text{Hz} = 2.00\ \text{kHz}$$

$$\lambda = \frac{c}{f_{\text{source}}} = \frac{3.00\times10^8\ \text{m/s}}{10.0\times10^9\ \text{Hz}} = 3.00\ \text{cm}$$

(d) $v = \frac{f_{\text{beat}}\lambda}{2}$ so,

$$\Delta v = \frac{\Delta f_{\text{beat}}\lambda}{2} = \frac{(5\ \text{Hz})(0.0300\ \text{m})}{2} = 0.0750\ \text{m/s} \approx 0.2\ \text{mi/h}$$

1-19 $u_{XA} = -u_{XB}$; $u'_{XA} = 0.7c = \frac{u_{XA} - u_{XB}}{1 - u_{XA}u_{XB}/c^2}$; $0.70c = \frac{2u_{XA}}{1 + (u_{XA}/c)^2}$ or $0.70u_{XA}^2 - 2cu_{XA} + 0.7c^2 = 0$.
Solving this quadratic equation one finds $u_{XA} = 0.41c$ therefore $u_{XB} = -u_{XA} = -0.41c$.

1-20 $$u = \frac{v + u'}{1 + vu'/c^2} = \frac{0.90c + 0.70c}{1 + (0.90c)(0.70c)/c^2} = 0.98c$$

1-21 $$u'_X = \frac{u_X - v}{1 - u_X v/c^2} = \frac{0.50c - 0.80c}{1 - (0.50c)(0.80c)/c^2} = -0.50c$$

1-22 (a) The speed as observed in the laboratory is found by using Equation 1.30:
$u_X = \frac{u'_X - v}{1 - u'_X v/c^2}$. But $u'_X = \frac{c}{n}$ (speed measured by an observer moving with the fluid),
therefore $u_X = \frac{(c/n) + v}{1 + v/(nc)} = \frac{c}{n}\frac{1 + nv/c}{1 + v/(nc)}$.

(b) $\frac{v}{c} << 1$. Use the binomial expansion,

$$u'_X \cong \frac{c}{n}\left[1 + n\left(\frac{v}{c}\right)\right]\left[1 - \frac{v}{nc}\right] \cong \frac{c}{n}\left[1 + \frac{nv}{c} - \frac{v^2}{c^2}\right] \cong \frac{c}{n} + v - \frac{v}{n^2}.$$

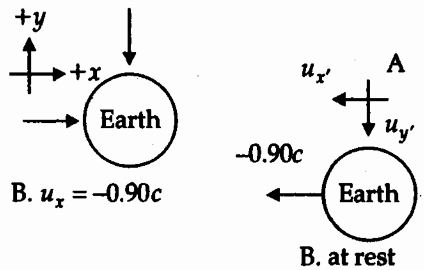
1-23 (a) Let event 1 have coordinates $x_1 = y_1 = z_1 = t_1 = 0$ and event 2 have coordinates $x_2 = 100$ mm, $y_2 = z_2 = t_2 = 0$. In S', $x'_1 = \gamma(x_1 - vt_1) = 0$, $y'_1 = y_1 = 0$, $z'_1 = z_1 = 0$, and $t'_1 = \gamma\left[t_1 - \left(\frac{v}{c^2}\right)x_1\right] = 0$, with $\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-1/2}$ and so $\gamma = \left[1 - (0.70)^2\right]^{-1/2} = 1.40$. In system S', $x'_2 = \gamma(x_2 - vt_2) = 140$ m, $y'_2 = z'_2 = 0$, and

$$t'_2 = \gamma\left[t_2 - \left(\frac{v}{c^2}\right)x_2\right] = \frac{(1.4)(-0.70)(100\ \text{m})}{3.00\times10^8\ \text{m/s}} = -0.33\ \mu\text{s}.$$

(b) $\Delta x' = x'_2 - x'_1 = 140$ m

(c) Events are not simultaneous in S', event 2 occurs 0.33 μs earlier than event 1.

1-24 A. $u_y = -0.90c$



$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0 - 0.90c}{1 - (0)(0.90c)/c^2} = -0.90c$$

$$u_y' = \frac{u_y}{\gamma\left(1 - u_x v/c^2\right)} = \frac{0 - 0.90c}{[1 - 0.81]^{-1/2}} \cong -0.392c$$

The speed of A as measured by B is

$$u_{AB} = \left[(u_x')^2 + \left(u_y'\right)^2\right]^{1/2} = \left[(-0.90c)^2 + (-0.392c)^2\right]^{1/2} = 0.982c.$$

Classically, $u_{AB} = 1.3c$.

1-25 We find Carpenter's speed: $\dfrac{mGM}{r^2} = \dfrac{mv^2}{r}$

$$v = \left[\frac{GM}{R+h}\right]^{1/2} = \left[\frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}{6.37\times10^6 + 0.16\times10^6}\right]^{1/2} = 7.82\ \text{km/s}.$$

Then the period of one orbit is $T = \dfrac{2\pi(R+h)}{v} = \dfrac{2\pi\left(6.53\times10^6\right)}{7.82\times10^3} = 5.25\times10^3\ \text{s}$.

(a) The time difference for 22 orbits is $\Delta t - \Delta t' = (\gamma - 1)\Delta t' = \left[\left(1 - \dfrac{v^2}{c^2}\right)^{-1/2} - 1\right](22)(T)$.

Using the binomial expansion one obtains

$$\left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22)(T) = \frac{1}{2}\left[\frac{7.82\times10^3\ \text{m/s}}{3\times10^8\ \text{m/s}}\right](22)\left(5.5\times10^3\ \text{s}\right) = 39.2\ \mu\text{s}.$$

(b) For one orbit, $\Delta t - \Delta t' = \dfrac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s} \approx 2\ \mu\text{s}$. The press report is accurate to one significant figure.

1-26 The observed length of an object moving with speed v is $L = L'\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2}$ with L' being the proper length. For the two ships, we know that $L_2 = L_1$, $L_2' = 3L_1'$ and $v_1 = 0.35c$. Thus $L_2^2 = L_1^2$ and $(9L_1'^2)\left[1-\left(\frac{v_2}{c}\right)^2\right] = L_1'^2\left[1-(0.35)^2\right]$, giving $9-9\left(\frac{v_2}{c}\right)^2 = 0.8775$, or $v_2 = 0.95c$.

1-27 For the pion to travel 10 m in time Δt in our frame,

$$10\text{ m} = v\Delta t = v(\gamma\Delta t') = v\left(26\times10^{-9}\text{ s}\right)\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2}$$

$$\left(3.85\times10^{8}\text{ m/s}\right)^2\left(1-\frac{v^2}{c^2}\right) = v^2$$

$$1.46\times10^{17}\text{ m}^2/\text{s}^2 = v^2(1+1.64)$$

$$v = 2.37\times10^{8}\text{ m/s} = 0.789c$$

1-28 For Astronauts approaching Alpha Centauri $\gamma^{-1} = \left[1-\left(\frac{v}{c}\right)^2\right]^{1/2} = [1-0.902]^{1/2} = 0.312$.

(a) astronauts' time $t' = \gamma^{-1}t = (0.312)(4.4\text{ y}) = 1.37$ years,

(b) astronauts' distance $d' = (0.312)(4.2\text{ light years}) = 1.31$ light years.

1-29 (a) A spaceship, reference frame S', moves at speed v relative to the Earth, whose reference frame is S. The space ship then launches a shuttle craft with velocity v in the forward direction. The pilot of the shuttle craft then fires a probe with velocity v in the forward direction. Use the relativistic compounding of velocities as well as its inverse transformation: $u_x' = \frac{u_x - v}{1-\left(u_x v/c^2\right)}$, and its inverse $u_x = \frac{u_x' + v}{1+\left(u_x' v/c^2\right)}$. The above variables are defined as: v is the spaceship's velocity relative to S, u_x' is the velocity of the shuttle craft relative to S', and u_x is the velocity of the shuttle craft relative to S. Setting u_x' equal to v, we find the velocity of the shuttle craft relative to the Earth to be: $u_x = \frac{2v}{1+(v/c)^2}$.

(b) If we now take S to be the shuttle craft's frame of reference and S' to be that of the probe whose speed is v relative to the shuttle craft, then the speed of the probe relative to the spacecraft will be, $u_x' = \frac{2v}{1+(v/c)^2}$. Adding the speed relative to S yields:

$u_x = \left[\frac{3+(v/c)^2}{1+2(v/c)^2}\right] = \frac{3v+v^3/c^3}{1+2v^2/c^2}$. Using the Galilean transformation of velocities, we see that the spaceship's velocity relative to the Earth is v, the velocity of the shuttle craft relative to the space ship is v and therefore the velocity of the shuttle craft relative to the Earth must be $2v$ and finally the speed of the probe must be $3v$. In the limit of low $\left(\frac{v}{c}\right)^2$, u_x reduces to $3v$. On the other hand, using relativistic addition of velocities, we find that $u_x = c$ when $v \to c$.

1-35 In the Earth frame, Speedo's trip lasts for a time $\Delta t = \frac{\Delta x}{v} = \frac{20.0 \text{ ly}}{0.950 \text{ ly/yr}} = 21.05$ Speedo's age advances only by the proper time interval: $\Delta t_p = \frac{\Delta t}{\gamma} = 21.05 \text{ yr}\sqrt{1-0.95^2} = 6.574 \text{ yr}$ during his trip. Similarly for Goslo, $\Delta t_p = \frac{\Delta x}{v}\sqrt{1-\frac{v^2}{c^2}} = \frac{20.0 \text{ ly}}{0.750 \text{ ly/yr}}\sqrt{1-0.75^2} = 17.64 \text{ yr}$. While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0 \text{ ly}}{0.750 \text{ ly/yr}} - \frac{0.20 \text{ ly}}{0.950 \text{ ly/yr}}\sqrt{1-0.75^2} = 17.64 \text{ yr}.$$

Then Goslo ends up older by $17.64 \text{ yr} - (6.574 \text{ yr} + 5.614 \text{ yr}) = 5.45 \text{ yr}$.

1-36 Let Suzanne be fixed in reference from S and see the two light-emission events with coordinates $x_1 = 0$, $t_1 = 0$, $x_2 = 0$, $t_2 = 3\ \mu\text{s}$. Let Mark be fixed in reference frame S' and give the events coordinate $x_1' = 0$, $t_1' = 0$, $t_2' = 9\ \mu\text{s}$.

(a) Then we have

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = 9\ \mu\text{s} = \frac{1}{\sqrt{1-v^2/c^2}}(3\ \mu\text{s} - 0) = \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{3} = \frac{v^2}{c^2} = \frac{8}{9} = v = 0.943c.$$

(b) $$x_2' = \gamma(x_2 - vt_2) = 3\left(0 - 0.943c \times 3\times10^{-6}\text{ s}\right)\left(\frac{3\times10^8 \text{ m/s}}{c}\right) = 2.55\times10^3 \text{ m}$$

1-37 Einstein's reasoning about lightning striking the ends of a train shows that the moving observer sees the event toward which she is moving, event B, as occurring first. We may take the S-frame coordinates of the events as $(x = 0, y = 0, z = 0, t = 0)$ and $(x = 100 \text{ m}, y = 0, z = 0, t = 0)$. Then the coordinates in S' are given by Equations 1.23 to 1.27. Event A is at $(x' = 0, y' = 0, z' = 0, t' = 0)$. The time of event B is:

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{1}{\sqrt{1-0.8^2}}\left(0 - \frac{0.8c}{c^2}(100 \text{ m})\right) = 1.667\left(\frac{80 \text{ m}}{3\times10^8 \text{ m/s}}\right) = -4.44\times10^{-7} \text{ s}.$$

The time elapsing before A occurs is 444 ns.