Today's Lecture

Vector Description of Motion
Motion in More Than One-D

Example: Velocity/Acceleration in 2D

An object undergoes acceleration of $\vec{a} = \left(2.3\hat{i} + 3.6\hat{j}\right)m/s^2$ over a period of 10*s*. After these 10*s* its velocity is $\vec{v}_f = \left(33\hat{i} + 15\hat{j}\right)m/s$.

a) What is the initial velocity?

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \rightarrow \vec{v}_i = \vec{v}_f - \vec{a}\Delta t$$

$$\vec{v}_i = 33\hat{i} + 15\hat{j} - \left(2.3\hat{i} + 3.6\hat{j}\right)10$$

$$\vec{v}_i = \left(10\hat{i} - 21\hat{j}\right)m/s$$

b) By how much did its speed (magnitude of velocity) change?

$v_f = \sqrt{33^2 + 15^2} = 36.25m/s$ and $v_i = \sqrt{10^2 + 21^2} = 23.26m/s$

Hence,

$$\Delta v = v_f - v_i \simeq 13m/s$$

Example: Velocity/Acceleration 2D

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c) By how much did its direction change?

$$\theta_f = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\frac{15}{33} = .426rad = 24.4^\circ$$

$$\theta_i = \tan^{-1}\frac{v_{yi}}{v_{xi}} = \tan^{-1}\frac{-21}{10} = -1.126rad = -64.5^\circ$$

Hence, $\Delta\theta = \theta_f - \theta_i = 88.1^\circ$

d) Compare the change in speed with the magnitude of the acceleration times the time, 10*s*. First we note that the magnitude of the acceleration is

$$a = \sqrt{2.3^2 + 3.6^2} = 4.27m/s^2$$

Since the acceleration and the velocity are not parallel there is no reason to believe that the change in speed is equal to this acceleration times 10*s*.

Example: Velocity/Acceleration 2D

An object's position is given by: $\vec{r} = (ct - bt^3)\hat{i} + dt^2\hat{j}$

$$c = 6.7 m/s, \quad b = .81 m/s^3, \quad d = 4.5 m/s^2$$

a) What is its initial velocity?

$$\vec{v}(t=0) = \left.\frac{d\vec{r}}{dt}\right|_{t=0} = \left[(c - 3bt^2)\hat{i} + 2dt\hat{j}\,\right]_{t=0}$$

$$\vec{v}(t=0) = c\hat{i} = 6.7\hat{i}\, m/s$$

b) How long does it take for its velocity to rotate 90º?

For this to happen the *x* component of the velocity must vanish, or:

$$c - 3bt^2 = 0 \rightarrow t = \sqrt{c/3b} = 1.66\,\mathrm{sec}$$

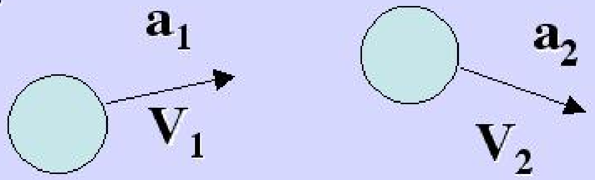
c) How much does its speed change during this time?

$$\vec{v}\left(t = \sqrt{c/3b}\right) = 2d\sqrt{c/3b}\,\hat{j} = 14.94\hat{j}$$

$$\Delta v = 14.94 - 6.7 = 8.24 m/s$$

Displacement, Velocity and Acceleration are Relative

Just as all displacement is relative, so are the velocity and acceleration. They are measured relative to some coordinate system origin which is assumed to be fixed.



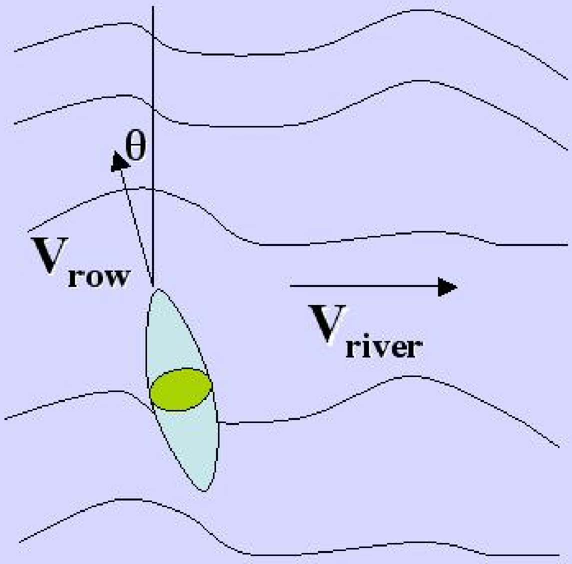
If body 1 is moving with velocity V_1 and body 2 with velocity V_2, then body 1 is moving relative to body 2 with velocity that id the differenec between V_1 and V_2

$$\vec{V}_r = \vec{V}_1 - \vec{V}_2$$

The same holds for acceleration $\vec{a}_r = \vec{a}_1 - \vec{a}_2$

The intertial reference frame is a reference frame with no acceleration.

Example: Velocity Vectors



A person can row with speed V_{row} in still water. In a river flowing at speed V_{river}, at what angle should they row to go straight across?

$$\vec{V}_{row} = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}}$$

$$\vec{V}_{river} = V_{river}\hat{\mathbf{i}}$$

$$V_x = -V_{row}\sin(\theta)$$

$$V_y = V_{row}\cos(\theta)$$

Note the signs!

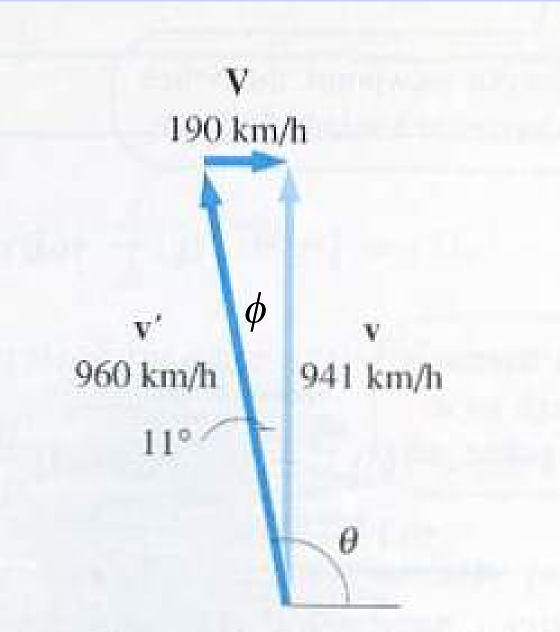
The velocity of the boat is $\vec{V}_{boat} = \vec{V}_{river} + \vec{V}_{row} = (V_x + V_{river})\hat{i} + V_y\hat{j}$

For zero x velocity $V_x = -V_{river} = -V_{row}\sin(\theta)$

$$\theta = \arcsin\left(\frac{V_{river}}{V_{row}}\right)$$

Note the signs!

Example: A Jetliner and the Jet Stream



A Jetliner with an airspeed of 960km/h flies from Houston to Omaha, a distance of 1290km. The jetstream at altitude is a steady v_{js} = 190km/h from west to east.

(a) In what direction should the jet fly?

(b) What is the ground speed of the jet?

(c) How long does the trip take?

(a) To ensure that the jet travels due north it must take a heading to the east so that its east-west velocity component counteracts the jet stream.

$$\vec{v}' = v' \cos\phi \,\hat{j} - v' \sin\phi \,\hat{i} \quad \text{and} \quad \vec{v}_{js} - v' \sin\phi \,\hat{i} = 0$$

Solving for ϕ we find: $960 \sin\phi = 190 \rightarrow \phi = \sin^{-1} \dfrac{190}{960} = .199 rad = 11.4^\circ$

(b) The ground speed is: $v' \cos\phi = 960 \cos 11.4^\circ = 941 km/h$

(c) The trip time is: $t = \dfrac{d}{v_y} = \dfrac{1290}{941} = 1.37 hr = 1 hr\ 22 \min$

Acceleration in Two Different Frames

Consider two different frames *S* and *S'* where the relative velocity between the two frames is $\vec{V}$.

If an object has a velocity $\vec{v}$ in frame S then its velocity in *S'* is $\vec{v}' = \vec{v} - \vec{V}$. Differentiating this expression yields

$$\vec{a}' = \frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{V}}{dt}.$$

Generally we will only consider frames that are not themselves accelerating. If neither *S* nor *S'* are accelerating then their relative velocity is constant. This means that for any object is S (or S'):

$$\vec{a}' = \vec{a}$$

Frames in uniform motion are called **inertial frames**! and
The laws of motion are the same in all inertial frames of reference.

Motion in More Than One Dimension

Chapter 4

Motion in More Than One Dimension

The equations of motion can be written as vector equations.

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j} + \frac{dV_z}{dt}\hat{k}$$

The concepts of single dimensional motion discussed so far apply to each dimension individually in the vector equations. Thus the vector equations each represent three scalar equations, one for each dimension

$$\vec{r} = \vec{r}_o + \vec{V}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{V}_f = \vec{V}_i + \vec{a}t$$

Motion in More Than One Dimension

Treating each dimension independently, we can break down each position, velocity and acceleration in any problem into it's components along those dimensions, and solve the scalar problem in each dimension.

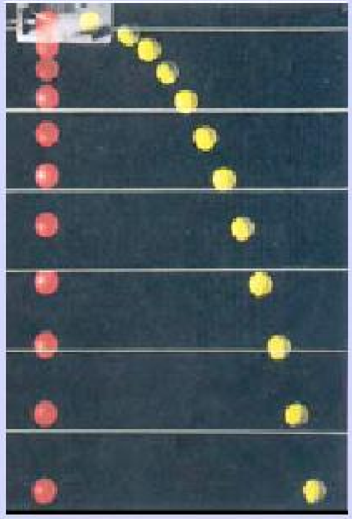
In the example of the asteroid and the Earth we did this. The x dimension was solved independently of the y dimension for the motion, where the time was the only quantity that was used in common with both dimensions.

$$\vec{r} = \vec{r}_o + \vec{V}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{V}_f = \vec{V}_i + \vec{a}t$$

Example: Gravity

A red ball is dropped from rest at the same time that a yellow ball is projected horizontally.

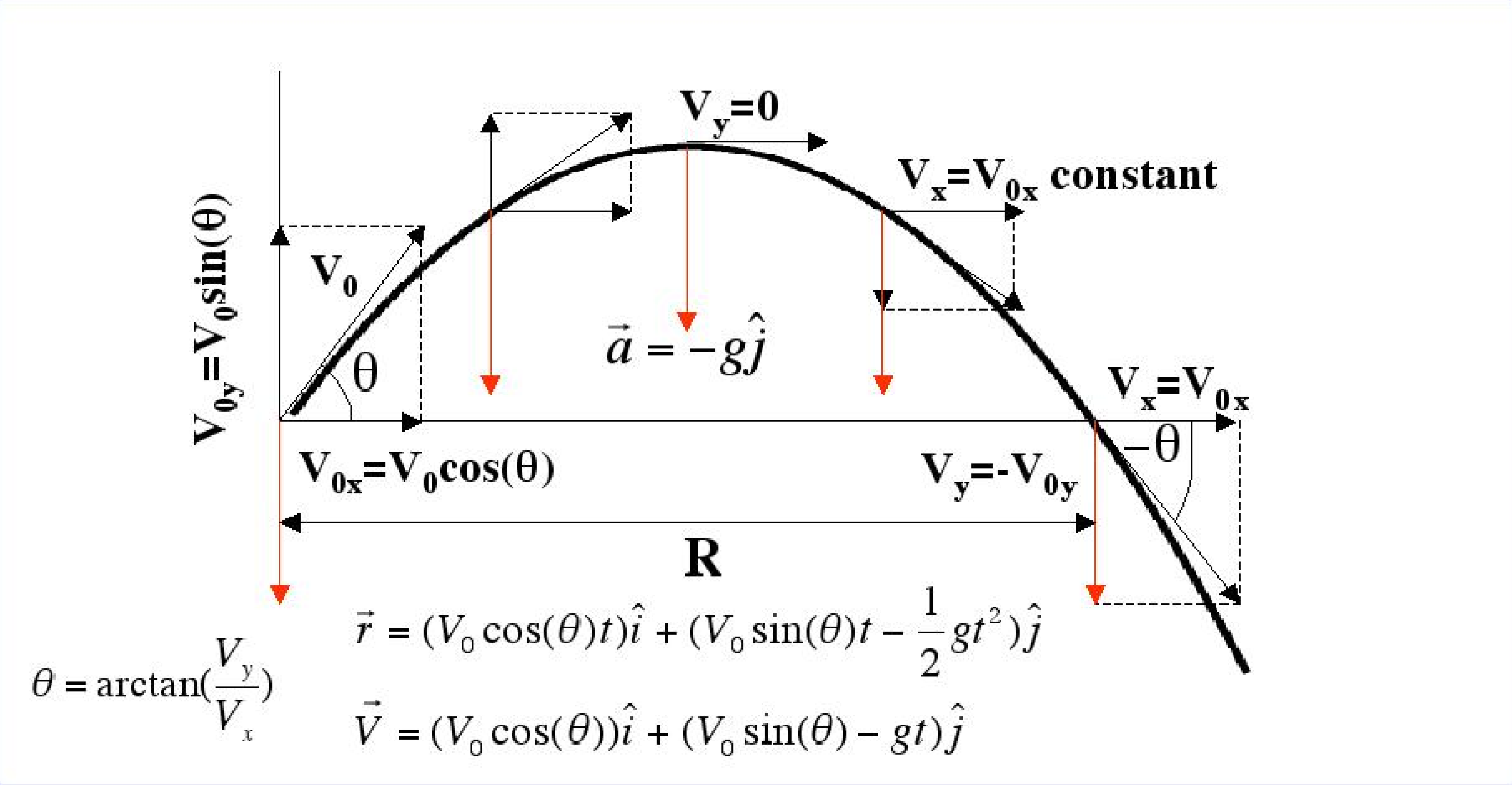


Both balls will have the same y(t) and $V_y(t)$ at all times. However, the x(t) and $V_x(t)$ will differ.

The vector or the scalar equations for the y direction can be used for the red ball. The vector or two sets of scalar equations, one for each dimension with motion, can be used to describe the yellow ball's trajectory. The vector equations describe both cases in the most concise format.

Example: Projectile

The same principles apply to a projectile launched at some angle to the ground.



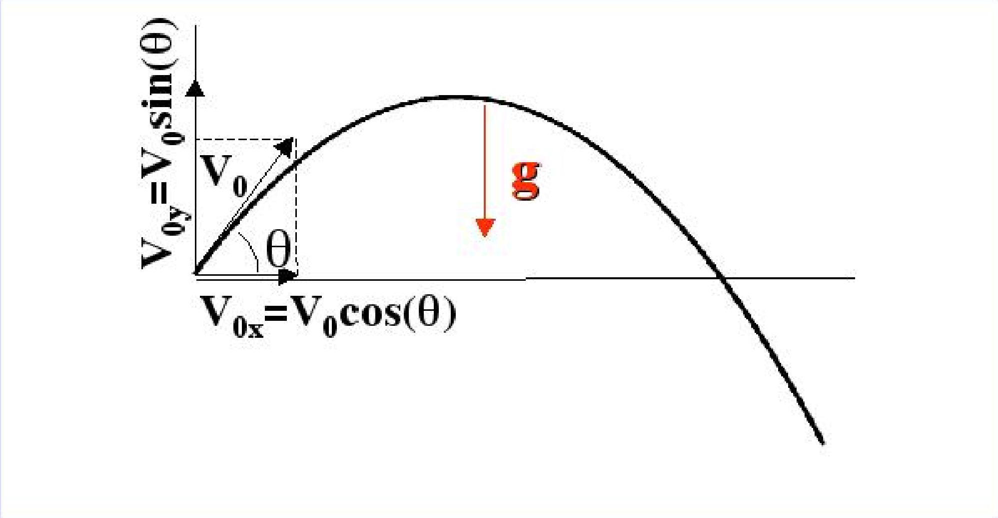
The motion in x has constant velocity because there is no acceleration in x.

Projectile Trajectory is Parabolic

We can show this by eliminating time from the problem and deriving the function y(x). First, solve for the time from the position equation in x

$$x = V_0 \cos(\theta) t$$

$$t = \frac{x}{V_0 \cos(\theta)}$$



Then substitute in for t in position equation for y

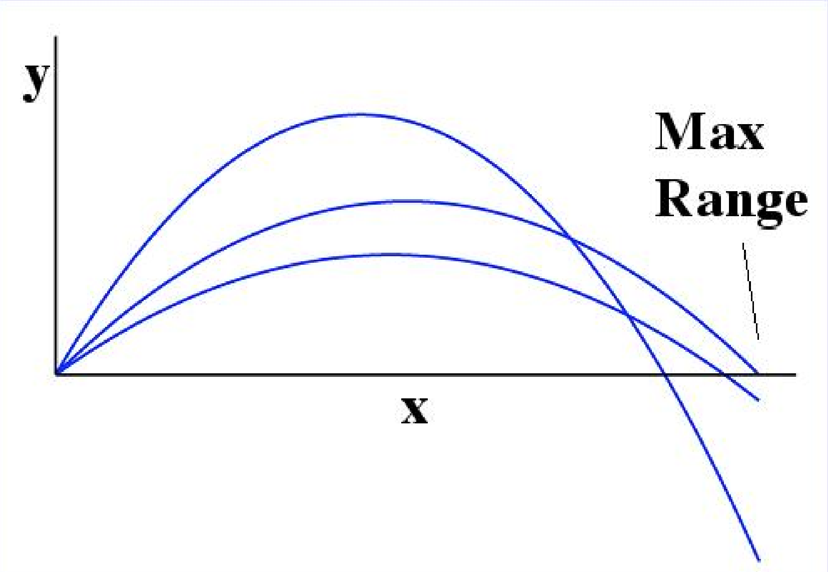
$$y = V_0 \sin(\theta)\left(\frac{x}{V_0 \cos(\theta)}\right) - \frac{1}{2} g \left(\frac{x}{V_0 \cos(\theta)}\right)^2$$

$$y = \tan(\theta) x - \frac{g}{2V_0^2 \cos^2(\theta)} x^2$$

Trajectory is always parabolic in x

Height and Range of Projectiles

Maximum height and range of the projectile depends on the launch angle.



The projectile is at its max height when V_y=0 (remember Lecture 1?)

$$V_y = V_0 \sin(\theta) - g t_{\max} = 0$$

$$t_{\max} = \frac{V_0 \sin(\theta)}{g}$$

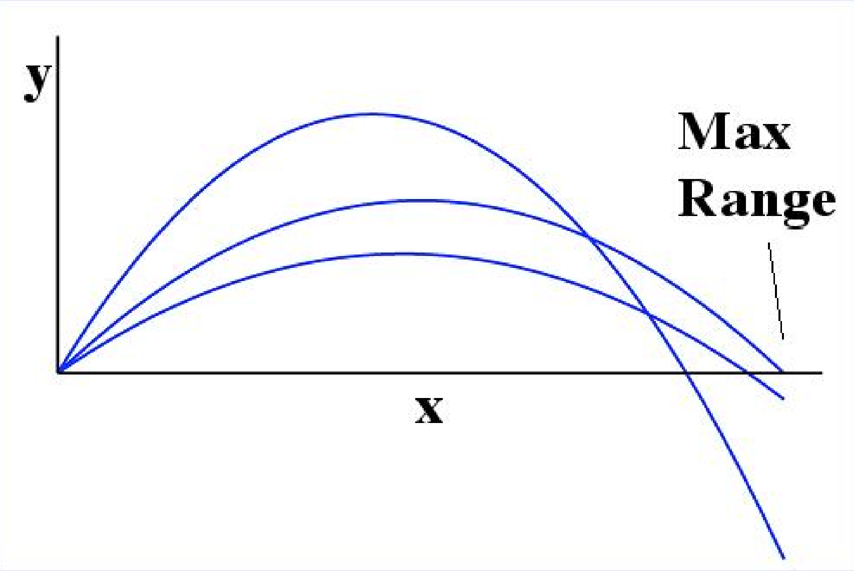
At this time

$$y_f = V_0 \sin(\theta) \frac{V_0 \sin(\theta)}{g} - \frac{1}{2} g \left(\frac{V_0 \sin(\theta)}{g}\right)^2 = \boxed{\frac{(V_0 \sin(\theta))^2}{2g}}$$

Which is largest at θ=90°, or vertical launch

Height and Range of Projectiles

Maximum height and range of the projectile depends on the launch angle.



The projectile has reached its range at time t_f when y=0

$$y_f = V_0 \sin(\theta) t_f - \frac{1}{2} g t_f^2 = 0$$

$$t_f = \frac{2V_0 \sin(\theta)}{g}$$

At this time the x location is

$$x_f = V_0 \cos(\theta)\left(\frac{2V_0 \sin(\theta)}{g}\right) = \boxed{\frac{V_0^2 \sin(2\theta)}{g}}$$

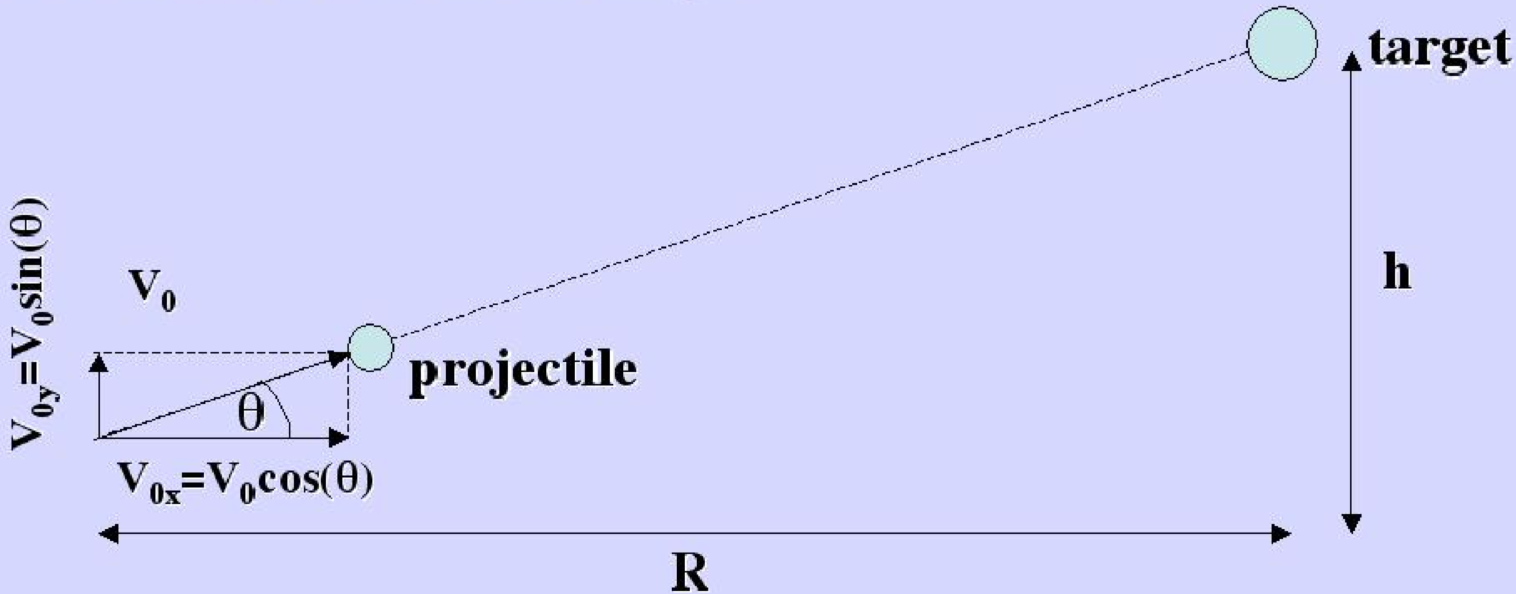
Which is largest at θ=45°

Using trigonometric identity

$$2\cos(\theta)\sin(\theta) = \sin(2\theta)$$

Example: Shoot the Monkey!

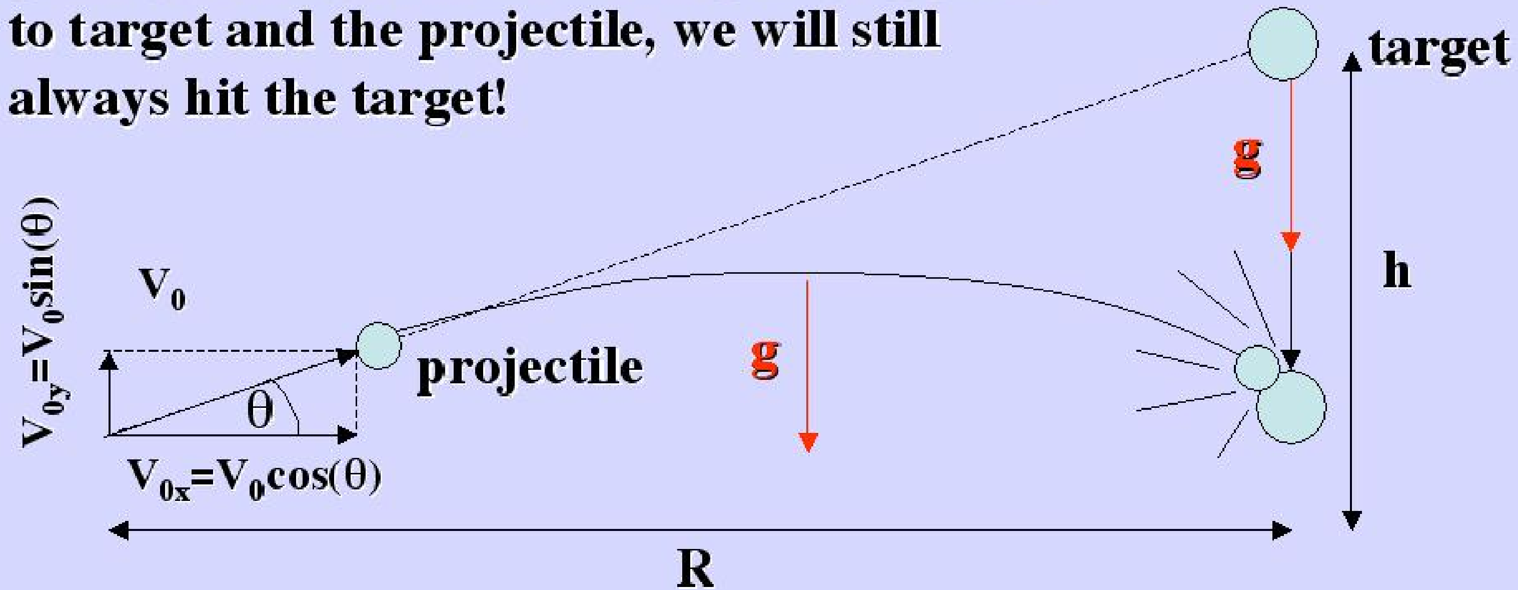
Suppose that without gravity we aim a projectile at a target and shoot. It will hit the target.



We can easily calculate components of the initial velocity and the time to the target given the distance R.

Example: Shoot the Monkey!

What's interesting is that even WITH gravity, or any acceleration equally applied to target and the projectile, we will still always hit the target!



We can prove this by showing that there is a time t for which $\vec{r}_t = \vec{r}_p$ or

$$\vec{x}_t = \vec{x}_p$$

$$\vec{y}_t = \vec{y}_p$$

Here "t" stands for target and "p" for projectile

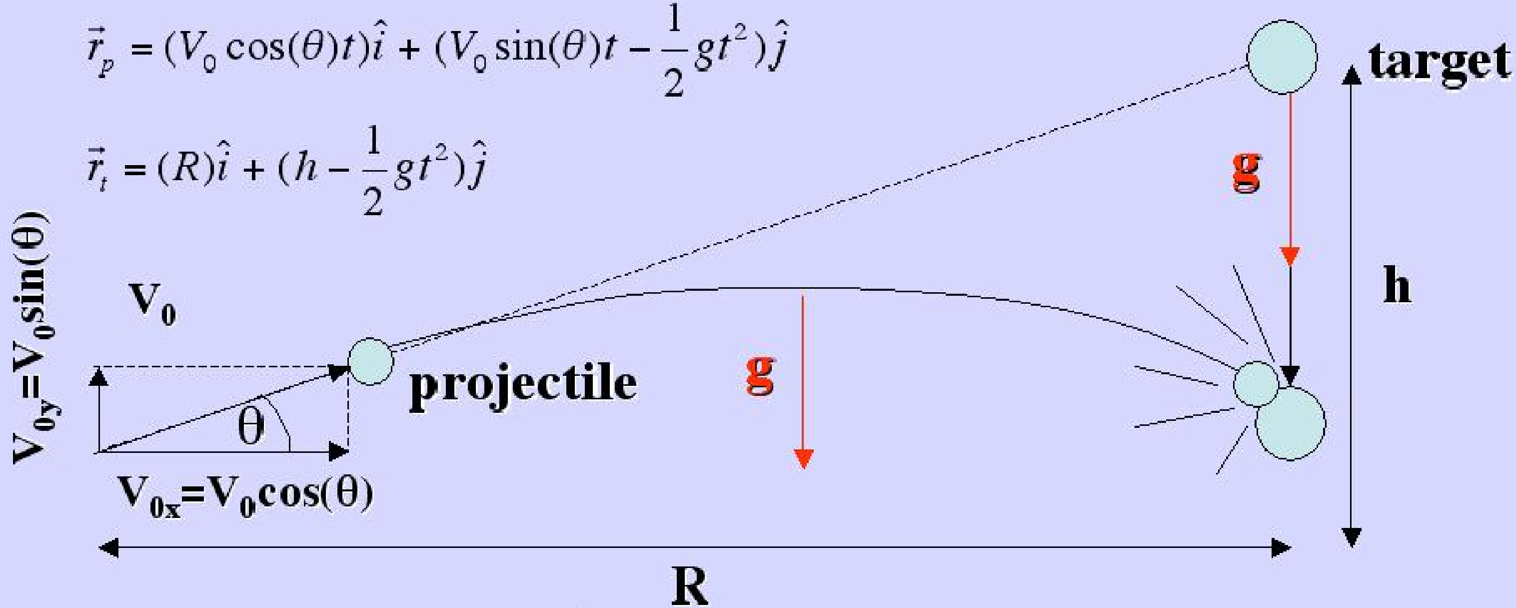
Example: Shoot the Monkey!

We first construct the position vectors

where $h = R\tan(\theta)$

$$\vec{r}_p = (V_0\cos(\theta)t)\hat{i} + (V_0\sin(\theta)t - \frac{1}{2}gt^2)\hat{j}$$

$$\vec{r}_t = (R)\hat{i} + (h - \frac{1}{2}gt^2)\hat{j}$$



When projectile reaches R,

$$R = V_0\cos(\theta)t$$

$$t = \boxed{\frac{R}{V_0\cos(\theta)}}$$

At what time are the y positions equal?

$$h - \frac{1}{2}gt^2 = V_{0y}t - \frac{1}{2}gt^2$$

$$t = \frac{h}{V_{0y}} = \frac{R\tan(\theta)}{V_0\sin(\theta)} = \boxed{\frac{R}{V_0\cos(\theta)}}$$

The Same Time! They Collide!

Next week we will review projectile motion and begin our study of circular motion

We’ll then begin Newton’s Law’s of Motion

First Quiz is this Friday! It will not cover material on circular motion!

Demo Tomorrow!

Tomorrow we will have a live demonstration of the laws of motion in two dimensions, namely "Shoot the Monkey"! So, don't miss it.

Note: No monkeys will be harmed in this demo in any way, only a teddie bear (unless we miss).