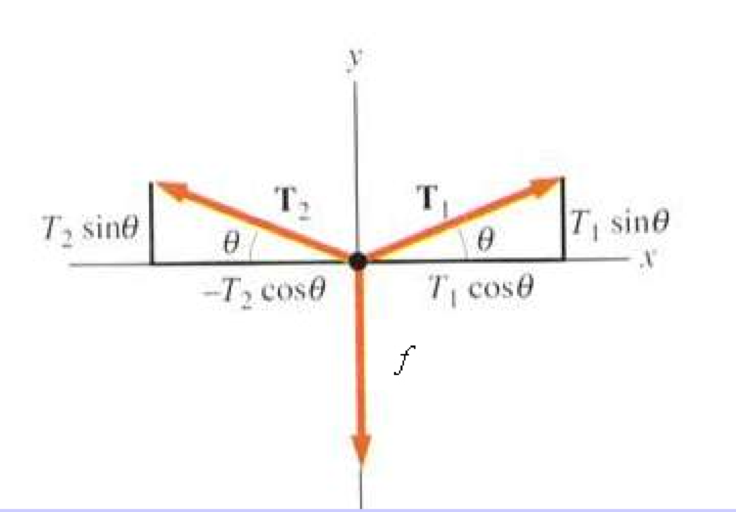
Today's Lecture

Lecture 11: Chapter 6,
Using Newton's Laws
Rotational Examples
Drag

First Example: Two Football Players and the Female Gymnast

There is a standard demonstration in a mechanics class where two large football players pull on a rope putting as much tension as possible in the rope. Then the smallest girl in the class is asked to pull (in the transverse direction) on the middle of the rope with her smallest pinky finger. Let's examine what happens. The free-body diagram is:



The EOM's for both the *x* and *y* direction:

$$T_{1x} - T_{2x} = T_1 \cos\theta_1 - T_2 \cos\theta_2 = 0$$
$$T_{1y} + T_{2y} - f = T_1 \sin\theta_1 - T_2 \sin\theta_2 - f = 0$$

Since she pulls in the middle of the rope $\theta_1 = \theta_2$

From the *x* equation this implies that $\boldsymbol{T_1 = T_2 = T}$.

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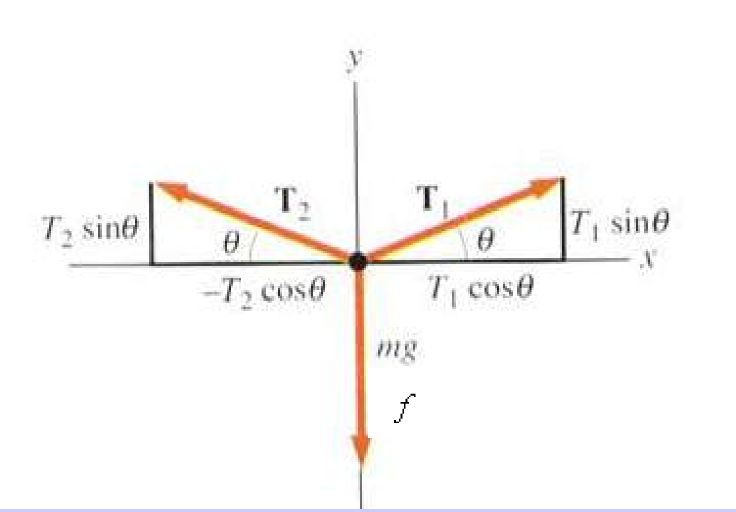


The equation for the *y* components simplifies to:

$$T_1 \sin\theta_1 - T_2 \sin\theta_2 - f = 0 \rightarrow 2T\sin\theta = f$$

For θ small, $\sin\theta = \tan\theta$, and the transverse displacement of the rope, δy, is:

$$\delta y = \frac{f}{4T} L$$

We see that when *L*, the length of the rope, is large, the girl with her pinky will be able to deflect the rope in the transverse direction even when $f/T << 1$.

Second Example: Two Blocks & Incline Plane

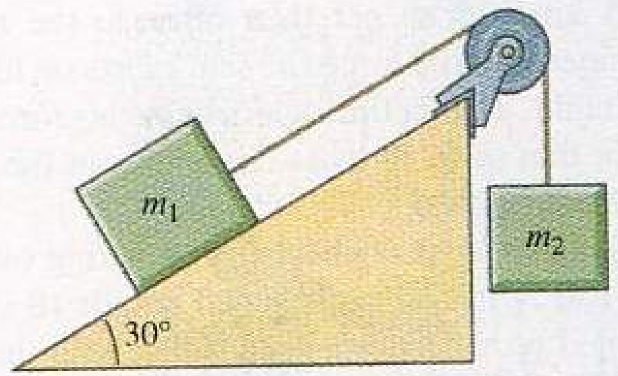


FIGURE 6-76 Problems 67, 68.

Assuming frictionless surfaces, find the acceleration of $\boldsymbol{m_2}$.

From a free-body diagram the vector EOM for $\boldsymbol{m_1}$ is:

$$\vec{T} + \vec{N} + \vec{F}_g = m\vec{a}$$

Choosing a coordinate system in which the *x* axis is parallel to the incline the component equations are:

$$T - m_1 g \sin\theta = m_1 a \quad \text{and} \quad N - m_1 g \cos\theta = 0$$

The EOM for $\boldsymbol{m_2}$ is particularly simple:

$$m_2 g - T = m_2 a$$

It is important to note that we have assumed that $\boldsymbol{m_2}$ is accelerating down and $\boldsymbol{m_1}$ is accelerating up the incline. We could have done the reverse, but we must be consistent. That is, $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ cannot both accelerate up (or down).

Second Example: Two Blocks & Incline Plane

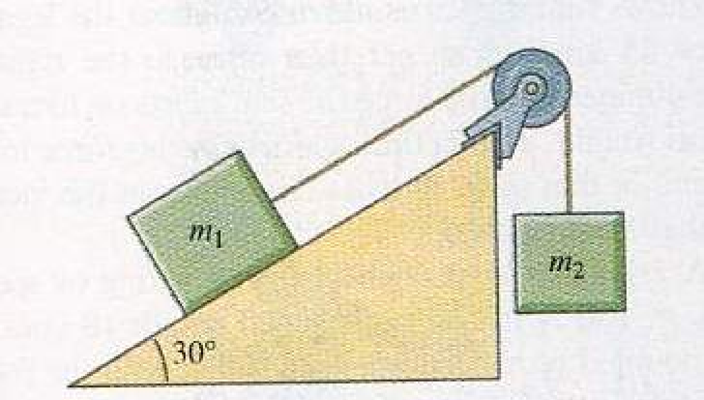


FIGURE 6-76 Problems 67, 68.

Assuming frictionless surfaces, find the acceleration of $\boldsymbol{m_2}$.

The two relevant equations are:

$$T - m_1 g \sin\theta = m_1 a \quad \text{and} \quad m_2 g - T = m_2 a$$

Taking into account that ***sin(θ)* = ½,** the acceleration is:

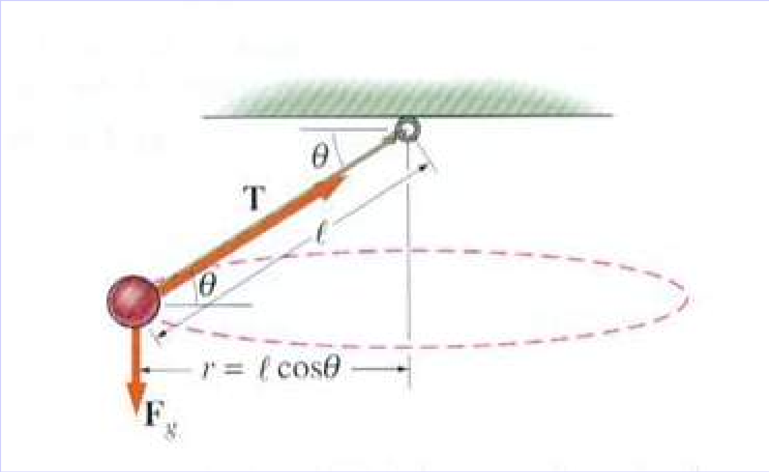
$$a = \frac{m_2 - m_1/2}{m_1 + m_2} g$$

As long as $\mathbf{m_2 > ½\, m_1}$ then the acceleration is consistent with $\boldsymbol{m_1}$ moving up the incline. If $\boldsymbol{m_2 < ½\, m_1}$ then $\boldsymbol{m_1}$ will accelerate down the incline.

What if there is friction on the surface of the incline?

$$T - m_1 g \sin\theta \mp \mu_k m_1 g \cos\theta = m_1 a \quad \text{and} \quad m_2 g - T = m_2 a$$

Third Example: Ball on a String

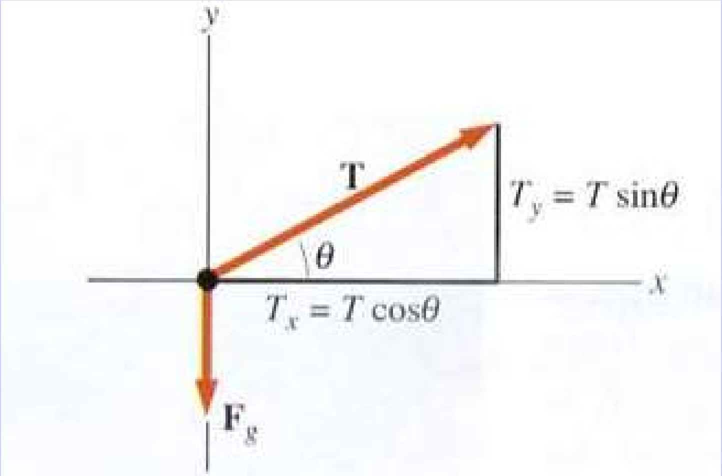


Consider a ball swinging in a horizontal circle on a string of length ℓ at an angle θ with the horizontal. Find

(a) the tension in the string and

(b) the speed of the ball.

There are only two forces acting on the ball, the tension T of the string and gravity F_g. The free-body diagram is

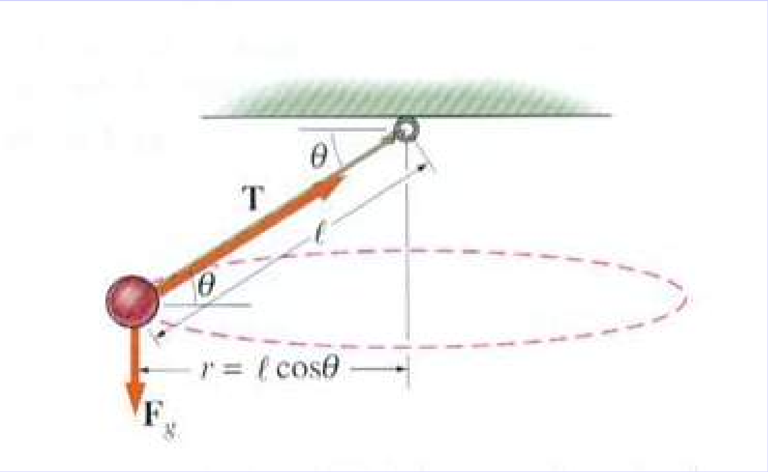


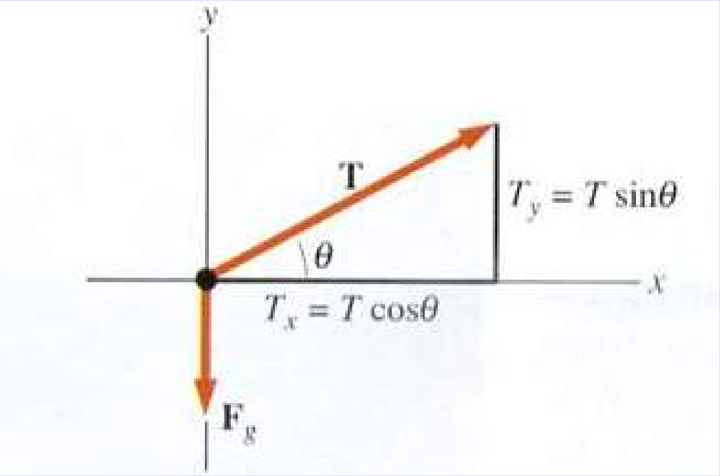
The only radial force is the radial component of the tension in the string! It is this force that induces the radial acceleration, $\boldsymbol{a_r = mv2/r}$. The EOM's in the radial and vertical direction are:

$$T_y = T\sin\theta = F_g = mg$$

$$T_r = T\cos\theta = mv^2/r = mv^2/\ell\cos\theta$$

Third Example: Ball on a String





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(a) the tension in the string and

(b) the speed of the ball.

$$T_y = T\sin\theta = F_g = mg$$
$$T_r = T\cos\theta = mv^2/r = mv^2/\ell\cos\theta$$

The tension comes from the *y* equation:

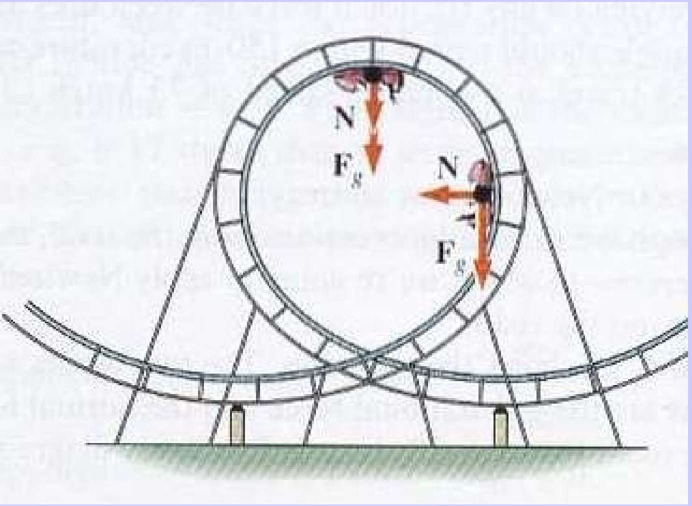
$$T = mg/\sin\theta$$

The velocity can then be found from the *x* EOM:

$$T\cos\theta = mg\cos\theta/\sin\theta = mv^2/\ell\cos\theta$$
$$v^2 = g\ell\cos^2\theta/\sin\theta$$

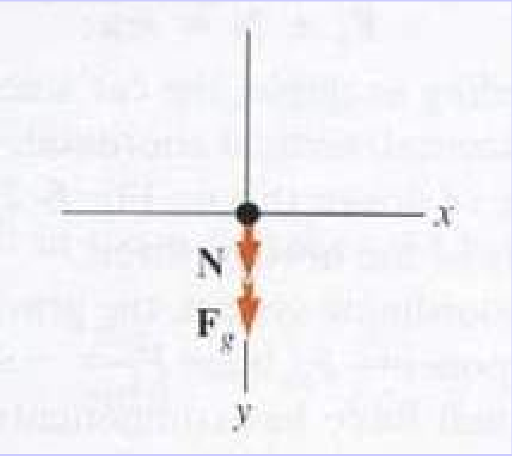
Do these results make physical sense?

Fourth Example: Loop-the-Loop



Consider a loop-the-loop roller coaster with a *6.3m* radius at the top. Find the minimum speed for the roller coaster to stay on the track at the top of the loop.

First consider the free-body diagram (at the top)



The EOM is:

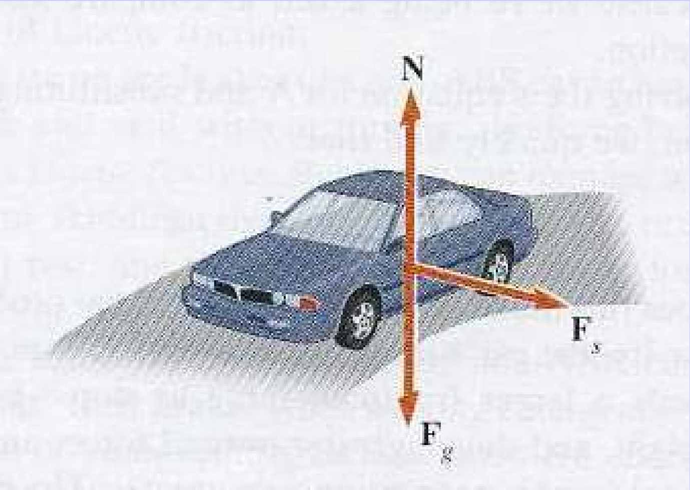
$$F_g + N = mg + N = ma_r = mv^2/r$$

The roller coaster looses contact with the track when ***N=0***. The solution for the minimum velocity is then:

$$v_{\min}^2 = gr \rightarrow v_{\min} = \sqrt{gr}$$

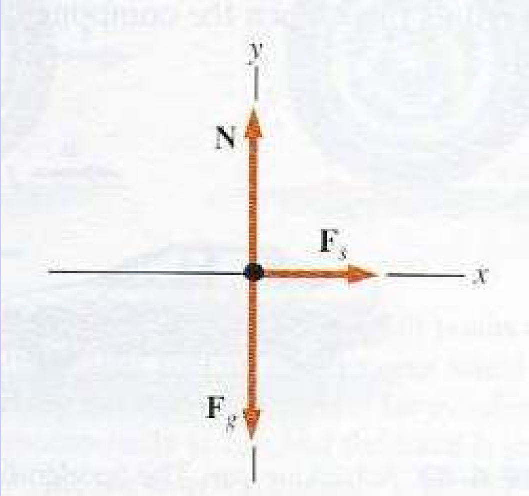
Does this result make sense?

Fifth Example: Rounding a Curve



A level road makes a 90° turn with a ***73m*** radius of curvature. What is the maximum speed for a car to negotiate this turn for a given coefficient of static friction, $\boldsymbol{\mu_s}$**=.88**?

As usual we first consider the free-body diagram.

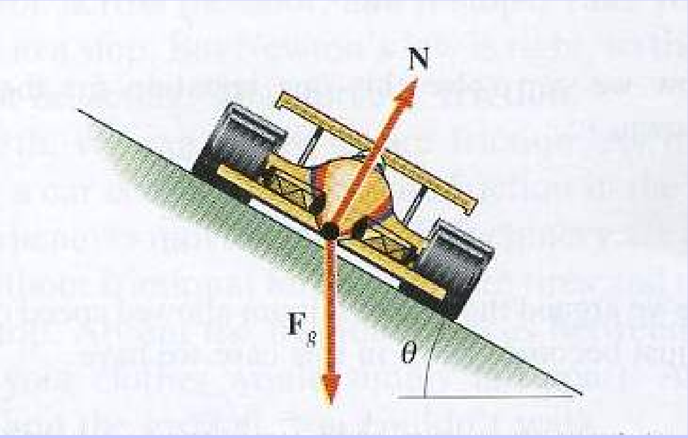


It is the force of static friction that acts to accelerate the car in the radial direction. The maximum frictional force is $\boldsymbol{\mu_s N}$. Since the normal force, ***N***, is simply $\boldsymbol{F_g}$, the radial EOM is:

$$\mu_s N = \mu_s mg = mv_{\max}^2/r$$

The maximum speed is: $v_{\max} = \sqrt{\mu_s gr} = \sqrt{.88(9.8)73} = 25.1 m/s = 90 km/hr$

Sixth Example: Rounding an Incline Curve

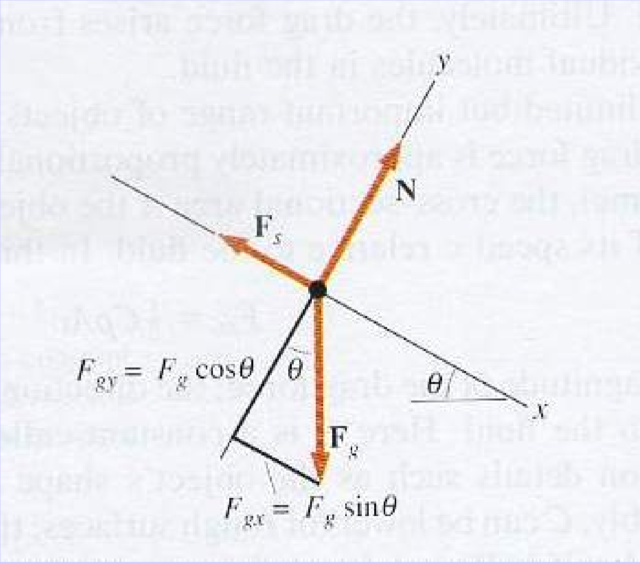


Suppose that a race car rounds a banked curve (on a track). Further suppose that the incline is steep enough so that if the car is traveling too slow it will slide down the incline. What is the range in speeds for a car to negotiate this turn for a given coefficient of static friction, $\boldsymbol{\mu}_{\mathbf{s}}$?

As usual we first consider the free-body diagram.

This is the correct diagram assuming that the car is traveling slow enough so that it requires a frictional force to keep it from sliding down the banked turn. So first we will solve for the minimum speed. Newton's law in vector form is:

$$\vec{F}_g + \vec{N} + \vec{F}_s = m\vec{a}$$



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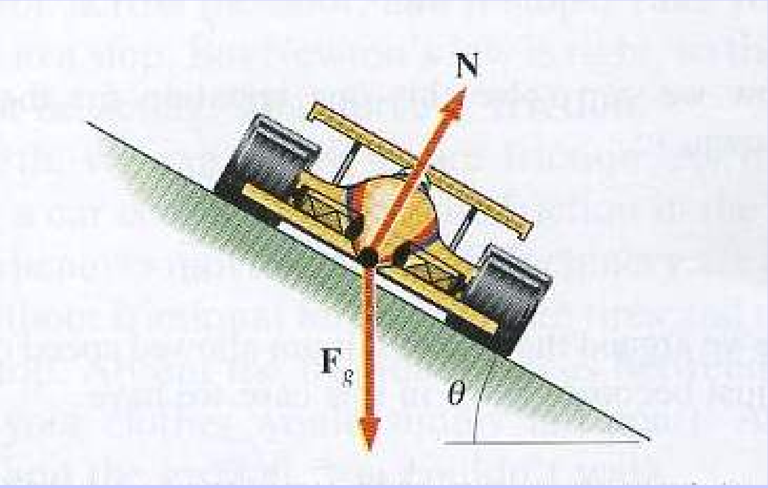
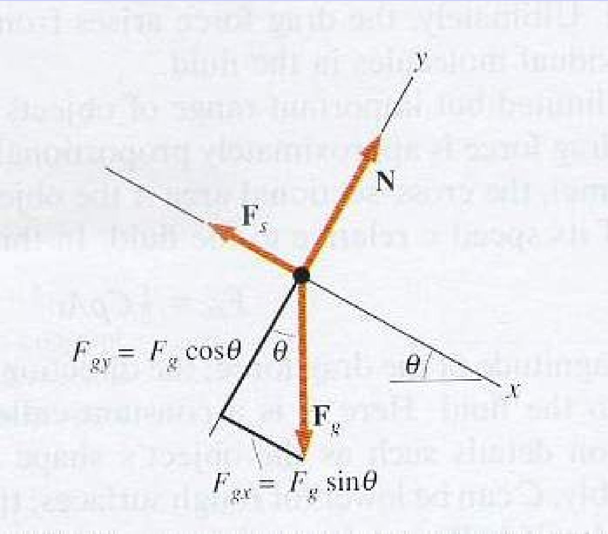


What is the range in speeds for a car to negotiate the banked turn given a coefficient of static friction?

As usual we first consider the free-body diagram.



This choice of coordinates is fine, but the the radial acceleration is horizontal. Thus the EOM's in these coordinates are:

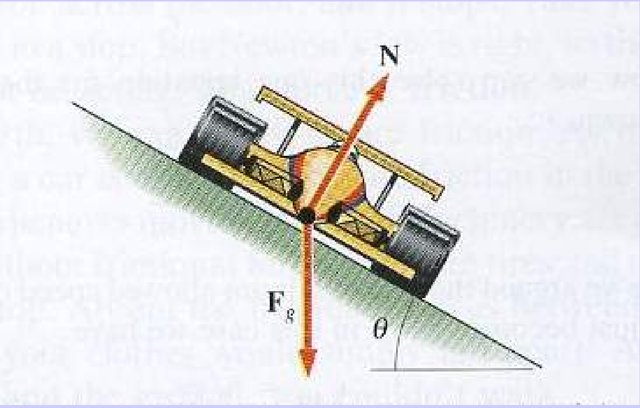
$$mg\sin\theta - \mu_s N = (mv_{\min}^2/r)\cos\theta$$
$$N - mg\cos\theta = (mv_{\min}^2/r)\sin\theta$$

Solving for N in the y equation and substituting into the x equation yields

$$g(\sin\theta - \mu_s\cos\theta) = (v_{\min}^2/r)(\cos\theta + \mu_s\sin\theta)$$

Note that θ is greater than that required for static equilibrium.

Sixth Example: Rounding an Incline Curve



What is the range in speeds for a car to negotiate the banked turn given a coefficient of static friction?

$$g(\sin\theta - \mu_s \cos\theta) = (v_{\min}^2/r)(\cos\theta + \mu_s \sin\theta)$$

Solving for the minimum velocity yields

$$v_{\min} = \sqrt{\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta} gr}$$

The solution for the maximum velocity can be obtained from this solution by replacing $\boldsymbol{\mu_s}$ with $\boldsymbol{-\mu_s}$. The result is:

$$v_{\max} = \sqrt{\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta} gr}$$

Assuming $\boldsymbol{\mu_s = .5, r = 80m}$, and an incline angle of 40^{o} these two velocities are:

$$v_{\min} = 13.7 m/s = 49.3 km/h \text{ and } v_{\max} = 42.5 m/s = 153 km/h$$

Seventh Example: Aerodynamic Drag

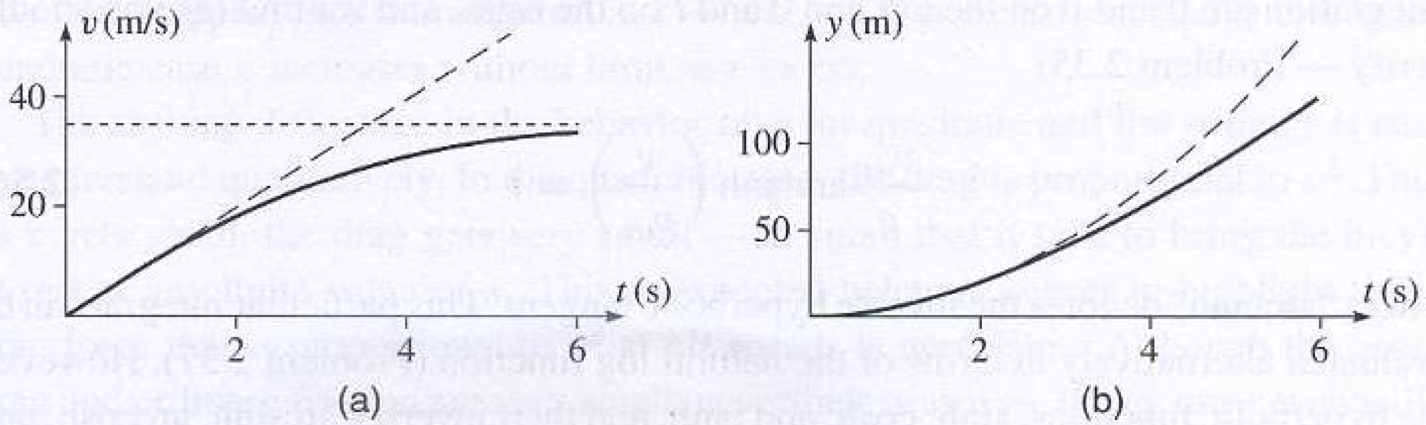


Figure 2.9 The motion of a baseball dropped from the top of a high tower (solid curves). The corresponding motion in a vacuum is shown with long dashes. **(a)** The actual velocity approaches the ball's terminal velocity $v_{ter} = 35$ m/s as $t \to \infty$. **(b)** The graph of position against time falls further and further behind the corresponding vacuum graph. When $t = 6$ s, the baseball has dropped about 130 meters; in a vacuum, it would have dropped about 180 meters.

The force of drag is (for most objects & speeds) of the form: $F_D = \frac{1}{2} C\rho A v^2.$

Here $\boldsymbol{\rho}$ is the density of the medium.

The terminal velocity for a vertically falling object is given by: $v_t = \sqrt{\frac{2mg}{C\rho A}}\,.$

Seventh Example: Aerodynamic Drag

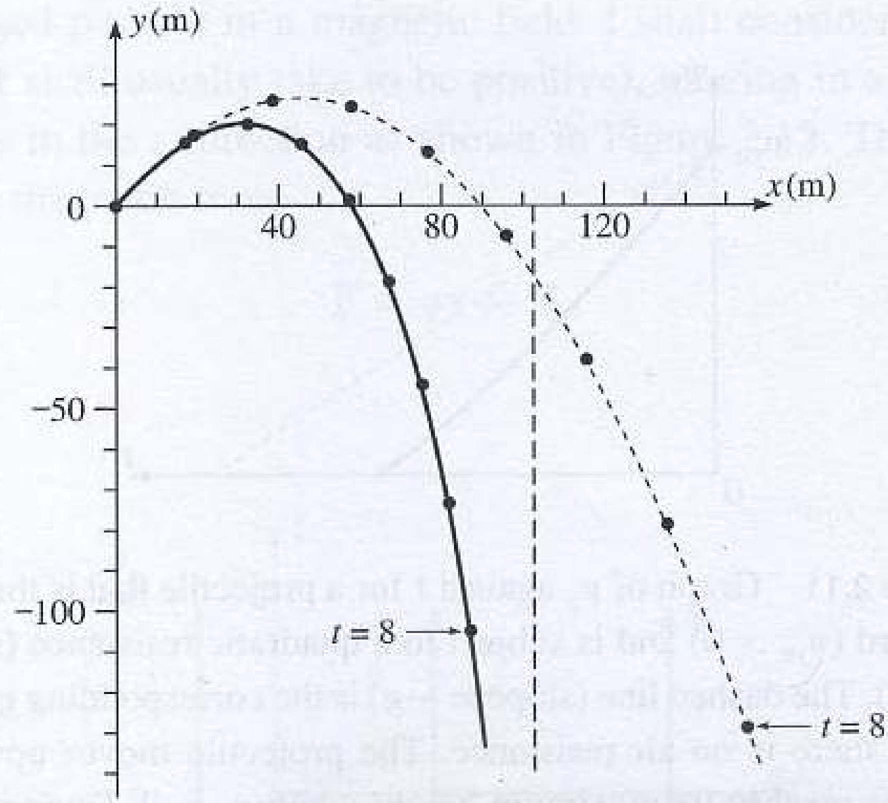
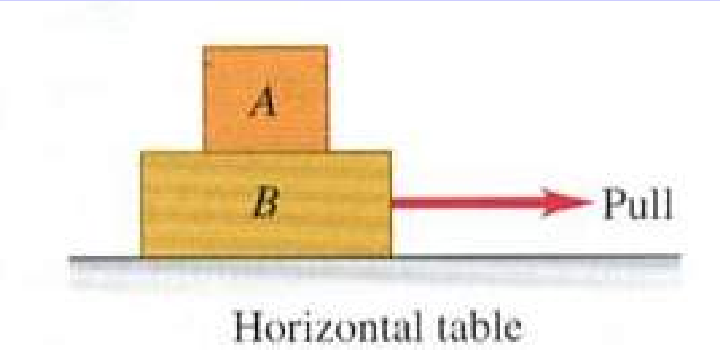


Figure 2.10 Trajectory of a baseball thrown off a cliff and subject to quadratic air resistance (solid curve). The initial velocity is 30 m/s at 50° above the horizontal; the terminal speed is 35 m/s. The

The trajectory is no longer parabolic and in general must be solved for numerically.

Eighth Example: Block on Block



Assume a frictionless surface between block B and the table, and a coefficient of static friction μ_s between block A and block B. What is the maximum force that you can pull on block B before block A slides off ?

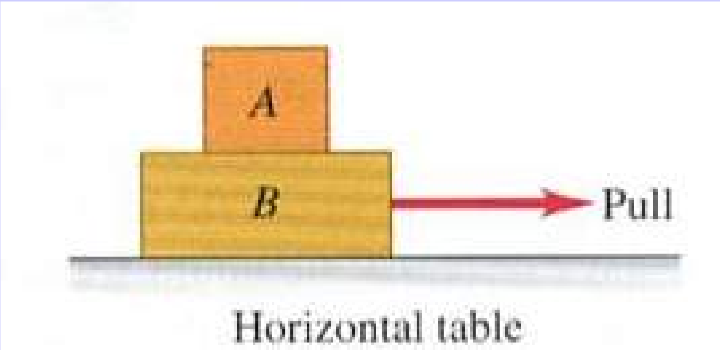
Again we need to consider the constraints to this problem before we consider a Free-Body diagram and the resulting EOM's. The maximum frictional force exerted on block A is $\mu_s N$ and it is this force that allows block A to be accelerated along with block B. Newton's third law tells us that this frictional force retards the acceleration of block B. Hence our vector EOM's are:

$$\vec{F} - \vec{F}_f = m_B \vec{a} \quad \text{and} \quad \vec{F}_f = m_A \vec{a}$$

When the frictional force is maximum the component form of these EOM's are:

$$F - \mu_s N = F - \mu_s m_A g = m_B a \quad \text{and} \quad \mu_s N = \mu_s m_A g = m_A a$$

Eighth Example: Block on Block



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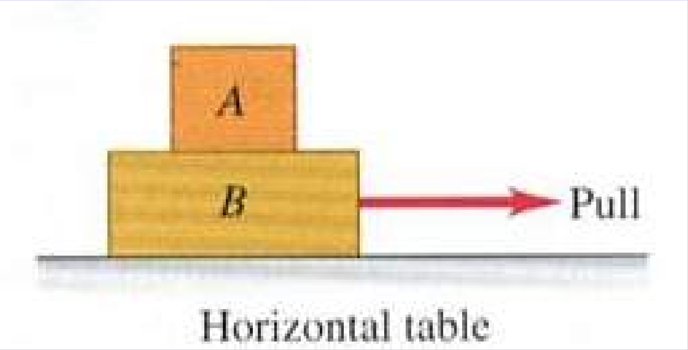
$$F - \mu_s N = F - \mu_s m_A g = m_B a \text{ and } \mu_s N = \mu_s m_A g = m_A a$$

First we note that solving for ***F*** yields $\boldsymbol{F = (m_A + m_B)a}$ which is exactly what we would expect. Since the maximum acceleration is given by $\boldsymbol{a = \mu_s g}$, the maximum force is:

$$F = (m_A + m_B)\mu_s g$$

Something interesting (??) happens when the force pulling block B increases beyond this limit. At that point the frictional force is reduced to $\boldsymbol{\mu_k N}$ and block A will slide backward at an accelerating rate relative to block B. Also the acceleration of block B will increase.

Eighth Example: Block on Block



When the force pulling block B increases beyond this limit, the frictional force is reduced to $\mu_{\mathbf{k}} N$. Block A then slides backward at an accelerating rate relative to block B. Also the acceleration of block B will increase. (a) Find the new acceleration of block B and compare it to the acceleration at the moment block A begins to slide. (b) How long will it take for block A to slide back a distance ***l*** after it begins to slide?

(a) The EOM for block B at threshold is:

$$F - \mu_k m_A g = m_B a_B \rightarrow (m_A + m_B)\mu_s g - \mu_k m_A g = m_B a_B$$

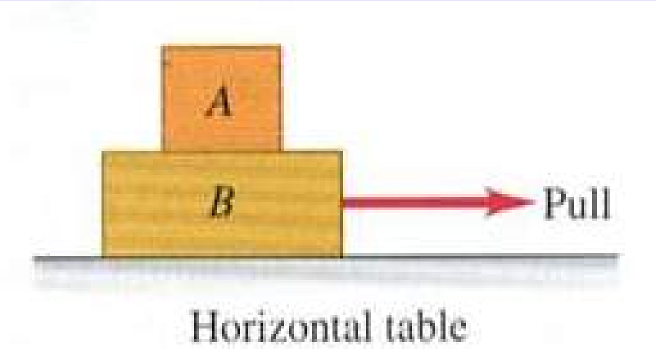
Solving for the acceleration of block B we find and we see that its acceleration has increased!

$$a_B = \mu_s g + \frac{m_A}{m_B}(\mu_s - \mu_k)g > \mu_s g$$

(b) The EOM for block A at threshold is:

$$\mu_k m_A g = m_A a_A \rightarrow a_A = \mu_k g$$

Eighth Example: Block on Block



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$$a_B = \mu_s g + \frac{m_A}{m_B}(\mu_s - \mu_k)g > \mu_s g$$

$$\mu_k m_A g = m_A a_A \rightarrow a_A = \mu_k g$$

The relative acceleration between the blocks is:

$$a = a_B - a_A = \mu_s g + \frac{m_A}{m_B}(\mu_s - \mu_k)g - \mu_k g$$
$$a = (\mu_s - \mu_k)\frac{m_A + m_B}{m_B} g$$

To find the time, it is now a simple matter of substituting this acceleration into:

$$t = \sqrt{2l/a}$$