Physics 214 UCSD/225a UCSB

Lecture 7

Finish Chapter 2 of H&M

- November revolution, charm and beauty

CP symmetry and violation

- Simple example
- Unitarity matrix for leptons and quarks

Beginning of Neutrino Physics

Missed a week due to fire in SD. Let's skip some stuff!

- Magnetic moment of proton etc.
- November revolution
  - Charm
  - Beauty
  - OZI suppression
- I encourage you to read up on this in chapter 2 of H&M

CP Symmetry

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu}$$

$$J^+_\mu = (\bar{\nu}_{eL} \bar{\nu}_{\mu L} \bar{\nu}_{\tau L}) \gamma_\mu \begin{pmatrix} e^-_L \\ \mu^-_L \\ \tau^-_L \end{pmatrix} + (\bar{u}_L \bar{c}_L \bar{t}_L) \gamma_\mu \mathbf{V_{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Note:

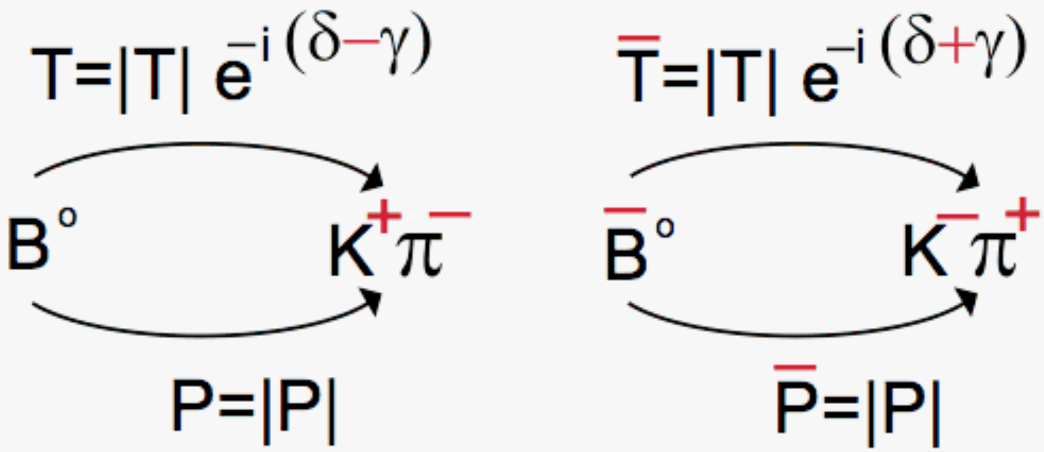
-> This requires CP because weak interactions maximally violate parity.

-> We will ignore subtleties in the difference between lepton and quark sector.

⇒We'll get back to this next quarter.

⇒All we care for now is that there's a *3x3 unitary matrix of couplings* involved.

Breaking CP is easy



$\delta =$ strong phase shift

$\gamma =$ difference in weak phase

CP $\gamma = -\gamma$ CP $\delta = +\delta$

$$A_{cp} = \frac{\mathcal{B}(B^0 \to K^+\pi^-) - \mathcal{B}(\bar{B}^0 \to K^-\pi^+)}{\mathcal{B}(B^0 \to K^+\pi^-) + \mathcal{B}(\bar{B}^0 \to K^-\pi^+)}$$

$$= \frac{-2|TP| \sin\gamma \sin\delta}{|T|^2 + |P|^2 + 2|TP| \cos\gamma \cos\delta}$$

⇒Add complex coupling to Lagrangian.

⇒Allow 2 or more channels

⇒Add CP symm. Phase, e.g. via dynamics.

Breaking CP in Standard Model

- Where does the CP violating phase come from?
  - 3x3 unitary matrix => 3 angles + 6 phases
    - $2N^2$ parameters, N^2 constraints from unitarity
  - 6 spinors with arbitrary phase convention
    - Only relative phase matters because only $|M|^2$ is physical.

    $\Rightarrow$Only 5 phases can be used to define a convention.

  $\Rightarrow$ One phase left in 3x3 matrix that has physical consequences.

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_x c_z & s_x c_z & s_z e^{-i\phi} \\ -s_x c_y - c_x s_y s_z e^{i\phi} & c_x c_y - s_x s_y s_z e^{i\phi} & s_y c_z \\ s_x s_y - c_x c_y s_z e^{i\phi} & -c_x s_y - s_x c_y s_z e^{i\phi} & c_y c_z \end{pmatrix}$$

x,y,z are euler angles. c=cos, s=sin.

Note: sin(z) = 0 <=> NO CP violating phase left !!!

CP violation summary

- CP violation is easy to add in field theory:
  - Complex coupling in Lagrangian
  - Interference of channels with:
    - Different CP violating phase
    - Different CP conserving phase
- Standard Model implements this via:
  - CP violating phase in charged current coupling across 3 families
  - CP conserving phase via:
    - Dynamics, e.g. Breit Wigner resonance lineshape
    - Flavor Mixing & oscillation in neutrino or quark sector

Let's look at neutrino sector in some detail !

Aside:

- If you want to know more about the details, please check out:

Lecture 9/20/2000 and further reading for it

- It constructs all possible conventions for the CKM matrix in probably more detail than you ever want to know.

Mixing in Standard Model

- Weak eigenstates not equal mass eigenstates.
  - Mass eigenstates responsible for propagation in time.
  - Weak eigenstates responsible for production and/or decay.

$\Rightarrow$Oscillation between weak eigenstates as a function of time.

$\Rightarrow$Discuss this in detail for Neutrino sector now.

Neutrino mixing

- At the W vertex an electron-neutrino is created together with a positron.
- That electron-neutrino is a superposition of mass eigenstates:

$$\left|\nu_e(t)\right\rangle = \sum_{i=1}^{3} U_{ei}^{*}\left|\nu_i(t)\right\rangle$$

- The time evolution of the mass eigenstate can be described either in its rest-frame or in the labframe:

$$\left|\nu_i(t)\right\rangle = e^{-im_i t_i}\left|\nu_i(0)\right\rangle = e^{-i(E_i t - p_i L)}\left|\nu_i(0)\right\rangle$$

- For interference among the mass eigenstates to be possible, they all have to have the same E because experimentally we average over time.

Time average demands E_i=E

Oscillation Amplitude

$$Amp(\nu_\mu \rightarrow \nu_\tau) = \left\langle \nu_\tau \middle| e^{-iEt} \sum_{i=1}^{3} e^{ip_i L} U_{\mu i}^{*} \middle| \nu_i \right\rangle$$

$$Amp(\nu_\mu \rightarrow \nu_\tau) = e^{-iEt} \sum_{i,j=1}^{3} e^{ip_i L} U_{\mu i}^{*} U_{\tau j} \left\langle \nu_j \middle| \nu_i \right\rangle$$

$$Amp(\nu_\mu \rightarrow \nu_\tau) = e^{-iEt} \sum_{i=1}^{3} e^{ip_i L} U_{\mu i}^{*} U_{\tau i}$$

Next we taylor expand p_i using:

$$p_i = \sqrt{E^2 - m_i^2} = E - \frac{m_i^2}{2E} + \ldots$$

Oscillation Probability

$$Amp(\nu_\mu \rightarrow \nu_\tau) = e^{-iE(t-L)} \sum_{i=1}^{3} e^{-i\frac{m_i^2}{2E}L} U_{\mu i}^{*} U_{\tau i}$$

$$\Pr ob(\nu_\mu \rightarrow \nu_\tau) = \sum_{i=1}^{3} \left| e^{-i\frac{m_i^2}{2E}L} U_{\mu i}^{*} U_{\tau i} \right|^2$$

In homework, you do this for the general case of N flavors.
Here we do it for the simpler case of 2 flavors only.

Simple math aside

$$|1-e^{ix}|^2 = (1-[\cos x + i\sin x])(1-[\cos x - i\sin x])$$

$$= [1-\cos x]^2 + \sin^2 x$$

$$= 2(1-\cos x)$$

We'll need this is a second.

2 flavor oscillation probability

$$\left|U_{11}U_{21}e^{-im_1^2\frac{L}{2E}} + U_{12}U_{22}e^{-im_2^2\frac{L}{2E}}\right|^2 = \left|U_{11}U_{21} + U_{12}U_{22}e^{i(m_1^2-m_2^2)\frac{L}{2E}}\right|^2$$

$$= \left|-\cos\theta\sin\theta + \cos\theta\sin\theta e^{i(m_1^2-m_2^2)\frac{L}{2E}}\right|^2 = \cos^2\theta\sin^2\theta\left|1 - e^{i(m_1^2-m_2^2)\frac{L}{2E}}\right|^2$$

$$= \cos^2\theta\sin^2\theta\left[(1-\cos\Delta)^2 + \sin^2\Delta\right] = 2\cos^2\theta\sin^2\theta\left[1-\cos\Delta\right]$$

$$= \frac{1}{2}\sin^2 2\theta\left[2\sin^2\frac{\Delta}{2}\right]$$

$$\Delta = (m_1^2 - m_2^2)\frac{L}{2E}$$

This is a bit simplistic, as it ignores matter effects. We'll discuss those on Wednesday.

Discussion of Oscillation Equation

$$\Pr ob(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \left[\sin^2 \frac{(m_1^2 - m_2^2)L}{4E} \right]$$

- Depends on difference in mass squared.
  - No mixing if masses are identical
  - Insensitive to mass scale
  - Insensitive to mass hierarchy
- Depends on $\sin^2(2\theta)$
  - Need large angle to see large effect
- Depends on L/4E
  - Exp. with unfortunate L/E won't see any effect.
  - Exp. with variable L/E can measure both angle and mass squared difference.
  - Exp. with Δm^2 L/4E >>1 and some energy spread average over $\sin^2$ -> 1/2

Experimental situation

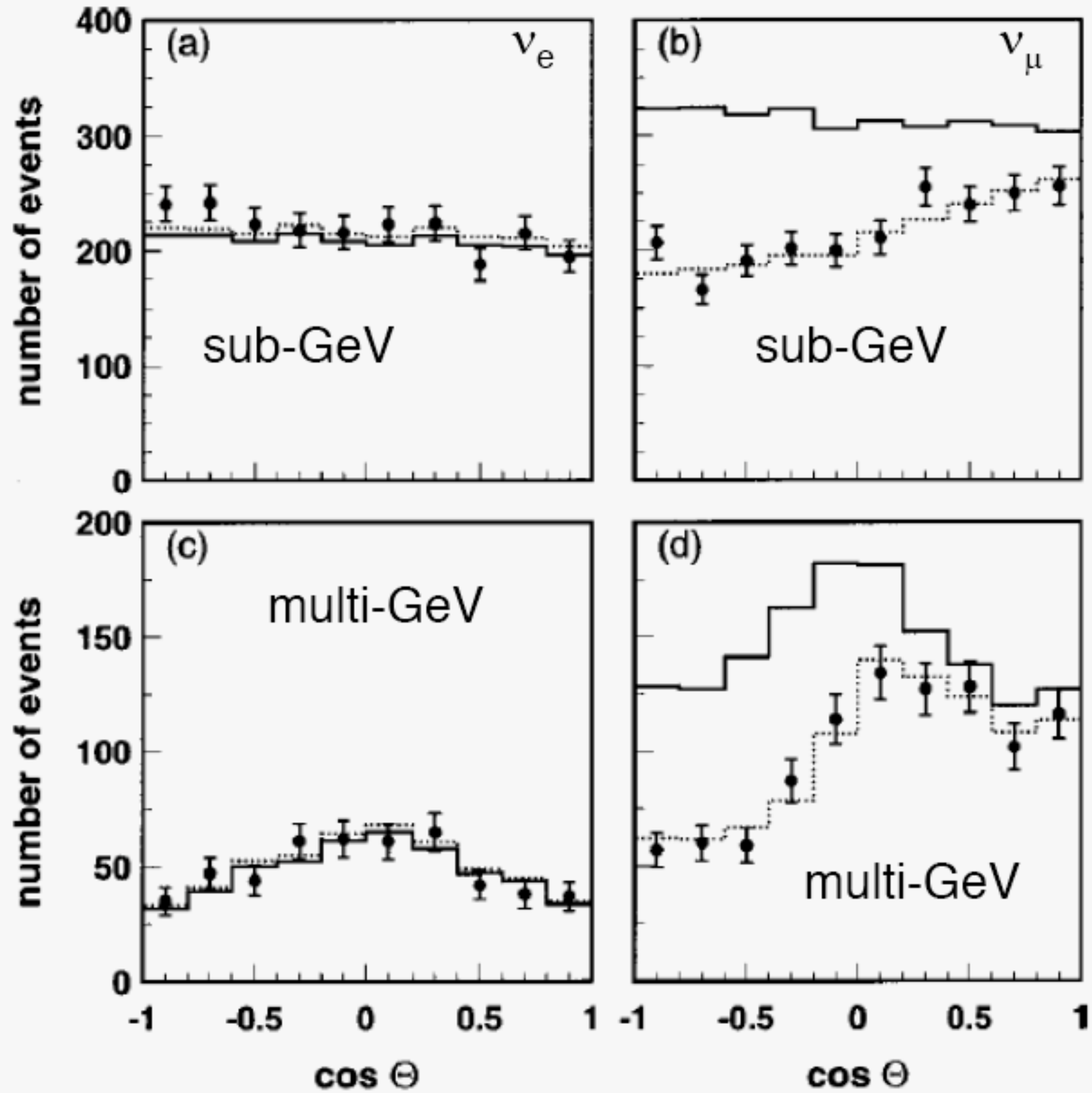
- Sources of electron neutrinos
  - Sun
  - Reactors

- Sources of muon neutrinos
  - From charged pion beams
  - From charged pion decay in atmosphere

Atmospheric neutrinos

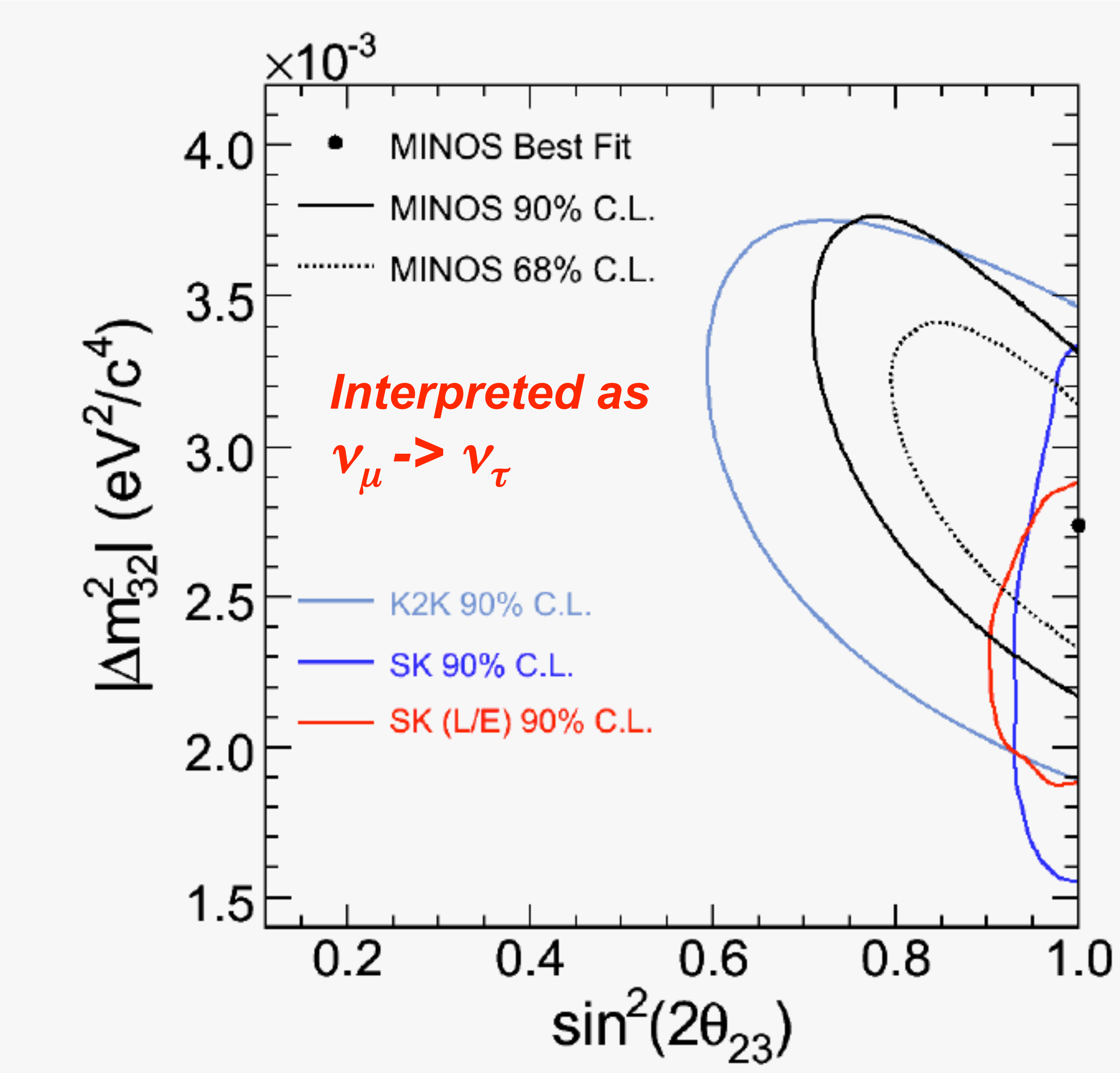
- Expect ν_μ anti-ν_μ in equal numbers
- Expect ν_e half as many as ν_μ + anti-ν_μ
- Can change L as a function of Zenith angle. (L ~ 15km to L ~ 13,000km)
- ν_e Oscillation to ν_μ
  => See excess of ν_μ vs zenith angle
- ν_μ Oscillation to ν_e
  => See excess of ν_e vs zenith angle
- ν_e Oscillation to ν_τ
  => Deficit of ν_e vs zenith angle
- ν_μ Oscillation to ν_τ
  => Deficit of ν_μ vs zenith angle

Super Kamiokande Results



cosθ=-1 up-going, L≈13,000 km

cosθ=1 down-going, L≈15 km

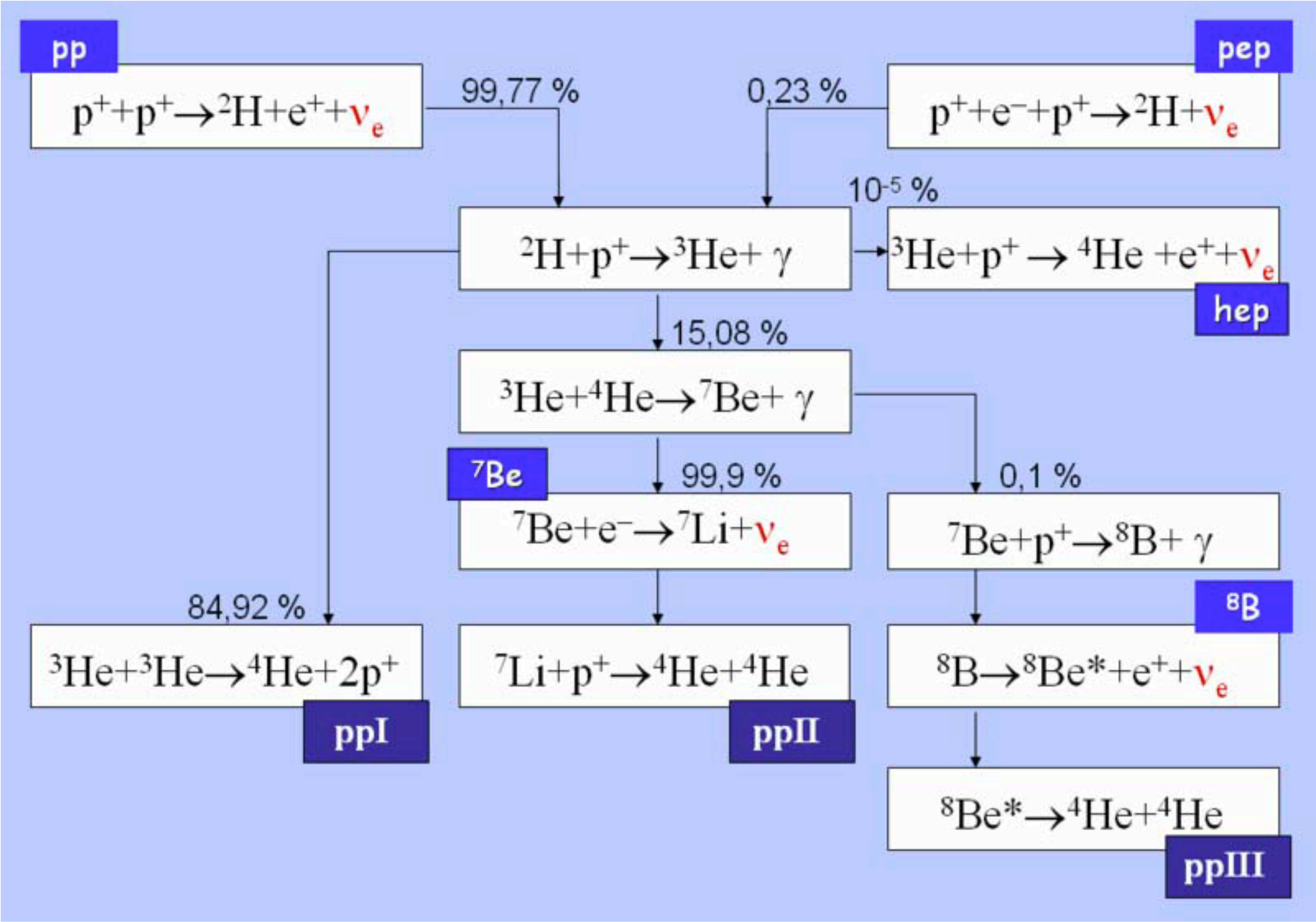


×10^-3
4.0
3.5
3.0
2.5
2.0
1.5
0.2
0.4
0.6
0.8
1.0
$|\Delta m^2_{32}|$ (eV²/c⁴)
$\sin^2(2\theta_{23})$
MINOS Best Fit
MINOS 90% C.L.
MINOS 68% C.L.
Interpreted as
ν_μ -> ν_τ
K2K 90% C.L.
SK 90% C.L.
SK (L/E) 90% C.L.

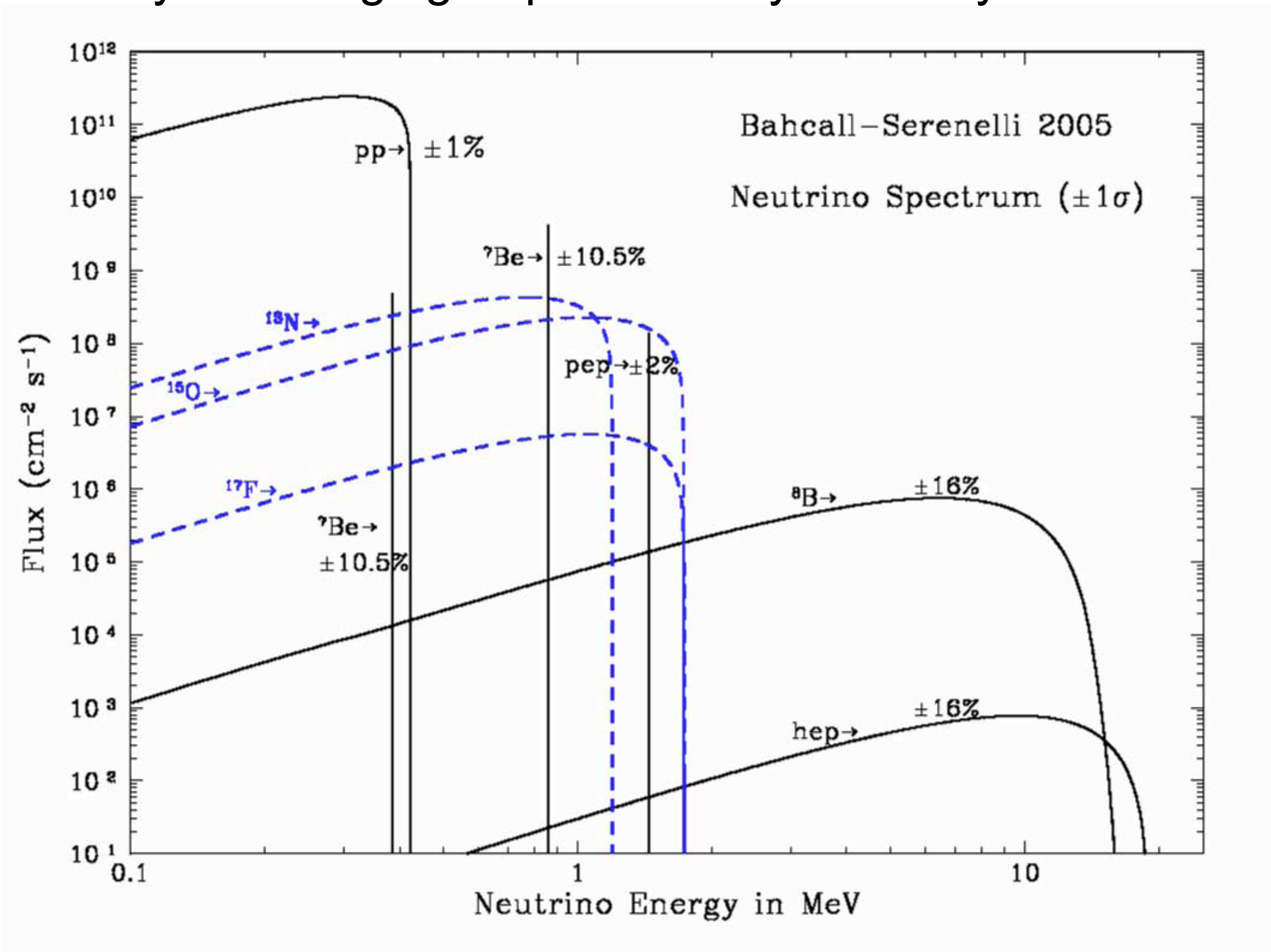
Neutrinos from the Sun

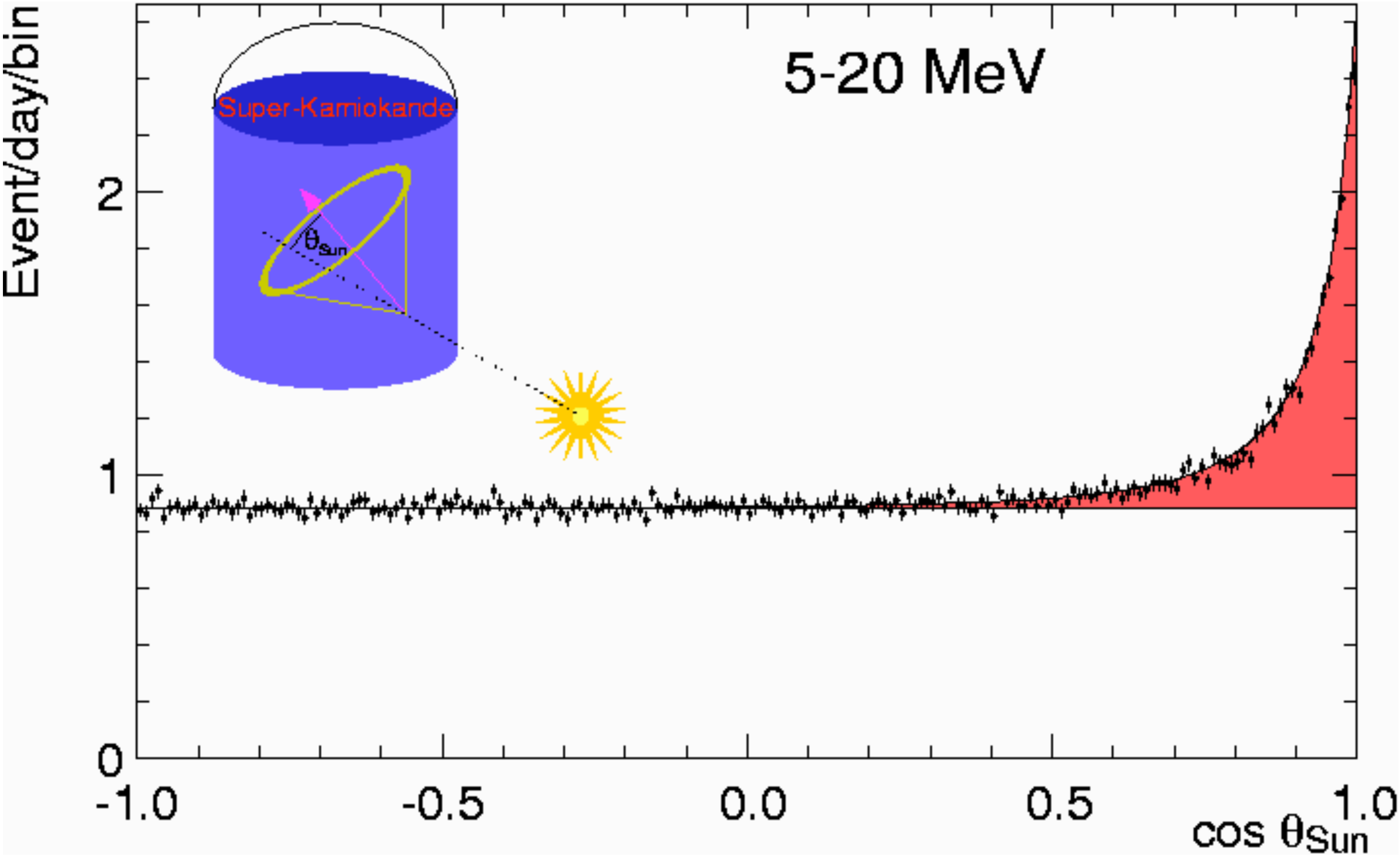
- Many mechanisms, all leading to electron neutrinos with varying energies.
  - Expect: 0.5 $\sin^2(2\theta)$ of solar model flux convolved with energy dependent efficiency.
- Neutrino energy too low to produce either muons or taus.
  - Electron disappearance experiments only in all but one experiment (SNO).

Solar Model is Quite Complex



Neutrino Energies are quite small
Very Challenging Experimentally for many decades

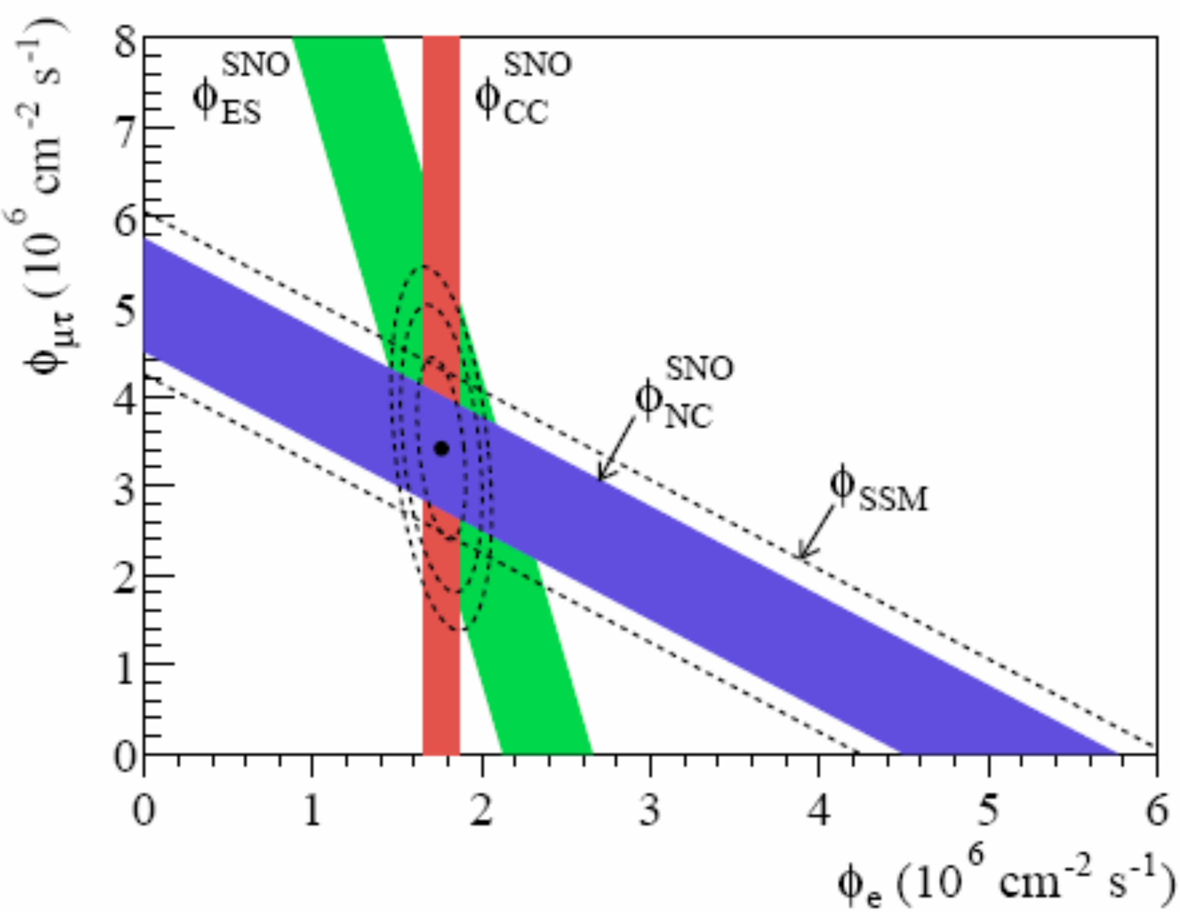




$0.465 \pm 0.005^{+0.016}_{-0.015}$ of expectation

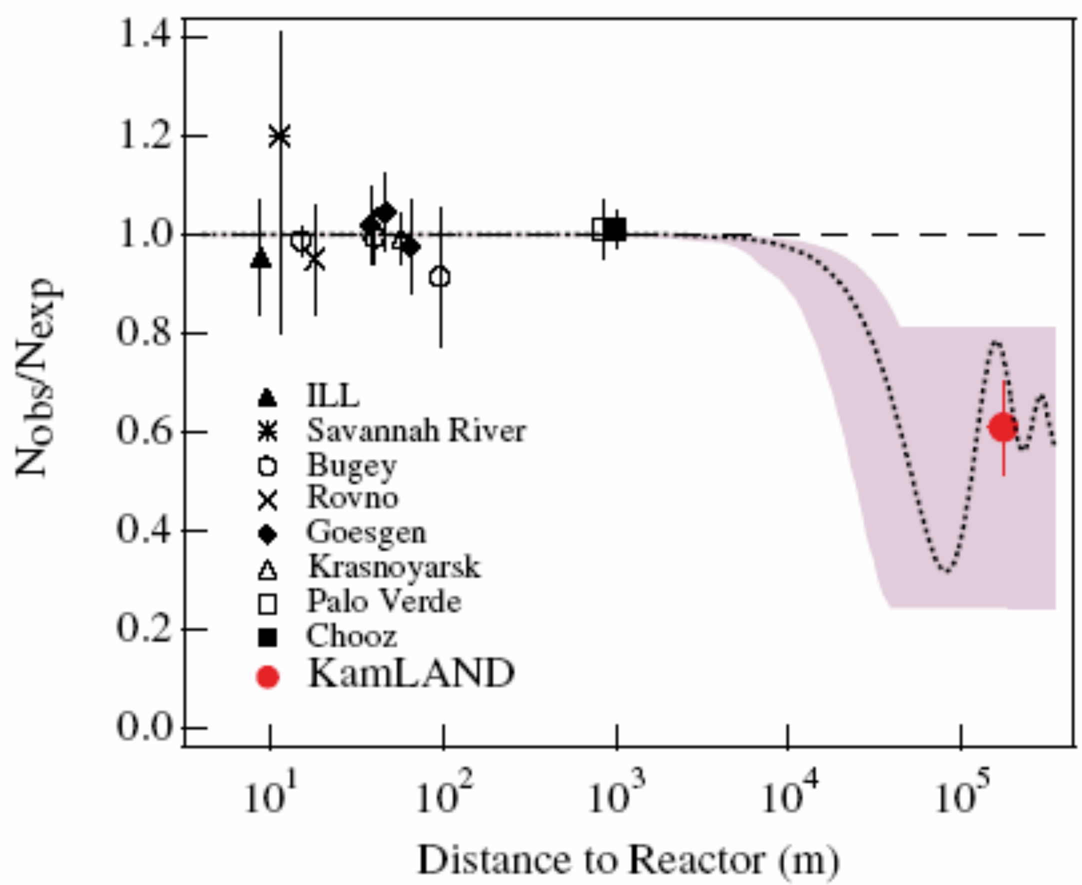
SNO allowed CC and NC, and was thus sensitive to all neutrino flavors => measures solar flux and electron neutrino flux.

$$\nu_e + \mathrm{d} \rightarrow \mathrm{p} + \mathrm{p} + \mathrm{e}^- \qquad \text{(CC)},$$
$$\nu_x + \mathrm{d} \rightarrow \mathrm{p} + \mathrm{n} + \nu_x \qquad \text{(NC)},$$
$$\nu_x + \mathrm{e}^- \rightarrow \nu_x + e^- \qquad \text{(ES)}.$$



Interpreted as $\nu_e \rightarrow \nu_\mu$

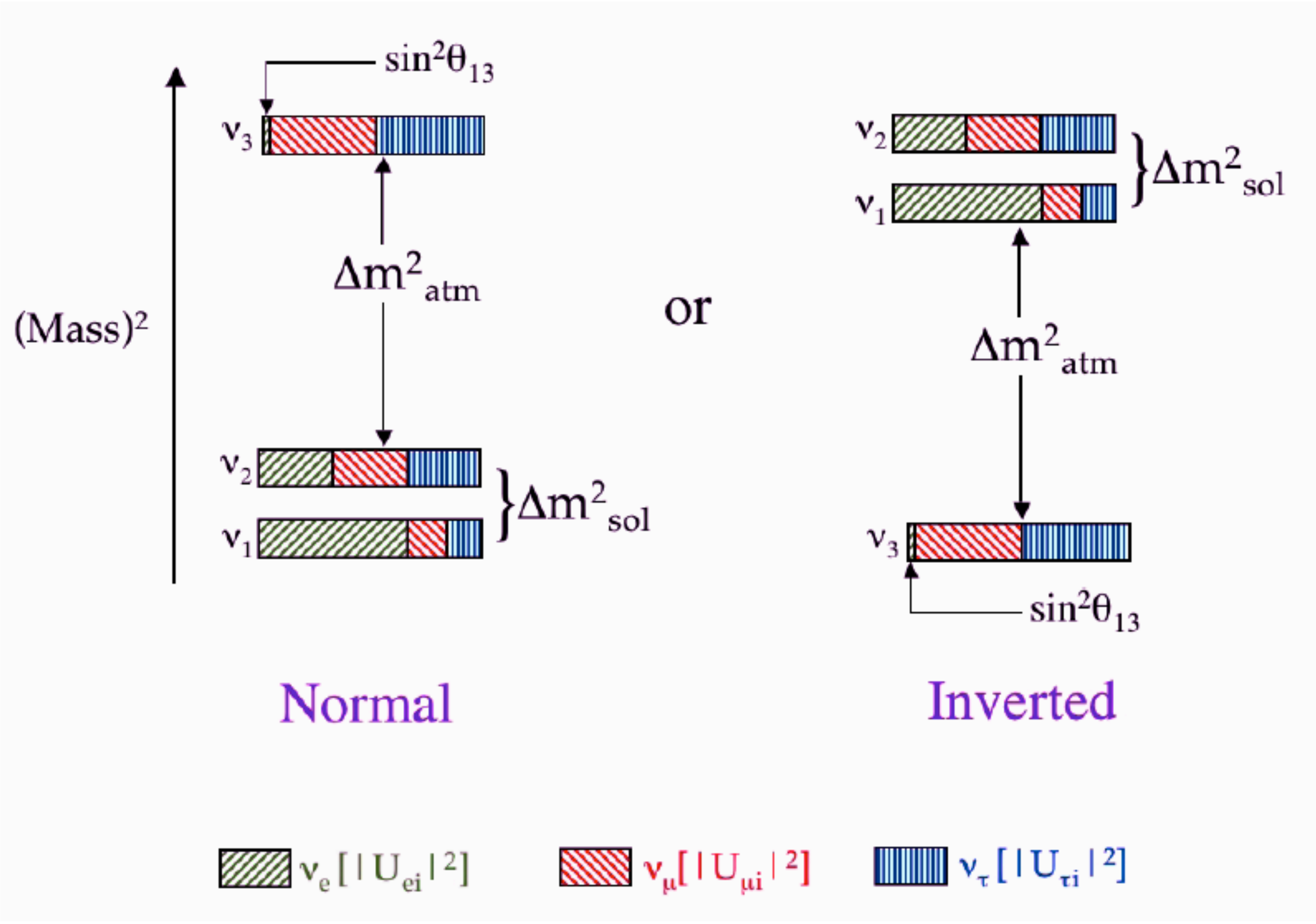
Reactor Experiments
All except KamLAND had L that is too small!
=> Only KamLAND saw oscillations !!!



Interpretation

- Atmospheric must be **ν_μ -> ν_τ**
  - Though tau appearance has never been seen.
  - However, electron appearance is ruled out.
  - The state that is far in mass from the other two must have very little electron neutrino content!

Two Possible Mass Hierarchies



Things we have not discussed yet.

- Majorana Neutrinos -> see homework
- “Size of CP violation” -> see homework
- Getting well collimated E via off-axis -> see homework
- Reactor neutrinos and sintheta13 -> see homework
- Resolving the mass hierarchy -> Wednesday.