Physics 214 UCSD/225a UCSB

Lecture 13

- Schedule for remaining Quarter
- Finish H&M Chapter 6
- Start H&M Chapter 8

Schedule for remaining Quarter

- Week of 11/26 - 11/30
  - Mo lecture
  - Tuesday 2-4pm Seminars
  - We lecture
  - Thursday 2-4pm Seminars
  - Thursday 6pm start of take-home final
- Week of 12/3 - 12/7
  - Mo lecture: hand-in take home final before lecture.
  - We lecture
  - Quarter Finished
- Week of 12/10 - 12/14
  - You get your grades before the week is over.

Propagators

Spinless: $$\frac{i}{p^2 - m^2}$$

Massive Vector Bosons:

$$\frac{i\left(-g^{\mu\nu} + p^{\mu}p^{\nu} / M^2\right)}{p^2 - M^2}$$

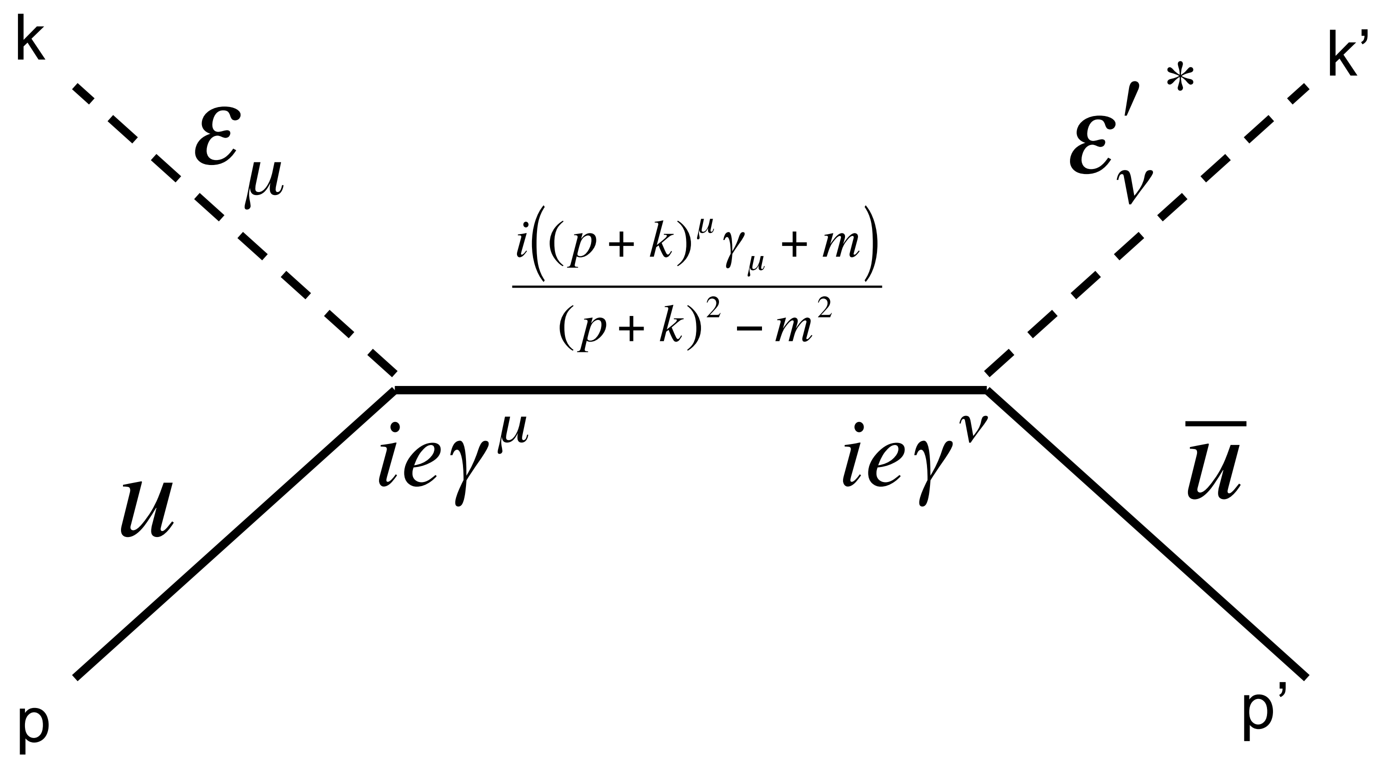
See H&M Ch.6.10ff for more details.

Spin 1/2, e.g. electron:

$$\frac{i\sum_s \bar{u}u}{p^2 - m^2} = \frac{i\left(p^{\mu}\gamma_{\mu} + m\right)}{p^2 - m^2}$$

Photon: $$\frac{-ig_{\mu\nu}}{q^2}$$

Compton Scattering: e- gamma -> e- gamma



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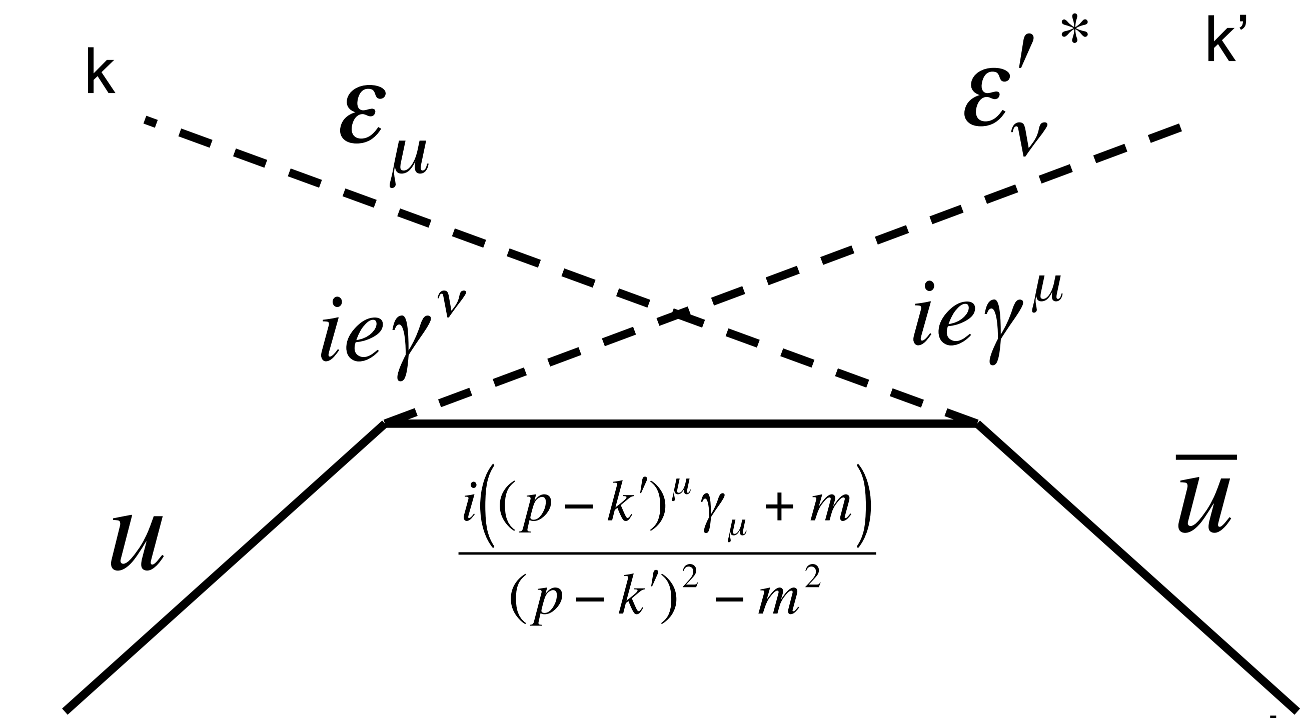


$$\overline{|M|^2} = \overline{|M_1 + M_2|^2} = 2e^4\left(-\frac{u}{s} - \frac{s}{u}\right)$$

Where we neglect the electron mass, and refer to H&M Chapter 6.14 for details.

Piar annihilation via crossing

- Like we've done before: u <-> t

$$\overline{|M|^2} = \overline{|M_1 + M_2|^2} = 2e^4\left(-\frac{t}{s} - \frac{s}{t}\right)$$

t = -2 kk' = -2 pp'
u = -2 kp' = -2 k'p

Ignoring the electron mass.

First step towards chapter 8

- In chapter 8 we investigate the structure of hadrons by scattering electrons of charge distributions that are at rest in the lab.
- As an initial start to formalism review e- mu- scattering with the initial muon at rest.
- Let's start with what we got last time, neglecting only terms with electron mass:

$$\overline{|M|^2} = \frac{8e^4}{t^2}\left[(k'p')(kp) + (k'p)(kp') - M^2 kk'\right]$$

$$\overline{|M|^2} = \frac{8e^4}{t^2}\left[(k'p')(kp) + (k'p)(kp') - M^2 kk'\right]$$

As we will want frame for which p = (M,0), it's worth rewriting this using q = k - k' .

As we won't care for the muon recoil p', we eliminate p' via:

p' = k - k' + p.

As we ignore the electron mass, we'll drop terms with k^2, or k'^2, and simplify $q^2 = -2kk'$.

We then get:

$$\overline{|M|^2} = \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2(kp - k'p) + 2(k'p)(kp) + \frac{1}{2}M^2 q^2\right]$$

I'll let you confirm this for yourself.

$$\overline{|M|^2} = \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2(kp - k'p) + 2(k'p)(kp) + \frac{1}{2}M^2q^2\right]$$

Now got to muon restframe: p = (M,0)
This means kp = EM and k'p = E'M .

$$\overline{|M|^2} = \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2M(E - E') + 2EE'M^2 + \frac{1}{2}M^2q^2\right]$$

$$= \frac{8e^4}{q^4}2EE'M^2\left[-\frac{q^2}{2M^2}\frac{M(E - E')}{2EE'} + 1 + \frac{q^2}{4EE'}\right]$$

$$= \frac{8e^4}{q^4}2EE'M^2\left[-\frac{q^2}{2M^2}\frac{M(E - E')}{2EE'} + 1 - \sin^2\frac{\theta}{2}\right]$$

$$= \frac{8e^4}{q^4}2EE'M^2\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

$q^2 = -2kk' = -4EE'\sin^2\theta/2$

$q^2 = -2pq = -2M(E-E')$

$$\overline{|M|^2} = \frac{8e^4}{q^4} 2EE'M^2 \left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2} \right]$$

Now recall how to turn this into dσ in the labframe:

$$d\sigma = \frac{1}{64\pi^2} \frac{\delta^{(4)}(p_D + p_C - p_A - p_B)}{\underbrace{v_A}_{=1} E_A E_B} |M|^2 \frac{dp_c^3 dp_D^3}{E_c E_D}$$

(slide 15 lecture 10)

$$d\sigma = \frac{1}{64\pi^2} \frac{\delta^{(4)}(p + k - p' - k')}{EM} |M|^2 \frac{d^3k' d^3p'}{E' p_0'}$$

$$= \frac{|M|^2}{4\pi^2} \frac{\delta^{(4)}(p + k - p' - k')}{4EM} \frac{E' dE' d\Omega}{2} \frac{d^3p'}{2p_0'}$$

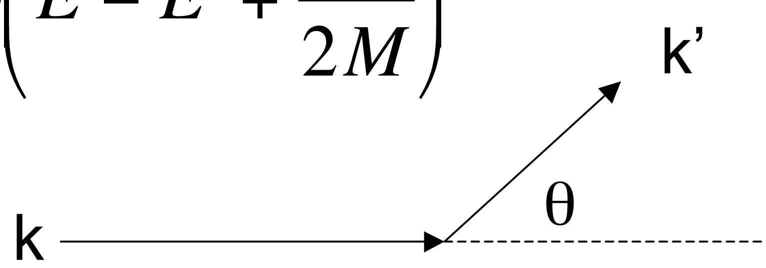
$$d\sigma = \frac{|M|^2}{4\pi^2}\,\frac{\delta^{(4)}(p+k-p'-k')}{4EM}\,\frac{E' d^3 E' d\Omega}{2}\,\frac{d^3 p'}{2p_0'}$$

What do we do about this ?

Recall, we are heading towards collissions between electron and hadron. The hadronic mess in the final state is not something we care to integrate over !!!

Exercise 6.7 in H&M:

$$\int \delta^{(4)}(p+q-p')\frac{d^3 p'}{2p_0'} = \frac{1}{2M}\delta\left(E - E' + \frac{q^2}{2M}\right)$$



Putting it all together:

$$\left\{\begin{aligned} \frac{d\sigma}{dE'd\Omega} &= \frac{|M|^2}{4\pi^2}\frac{E'}{8EM}\int \delta^{(4)}(p+q-p')\frac{d^3p'}{2p_0'} \\ \overline{|M|^2} &= \frac{8e^4}{q^4}2EE'M^2\left[\cos^2\frac{\theta}{2}-\frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right] \end{aligned}\right.$$

$$\frac{d\sigma}{dE'd\Omega} = 4\alpha^2\frac{2ME'^2}{q^4}[\ldots]\int \delta^{(4)}(p+q-p')\frac{d^3p'}{2p_0'}$$

$$\frac{d\sigma}{dE'd\Omega} = \frac{4E'^2\alpha^2}{q^4}\left[\cos^2\frac{\theta}{2}-\frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]\delta\left(E-E'+\frac{q^2}{2M}\right)$$

Now we perform the E' integration by noticing:

$$\frac{1}{2M}\delta\left(E - E' + \frac{q^2}{2M}\right) = \frac{\delta\left(E' - {E}/{A}\right)}{2MA}$$

$$A \equiv 1 + \frac{2E}{M}\sin^2\frac{\theta}{2}$$

Exercise 6.7 H&M

We then finally get:

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{4\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

This is completely independent of the target's final state !!!

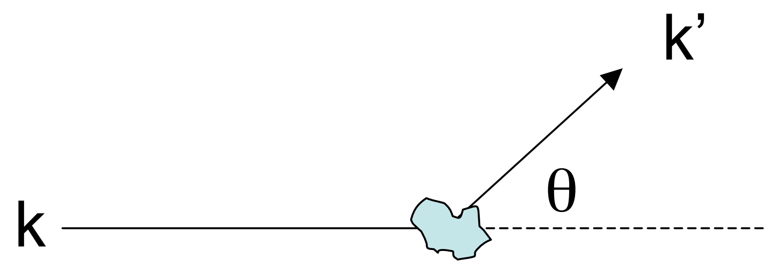
Experimental importance

- You measure only initial and final state electron.
- You have a prediction for electron scattering off a spin 1/2 point particle with charge = -1.
- Any deviation between measurement and prediction indicates substructure of your supposed point particle !!!

- Next: How does one describe scattering off a charge distribution, rather than a point particle?

=> Beginning of chapter 8 !

Probing the Structure of Hadrons with electron scattering



- All you measure is the incoming and outgoing electron 3 momentum.
- If you had a static target then you can show that this gives you directly the fourier transform of the charge distribution of your target:

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{point} \bullet \left|F(q)\right|^2$$

$$F(q) = \int \rho(x) e^{iqx} d^3x$$

Once the target is not static, we're best of using e-mu scattering as our starting point.

e-proton vs e-muon scattering

- What's different?
- If proton was a spin 1/2 point particle with magnetic moment e/2M then all one needs to do is plug in the proton mass instead of muon mass into:

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{4\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right]$$

- However, magnetic moment differs, and we don't have a point particle !!!

e-proton vs e-muon scattering

- Let's go back to where we started:
  - What's the transition current for the proton ?

$$J^{\mu} = -e\overline{u}(k')\gamma^{\mu}u(k)e^{i(k'-k)x}$$ electron

$$J^{\mu} = -e\overline{u}(p')\left[???\right]u(p)e^{i(p'-p)x}$$ proton

We need to find the most general parametrization for [???], and then measure its parameters.

What is [???]

- This is a two part problem:
  - What are the allowed 4-vectors in the current ?
  - What are the independent scalars that the dynamics can depend on ?
- Let's answer the second first:

$$M^2 = p'^2 = (p+q)^2 = M^2 + 2pq + q^2$$

- I can thus pick either pq or q^2 as my scalar variable to express dependence on kinematics.

What's are the 4-vectors allowed?

- Most general form of the current:

$$J^{\mu} = -e\overline{u}(p')\left[???\right]u(p)e^{i(p'-p)x}$$

$$\left[???\right] = \gamma^{\mu}K_1 + i\sigma^{\mu\nu}\left(p'-p\right)_{\nu}K_2 + i\sigma^{\mu\nu}\left(p'+p\right)_{\nu}K_3 +$$

$$+\left(p'-p\right)^{\mu}K_4 + \left(p'+p\right)^{\mu}K_5$$

- Gordon Decomposition of the current: any term with (p+p') can be expressed as linear sum of components with γ^{μ} and $\sigma^{\mu\nu}$ (p'-p).
- K_4 must be zero because of current conservation.

Gordon Decomposition

- Exercise 6.1 in H&M:

$$\overline{u}(p')\gamma^{\mu}u(p) = \overline{u}(p')\left[\left(p'+p\right) + i\sigma^{\mu\nu}\left(p'-p\right)_{\nu}\right]u(p)$$

- I leave it as a future homework to show this.

Current Conservation

$$q_\mu J^\mu = 0$$

$$0 = q_\mu \overline{u}(p')\left[\gamma^\mu K_1 + i\sigma^{\mu\nu}\left(p' - p\right)_\nu K_2 + \left(p' + p\right)^\mu K_5\right]u(p)$$

$$q_\mu \gamma^\mu \psi = m\psi \approx 0$$ because of relativistic limit.

$$q_\mu \sigma^{\mu\nu} q_\nu = 0$$ because sigma is anti-symmetric.

As a result, K_5 must be zero.

$$J^\mu = -e\overline{u}(p')\left[\gamma^\mu F_1\left(q^2\right) + i\sigma^{\mu\nu} q_\nu \frac{\kappa}{2M} F_2\left(q^2\right)\right]u(p)e^{i(p'-p)x}$$

Proton Current

$$J^{\mu} = -e\overline{u}(p')\left[\gamma^{\mu}F_1\left(q^2\right) + i\sigma^{\mu\nu}q_{\nu}\frac{\kappa}{2M}F_2\left(q^2\right)\right]u(p)e^{i(p'-p)x}$$

We determine F1, F2, and kappa experimentally, with the constraint that F1(0)=1=F2(0) in order for kappa to have the meaning of the anomalous magnetic moment.

The two form factors F1 and F2 parametrize our ignorance regarding the detailed structure of the proton.