Physics 214 UCSD/225a UCSB

Lecture 11

- Finish Halzen & Martin Chapter 4
  - Invariant variables s,t,u
  - Decay rate definition
  - origin of the propagator
- Halzen & Martin Chapter 5
  - Review of Dirac Equation

E-mu vs e-e vs e-ebar scattering

Electron - muon

$$M = -e^2\left(\frac{(p_A + p_C)^\mu (p_B + p_D)_\mu}{(p_D - p_B)^2}\right)$$

Electron - electron

$$M = -e^2\left(\frac{(p_A + p_C)^\mu (p_B + p_D)_\mu}{(p_D - p_B)^2} + \frac{(p_A + p_D)^\mu (p_B + p_C)_\mu}{(p_C - p_B)^2}\right)$$

Electron - positron

$$M = -e^2\left(\frac{(p_A + p_C)^\mu (-p_B - p_D)_\mu}{((-p_D) - (-p_B))^2} + \frac{(p_A - p_B)^\mu (-p_D + p_C)_\mu}{(p_C - (-p_D))^2}\right)$$

Need a convenient way of expressing the scalar products !!!

Invariant variables s,t,u

Example: e+ e- -> e+ e-

- $s = (p_A + p_B)^2$
- $= 4 (k^2 + m^2)$

- $t = (p_A - p_C)^2$
- $= -2 k^2 (1 - \cos\theta)$

- $u = (p_A - p_D)^2$
- $= -2 k^2 (1 + \cos\theta)$

$k = |k_i| = |k_f|$ $m = m_e$ θ = scattering angle, all in com frame.

N-body decay

$$d\Gamma = \frac{|M|^2}{F} dQ$$

$$d\Gamma = \frac{1}{2E_A} |M|^2 \frac{dp_1^3}{(2\pi)^3 2E_1} \frac{dp_2^3}{(2\pi)^3 2E_2} \bullet\bullet\bullet \frac{dp_n^3}{(2\pi)^3 2E_n} (2\pi)^4 \delta^{(4)}(p_A - p_1 - \bullet\bullet\bullet - p_n)$$

$$N_A(t) = N_A(0) e^{-\Gamma t}$$

Origin of propagator

- When we discussed perturbation theory a few lectures ago, we did what some call “old fashioned perturbation theory”.
  - It was not covariant
  - We required momentum conservation at vertex but not Energy conservation
  - At second order, we need to consider time ordered products.
- When you do this “more modern”
  - Fully covariant
  - 4-momentum is conserved at each vertex
  - However, “propagating particles” are off-shell

Spinless massive propagator

$$\frac{1}{\left(p_A + p_B\right)^2 - m^2} = \frac{1}{p^2 - m^2}$$

For more details see Halzen & Martin

H&M Chapter 5
Review of Dirac Equation

- Notation Reminder
- Dirac Equation for free particle
  - Mostly an exercise in notation
- Define currents
  - Make a complete list of all possible currents
- Aside on Helicity Operator
  - Solutions to free particle Dirac equation are eigenstates of Helicity Operator
- Aside on "handedness"

Notation Reminder (1)

- Sigma Matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix};\ \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};\ \ I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Gamma Matrices:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix};\ \ \gamma^{j=1,2,3} = \begin{pmatrix} 0 & \sigma_{j=1,2,3} \\ -\sigma_{j=1,2,3} & 0 \end{pmatrix};\ \ \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

- State Vectors:

$$\psi = \begin{pmatrix} . \\ . \\ . \\ . \end{pmatrix};\ \ \overline{\psi} \equiv \psi^{T*}\gamma^0 = \begin{pmatrix} . & . & . & . \end{pmatrix}$$

Notation Reminder (2)

- Obvious statements about gamma matrices

$$\gamma^{(j=0,2,5)T} = \gamma^{(j=0,2,5)}; \quad \gamma^{(j=1,3)T} = -\gamma^{(j=1,3)}$$

$$\gamma^{(j=0,5)T*} = \gamma^{(j=0,5)}; \quad \gamma^{(j=1,2,3)T*} = -\gamma^{(j=1,2,3)}$$

- Probability density $\overline{\psi}\gamma^0\psi = \psi^{T*}\psi = \# \geq 0$

- Scalar product of gamma matrix and 4-vector

$$\not{A} \equiv \gamma^\mu A_\mu = \gamma^0 A_0 - \gamma^1 A_1 - \gamma^2 A_2 - \gamma^3 A_3$$

Is again a 4-vector

Dirac Equation of free particle

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$$

$$i\partial_{\mu}\overline{\psi}\gamma^{\mu} + m\overline{\psi} = 0$$

Ansatz:

$$\psi = e^{-ipx}u(p)$$

Explore this in restframe of particle:

$$\psi_{+1/2} = \sqrt{2m}e^{-imt}\begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad \psi_{-1/2} = \sqrt{2m}e^{-imt}\begin{pmatrix}0\\1\\0\\0\end{pmatrix};$$

Normalization chosen to describe 2E particles, as usual.

Particle vs antiparticle in restframe

$$\psi_{+1/2} = \sqrt{2m}e^{-imt}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}; \quad \psi_{-1/2} = \sqrt{2m}e^{-imt}\begin{pmatrix}0\\1\\0\\0\end{pmatrix};$$

particle

Recall, particle -> antiparticle means E,p -> -E,-p

$$\left(\gamma^\mu p_\mu - m\right)u = 0$$

$$\left(\gamma^0 m\right)u = Eu$$

← for p=0 we get this equation to satisfy by the energy eigenvectors

$$\begin{pmatrix} mI & 0 \\ 0 & -mI \end{pmatrix} u = Eu$$

It is thus obvious that 2 of the solutions have E < 0, and are the lower two components of the 4-columns.

Particle & Anti-particle

$$\psi_{+1/2} = \sqrt{2m}e^{-imt}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}; \;\; \psi_{-1/2} = \sqrt{2m}e^{-imt}\begin{pmatrix}0\\1\\0\\0\end{pmatrix};$$

Positive energy solution

Negative energy solution

$$\psi_{+1/2} = \sqrt{2m}e^{-imt}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}; \;\; \psi_{-1/2} = \sqrt{2m}e^{-imt}\begin{pmatrix}0\\0\\0\\1\end{pmatrix};$$

(Anti-)Particle not in restframe

$$\psi_{+1/2} = Ne^{-ipx}\begin{pmatrix} 1 \\ 0 \\ \frac{\vec{\sigma}\vec{p}}{E+m}\begin{pmatrix}1\\0\end{pmatrix}\end{pmatrix}; \quad \psi_{-1/2} = Ne^{-ipx}\begin{pmatrix} 0 \\ 1 \\ \frac{\vec{\sigma}\vec{p}}{E+m}\begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix}$$

$$\psi_{+1/2} = Ne^{+ipx}\begin{pmatrix} \frac{\overrightarrow{-\sigma}\vec{p}}{|E|+m}\begin{pmatrix}1\\0\end{pmatrix} \\ 1 \\ 0 \end{pmatrix}; \quad \psi_{-1/2} = Ne^{+ipx}\begin{pmatrix} \frac{\overrightarrow{-\sigma}\vec{p}}{|E|+m}\begin{pmatrix}0\\1\end{pmatrix} \\ 0 \\ 1 \end{pmatrix}$$

I suggest you read up on this in H&M chapter 5 if you're not completely comfortable with it.

Currents

- Any bi-linear quantity can be a current as long as it has the most general form:

$$\overline{\psi}(4x4)\psi$$

- By finding all possible forms of this type, using the gamma-matrices as a guide, we can form all possible currents that can be within this formalism.

The possible currents

$\bar{\psi}\psi$ scalar

$\bar{\psi}\gamma^5\psi$ pseudo-scalar

$\bar{\psi}\gamma^\mu\psi$ vector

$\bar{\psi}\gamma^5\gamma^\mu\psi$ pseudo-vector

$\bar{\psi}\frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)\psi$ tensor