7.2 Wave Nature of Matter

De Broglie Wavelength
Diffraction of electrons
Uncertainty Principle
Wave Function
Tunneling

Wave properties of matter

Material particles behave as waves with a wavelength given by the De Broglie wavelength (Planck's constant/momentum)

$$\lambda = \frac{h}{p}$$

The particles are diffracted by passing through an aperture in a similar manner as light waves.

The wave properties of particles mean that when you confine it in a small space its momentum (and kinetic energy) must increase. (uncertainty principle) This is responsible for the size of the atom.

De Broglie Wavelength

Momentum of a photon.

$$p = \frac{E}{c}$$ Einstein's special relativity theory

since $E = \frac{hc}{\lambda}$

$$p = \frac{h}{\lambda}$$

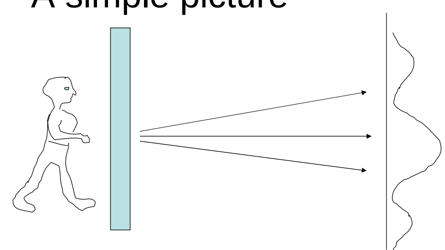
Wavelength of a particle

$$\lambda = \frac{h}{p}$$

Lois De Boglie

De Broglie proposed that this wavelength applied to material particles as well as for photons. (1924)

A simple picture



Suppose you walk into a room through a doorway.

In the wave picture you will be diffracted.

By a small amount since you are big.
But suppose you can shrink in size. Then the angle will increase.

Big particles

Find the De Broglie wavelength of a 100 kg man walking at 1 m/s.

p=mv

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\text{x}10^{-34}\,\text{Js}}{(100\text{kg})(1.0\text{m/s})} = 6.6\text{x}10^{-36}\,\text{m}$$

For macroscopic momenta the wavelengths are so small that diffraction effects are negligible.

Small particles

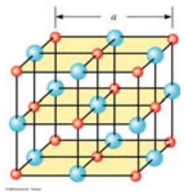
Find the wavelength of an electron traveling at 1.0 m/s (m_e =9.11x10^{-31} kg)

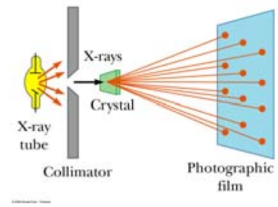
$$\lambda = \frac{h}{mv} = \frac{6.63\text{x}10^{-34}}{(9.11\text{x}10^{-31})(1)} = 7.3\text{x}10^{-4}\,\text{m}$$ = 0.73 mm

Diffraction effects should be observable for small particles.

The wavelength of the electron can be changed by varying it velocity.

Diffraction of light from crystals





Crystals act as a three-dimensional diffraction grating

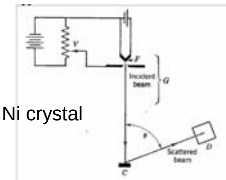
Light with wavelength close to the inter-atomic spacing (x-rays) is diffracted.

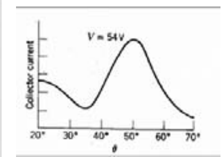
Diffraction of electrons from crystals

Davisson-Germer Experiment (1927)

An electron beam is scattered from a crystal

Ni crystal





The scattered beam shows a diffraction pattern expected for the crystal spacing.

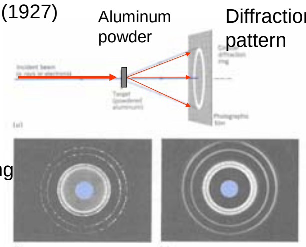
Comparison between electron diffraction and x-ray diffraction

George Thompson (1927)

Aluminum powder

Diffraction pattern

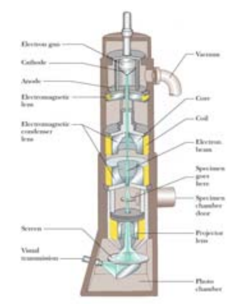
Either x-rays or electrons



The electron wavelength was adjusted to the same value as the x-ray by varying the voltage.
The diffraction pattern for x-rays and electrons are very similar.

X-ray pattern electron pattern

Electron microscopy



$$\lambda = \frac{h}{mv}$$

Shorter wavelength can be obtained by increasing v, the speed of the electron.

Wavelength in an electron microscope

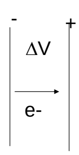
Suppose an electron microscope accelerated electrons across a potential of 10^4 V. What would the wavelength of the electron be?

$$KE = \frac{1}{2}mv^2 = e\Delta V$$

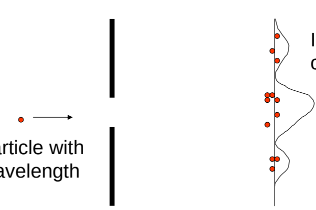
$$v = \sqrt{\frac{2e\Delta V}{m_e}}$$

$$\lambda = \frac{h}{mv} = \frac{h}{m}\sqrt{\frac{m}{2e\Delta V}} = h\sqrt{\frac{1}{2me\Delta V}}$$

$$\lambda = 6.63x10^{-34}\sqrt{\frac{1}{2(9.11x10^{-31})(1.60x10^{-19})(10^4)}} = 1.22x10^{-11}m$$



Diffraction of particles Probabilistic Interpretation of the wave amplitude.



particle with wavelength λ

Intensity $\propto$ Amplitude^2 of the diffracted wave.

But each particle hits a certain point on the screen

The amplitude^2 is interpreted as the probability of the particle hitting the screen at a certain position
This is true for electrons as well as photons.

Wavefunction

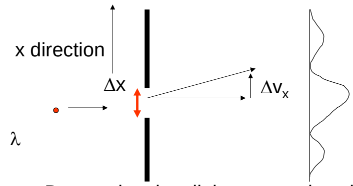
In quantum mechanics the result of an experiment is given in terms of a wavefunction Ψ. The square of the wavefunction Ψ^2 is the probability of the particle being at a certain position.

The wavefunction can be calculated using using the Schrödinger Equation. For instance for electrons in an atom.

Uncertainty Principle

The uncertainty in position is Δx

The uncertainty in the momentum is $\Delta p_x = m\Delta v_x$



Decreasing the slit increases the width of the diffraction.

When Δx decreases, Δp_x increases.

Uncertainty Principle

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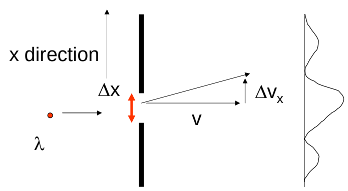


Diffraction

$$\Delta x \sin\theta = \lambda = \frac{h}{mv}$$

$$\sin\theta = \theta = \frac{v_x}{v}$$

$$\Delta x m \Delta v_x = \Delta x \Delta p_x = h$$

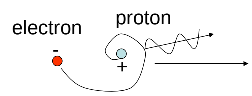
most often written as an inequality

$$\Delta x \Delta p_x \geq h$$

The position and velocity cannot be know with unlimited certainty.

The size of an atom

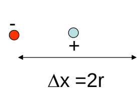
What accounts for the size of the hydrogen atom?



Classical theory would predict that an electron would spiral in to the proton.
The kinetic energy would be dissipated as radiation.

The finite size of the atom is a quantum mechanical effect.

Estimate from the Uncertainty Principle



$$\Delta x \Delta p_x \approx h$$

$$\Delta p_x \approx \frac{h}{\Delta x} = \frac{h}{2r} \approx p_x$$

An electron confined in a space Δx around a proton

As the size of the atom Δx decreases – the kinetic energy increases due to the uncertainty principle.

$$KE = \frac{1}{2} m v_x^2 = \frac{p_x^2}{2m} \approx \frac{h^2}{8r^2 m}$$

Size of atoms

A rough measure of the size of the atom is when the electrostatic potential energy balance PE+KE=0

$$\frac{-e^2}{4\pi\varepsilon_o r} + \frac{h^2}{8r^2 m} = 0$$

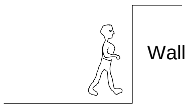
solve for r gives r=0.26 nm

this is approximately the size of atoms

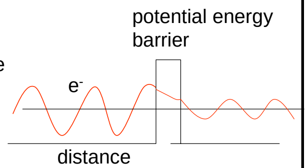
The size of atoms is a consequence of the wave nature of electrons.
They cannot be confined to a small space without increase in the momentum and Kinetic Energy.

Tunneling across a barrier

a macroscopic object impinging on a barrier the object cannot penetrate within the barrier.



a wave particle impinging on a barrier can penetrate within the barrier for distance. and go through the barrier if it is thin enough.



For instance electrons in a wire can cross an oxide insulating layer by tunneling.