HW ASSIGNMENT #3 - - due Tuesday Oct 30

Physics 171/271

From textbook:
Chapter 5;7
Chap 6: 5,6

Prob 1: In class, we discussed a simplified model for action potential propagation that relied on replacing the sodium channel current with a cubic function of voltage. An even simpler approach assumes that the sodium conductance is very small (approximately zero) below some threshold V_{th} and becomes large above that threshold. This leads to the equation

$$\tau \frac{\partial V}{\partial t} = \lambda^2 \frac{\partial^2 V}{\partial x^2} + (V_r - V) + g_{Na}\theta(V - V_{th})(V_{Na} - V)$$

The function $\theta(x)$ is zero for negative arguments, 1 for positive ones. Assume that we are looking for a propagating solution with velocity u which has a membrane potential which equals the threshold voltage at the point z=x-ut =0. Write down the solution for both positive and negative z by solving the relevant ODEs. Find an equation for the velocity by requiring that the solution slope be continuous across z=0.

Prob 2 (271 only) Under some conditions, the HH model predicts that a nerve membrane will behave like a resonant circuit. Let us consider only the delayed rectified potassium current, i.e. a model

$$C\frac{\partial V}{\partial t} = \bar{g}_L(V_L - V) + \bar{g}_K n^4 (V_K - V) + I_{ext}$$

$$\frac{dn}{dt} = \alpha(V)(1-n) - \beta(V)n$$

Let us assume that the external current is a steady-state constant I_0 plus a time-dependent small perturbation $\delta I(t)$. By linearizing everything about the steady-state operating point (i.e. keeping only linear terms in δI) show that the voltage response is the same as that of a circuit with a capacitor in parallel with a resistor in parallel with a resistor and inductor.