Physics 110A: Problem Set #4

Due Monday, November 5 by 12:30 pm

Reading: MT chapters 6-7 ; lecture notes #4

[**1**] A hoop of mass m and radius R rolls without slipping down an inclined plane of mass M which makes an angle α with the horizontal. Find the Euler-Lagrange equations and the integrals of the motion if the plane can slide without friction along a horizontal surface.

[**2**] Consider a particle moving in three dimensional space. The potential energy is $U(x, y, z) = U_1$ if $z < 0$ and $U(x, y, z) = U_2$ if $z \geq 0$. If a particle of mass m moving with speed v and at polar angle θ (*i.e.* the angle with respect to the $\hat{\mathbf{z}}$ axis) passes from the 'lower' half-space (*i.e.* the $z < 0$ region) into the 'upper' half space (*i.e.* the $z > 0$ region), show that in the latter region it moves with constant velocity v' and at polar angle θ'. Find v' and θ'. What is the optical analog to this problem?

[**3**] An enextensible massless string of length ℓ passes through a hole in a frictionless table. A point mass m on one end of the string moves on the table and a point mass m hangs from the other end.

(a) Write the Lagrangian for this system.

(b) Under what conditions will the hanging mass remain stationary?

(c) Starting from the situation in part (b), the hanging mass is pulled down slightly and released. State clearly what is conserved during this process.

(d) Compute the subsequent motion of the hanging mass.

[**4**] The point of suspension of a pendulum of mass m is itself a mass, M, allowed to move in the horizontal direction. The mass M is connected to springs of force constant k on either side, providing a net restoring force $-2kx$ on the point of suspension.

(a) Use the generalized coordinates x (the displacement from equilibrium of the support mass M) and θ (the angular displacement of the pendulum from the vertical) to write the Lagrangian of the system.

(b) Solve for the small oscillations of this system.

[**5**] A uniform ladder of length L and mass M has one end on a smooth horizontal floor and the other end against a smooth vertical wall. The ladder is initially at rest in a vertical plane perpendicular to the wall and makes an angle θ_0 with the horizontal. Make a convenient choice of generalized coordinates and find the Lagrangian. Derive the corresponding equations of motion. Prove that the ladder leaves the wall when its upper end has fallen to a height $\frac{2}{3}L\sin\theta_0$. Show how the subsequent motion can be reduced to explicit integrals. Does the ladder ever lose contact with the floor?