# Homework #5

Ch 6

(#Q7) For objects in circular orbits, the centripetal force is provided by gravity:

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$r = \frac{GM}{v^2}$$

The radius of the orbit is independent of the mass of the object. If the antenna maintains the same speed when it breaks off, it will maintain its orbit. If the antenna slowed down, the antenna would not be able to maintain its circular orbit and would land on the Earth.

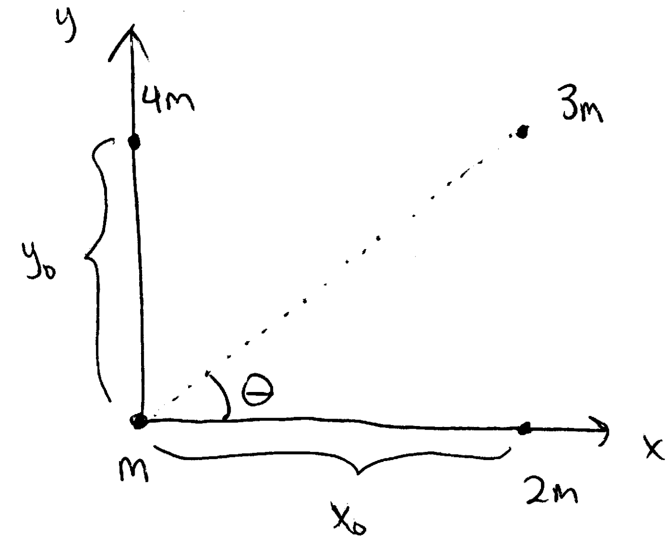
(Q16) If we could measure the period and radius of the orbit of the moon, and assume a circular orbit, then we could find the mass of Pluto. ($T$ = period, $r$ = radius of orbit)

$$\frac{GM_p m_{moon}}{r^2} = \frac{m_{moon} v^2}{r} = \frac{m_{moon}\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$\frac{GM_p}{r^2} = \frac{4\pi^2 r}{T^2} \rightarrow M_p = \frac{4\pi^2 r^3}{GT^2}$$

:)

(P9)




$$r_{4m} = y_0$$

$$r_{2m} = x_0$$

$$r_{3m} = \sqrt{x_0^2 + y_0^2}$$

$$\vec{F} = \left( F_{2m,m} + F_{3m,m} \cos\theta \right) \hat{i}$$

$$+ \left( F_{4m,m} + F_{3m,m} \sin\theta \right) \hat{j}$$

$$\cos\theta = \frac{x_0}{\sqrt{x_0^2 + y_0^2}} , \quad \sin\theta = \frac{y_0}{\sqrt{x_0^2 + y_0^2}}$$

$$F_{2m,m} = \frac{G(2m)m}{x_0^2}$$

$$F_{4m,m} = \frac{G(4m)m}{y_0^2}$$

$$F_{3m,m} = \frac{G(3m)m}{x_0^2 + y_0^2}$$

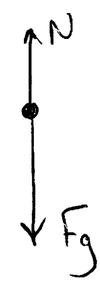
$$\vec{F} = \left[\frac{2Gm^2}{x_0^2} + \frac{3Gm^2}{x_0^2+y_0^2}\left(\frac{x_0}{\sqrt{x_0^2+y_0^2}}\right)\right]\hat{\imath}$$

$$+ \left[\frac{4Gm^2}{y_0^2} + \frac{3Gm^2}{x_0^2+y_0^2}\left(\frac{y_0}{\sqrt{x_0^2+y_0^2}}\right)\right]\hat{\jmath}$$

$$\vec{F} = Gm^2\left[\left(\frac{2}{x_0^2} + \frac{3x_0}{(x_0^2+y_0^2)^{3/2}}\right)\hat{\imath} \,,\, \left(\frac{4}{y_0^2} + \frac{3y_0}{(x_0^2+y_0^2)^{3/2}}\right)\hat{\jmath}\right]$$

(P24) Apparent weight is equal to the normal force experienced by the person.

(a) At the top




$$\Sigma F = F_g - N = \frac{mv^2}{r}$$

$$N = mg - \frac{mv^2}{r} = m\left(g - \frac{v^2}{r}\right)$$

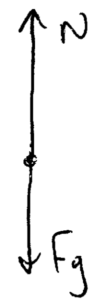
$$r = \frac{27.5\,m}{2} = 13.8\,m \,, \quad v = \frac{2\pi r}{T}$$

$$N_{top} = m\left(g - \frac{4\pi^2 r}{T^2}\right)$$

Fractional Change: $\dfrac{N_{Top} - mg}{mg} = \dfrac{mg - \frac{4\pi^2 mr}{T^2} - mg}{mg}$

$$= -\frac{4\pi^2 r}{gT^2} = -\frac{4\pi^2(13.8m)}{(9.8m/s^2)(10.5s)^2} = -0.504 \text{ (50.4\% less)}$$

(B) at the bottom




$$\Sigma F = N - F_g = \frac{mv^2}{r}$$

$$N_{Bottom} = mg + \frac{mv^2}{r}$$

Fractional Change:

$$\frac{N_{Bottom} - mg}{mg} = \frac{mg + \frac{mv^2}{r} - mg}{mg} = \frac{4\pi^2 r}{gT^2}$$

$$= \frac{4\pi^2(13.8m)}{(9.8m/s^2)(10.5s)^2} = +0.504 \text{ (50.4\% greater)}$$

(P28)(a) For orbits near the planet

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM_p\cancel{m}}{r^{\cancel{2}}}$$

$$M_p = \frac{rv^2}{G}$$

In terms of the Period T

$$V = \frac{2\pi r}{T} \quad \rightarrow \quad M_p = \frac{4\pi^2 r^3}{GT^2}$$

For orbits near the surface $r \approx R_p$

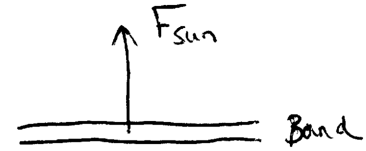
$$V = \frac{4}{3}\pi R_p^3 \approx \frac{4}{3}\pi r^3$$

$$\rightarrow \rho = \frac{M_p}{V} = \frac{\left(\frac{4\pi^2 r^3}{GT^2}\right)}{4/3\,\pi r^3}$$

$$\rho = \frac{3\pi}{GT^2}$$

(b) $$\rho_{Earth} = \frac{3\pi}{(6.67\times10^{-11}\ m^3/kg s^2)[(90 min)(60 sec/min)]^2} = 4800\ kg/m^3$$

(P41) The force of gravity exerted by the sun will provide the centripetal force for the people.



$$\frac{v^2}{r_{orbit}} = g$$

$$\frac{4\pi^2 r_{orbit}}{T^2} = g$$

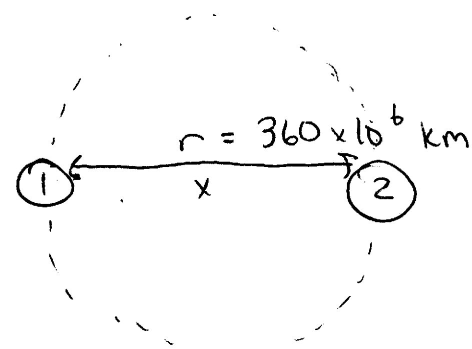
$$T = \sqrt{\frac{4\pi^2 r_{orbit}}{g}} = \sqrt{\frac{4\pi^2 (149.6\times10^6\,km)(1000\,m/km)}{9.8\,m/s^2}}$$

$$= 7.76\times10^5\,s \left(\frac{1\,day}{24\,hr}\right)\left(\frac{1\,hr}{3600\,s}\right)$$

$$= 9.0\ days$$

(P58)

For each planet $F_g = F_c$




$$\frac{Gm\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{(r/2)} = \frac{2\pi^2 \cancel{m} r}{T^2}$$

$m_1 = m_2 = m$

$T = 5.0$ years.

$$m = \frac{2\pi^2 r^3}{GT^2}$$

$$= \frac{2\pi^2 (360\times10^9\,m)^3}{(6.67\times10^{-11}\,m^3/kg\,s^2)\left[(5.0\,yr)\left(\frac{365\,days}{yr}\right)\left(\frac{24\,hr}{day}\right)\left(\frac{3600\,s}{hr}\right)\right]^2}$$

$$m = \frac{(3.68\times10^{36})/4}{1.66\times10^6\,m^3/kg} = 5.5\times10^{29}\,kg$$

(About the same as the sun!)

6

(P60)

$$\frac{G M_{\text{Milky Way}} \cancel{M_{\text{Sun}}}}{r^2} = \cancel{M_{\text{Sun}}} \frac{4\pi^2 r}{T^2}$$

$$M_{MW} = \frac{4\pi^2 r^3}{G T^2} = \frac{4\pi^2 [(30{,}000\ \text{ly})(9.5\times10^{15}\ \text{m})]^3}{(6.67\times10^{-11}\ \text{m}^3/\text{kg s}^2)[(200\times10^6\ \text{yrs})(365)(24)(3600)]^2}$$

$$M_{MW} = \frac{9.14\times10^{62}}{2.65\times10^{21}} = 3.45\times10^{41}\ \text{kg}$$

$$\#\ \text{of stars} = \frac{M_{MW}}{\text{mass/star}} = \frac{3.45\times10^{41}}{2\times10^{30}} = 1.7\times10^{11}\ \text{stars}$$

(Thats 170 billion)

$\boxed{\text{Ch 7}}$

(Q9) Work is like the product of the force and the distance. If we apply the same force to both springs, the force is the same, but the stiffer spring will move less, so more work is done on spring 2. If we push so that they move the same distance, we would have to push harder on the stiffer spring. In this case, the distance is the same, but the force on stiffer spring is greater, so more work is done on spring 1.

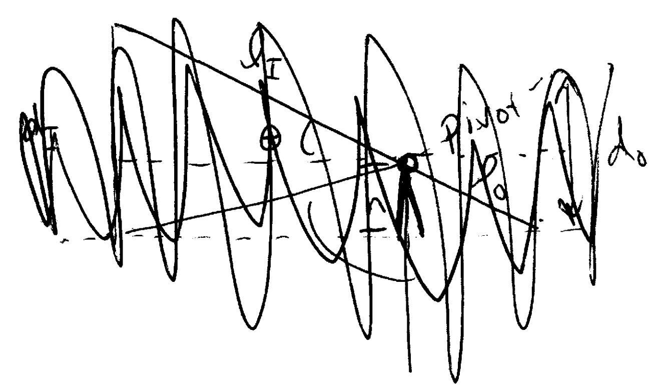
(Q14) $KE_1 = KE_2$

$$m_1 v_1^2 = m_2 v_2^2$$

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$ → So the lighter bullet will have the greater speed, by a factor of the square root of the mass

Since they have the same kinetic energy, they both can do the same amount of work (work-energy theorem)

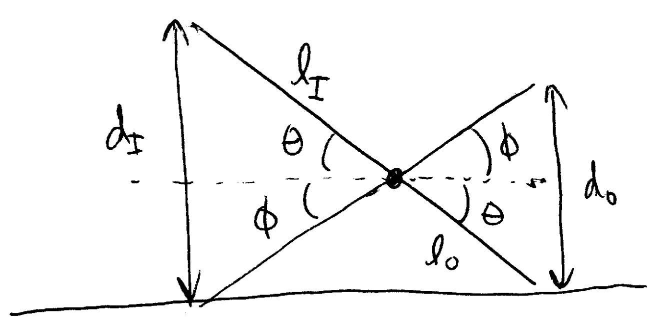
(P11) $W = \vec{F} \cdot \vec{d}$




The distance is

$$d_I = l_I \sin\theta + l_I \sin\phi$$
$$= l_I(\sin\theta + \sin\phi)$$

$$d_o = l_o(\sin\theta + \sin\phi)$$



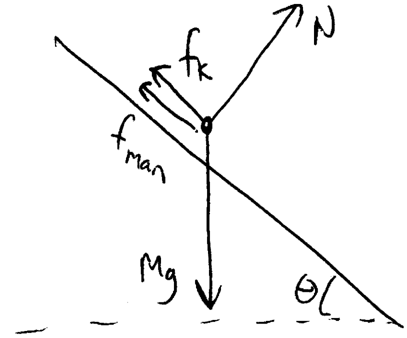

If $W_I = W_o$

$$F_I d_I = F_o d_o$$

$$\frac{F_I}{F_o} = \frac{d_o}{d_I} = \frac{l_o(\sin\theta + \sin\phi)}{l_I(\sin\theta + \sin\phi)} = \frac{l_o}{l_I}$$

$$\therefore \frac{F_o}{F_I} = \frac{l_I}{l_o}$$

(P13)




(a) If the piano is not accelerating the net force parallel to the incline is zero

$$\Sigma F_{\parallel} = F_{man} + f_k - mg\sin\theta = 0$$

$$F_{man} = mg\sin\theta - f_k$$

$$= mg\sin\theta - \mu_k mg\cos\theta$$

$$= mg(\sin\theta - \mu_k\cos\theta)$$

$$= (380\,kg)(9.8\,m/s^2)(\sin 27° - 0.40(\cos 27°))$$

$$F_{man} = 360\,N$$

The man exerts 360 N of force to keep the piano from accelerating.

(b) $W = \vec{F}\cdot\vec{d}$

$$W_{man} = -F_{man}(d) = (360N)(3.5m) = -1300\,J$$

(c) $W = \vec{F}\cdot\vec{d} = -f_k \cdot d$ (because they are in opposite directions)

$$= -\mu_k mg\cos\theta\, d = -(380\,kg)(9.8\,m/s^2)(\cos 27°)(3.5m)(0.4)$$

$$W_{friction} = -4600\,N$$

(d) $W = \vec{F} \cdot \vec{d} = mg \sin\theta \, d$

$= (380\,kg)(9.8\,m/s^2)(\sin 27)(3.5\,m)$

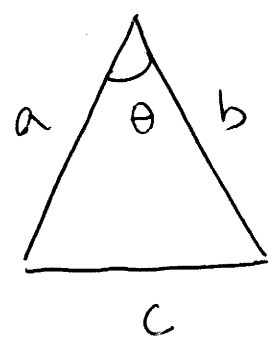
$W_{gravity} = 5900\,N$

(e) $W_{net} = W_{man} + W_{friction} + W_{gravity}$

$= -1300\,J - 4600\,J + 5900\,J$

$= 0\,J$ → Not surprising since the kinetic energy didn't change.

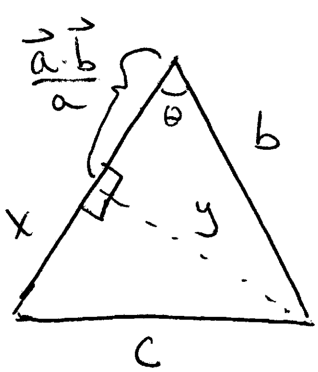
(P26)




so: $c^2 = x^2 + y^2$

$x = a - \frac{(\vec{a} \cdot \vec{b})}{a}$

$y^2 = b^2 - \frac{(\vec{a} \cdot \vec{b})^2}{a^2}$




$x^2 = a^2 + \left(\frac{\vec{a} + \vec{b}}{a}\right)^2 - 2(\vec{a} \cdot \vec{b})$

$c^2 = a^2 + \cancel{\left(\frac{\vec{a} \cdot \vec{b}}{a}\right)^2} - 2(\vec{a} \cdot \vec{b})$

$+ b^2 - \cancel{\left(\frac{\vec{a} \cdot \vec{b}}{a}\right)^2}$

$= a^2 + b^2 - 2(\vec{a} \cdot \vec{b})$

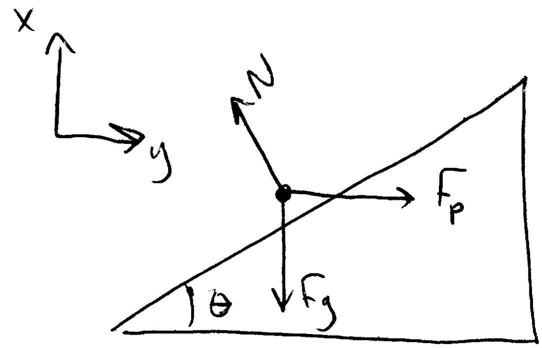
$c^2 = a^2 + b^2 - 2ab\cos\theta$

(#P36) $W = \int \vec{F} \cdot d\vec{x}$

$$W = \int_0^d kx^4 \, dx = \frac{kx^5}{5} \Big|_0^d = \frac{kd^5}{5}$$

(P55)

(a) Here its easier to work in the regular up-down coordinates:




$$\Delta x = (5.0\,m)\cos\theta$$

$$\Delta y = (5.0\,m)\sin\theta$$

$$W_{push} = \vec{F} \cdot \vec{d} = F_p \Delta x = (150N)(5.0m)\cos 30^\circ = 650J$$

(b) $W_{gravity} = \vec{F} \cdot \vec{d} = -mg\Delta y = (20kg)(9.8\,m/s^2)(5.0m)\sin 30^\circ$

$$W_{gravity} = -490J$$

(c) The normal force is alway perpendicular to the displacement

$$W_{normal} = 0$$

(d) $W_{net} = 650J - 490J = 160J = \Delta KE$

$$\Delta KE = \frac{1}{2} m v_f^2 - \cancel{KE_i}^{\,0}$$

$$V_f = \sqrt{\frac{2\Delta KE}{m}} = \sqrt{\frac{2(160J)}{20kg}} = 4\,m/s$$

(P64)

(a)

$$W = \Delta KE$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}m_i^2 = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$= \frac{1}{2}m((0.75v_0)^2 - v_0^2)$$

$$= \frac{1}{2}m v_0^2(-0.44)$$

$$W = -0.22\, m v_0^2$$

(b)

$$W = \vec{F}\cdot\vec{d}$$

$$W = -f_k(2\pi R)$$

$$f_k = \mu_k N = \frac{-W}{2\pi R}$$

$$\mu_k = \frac{-W}{2\pi mgR} = \frac{0.22\cancel{m}v_0^2}{2\pi \cancel{m}gR} = \frac{0.035\, v_0^2}{gR}$$

(c)

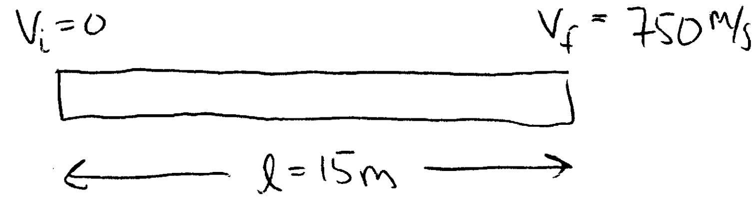
$$\#\text{ of Revolutions} = \frac{\text{Amount of energy available}}{\text{work per revolution}}$$

$$= \frac{KE_i}{W}$$

$$= \frac{\frac{1}{2}\cancel{m}\cancel{v_0^2}}{+0.22\cancel{m}\cancel{v_0^2}} =$$

$$= 2.3 \text{ revolutions}$$

(P66) Muzzle velocity → Speed with which the bullet leaves the end of the muzzle.




$$W = \vec{F}\cdot\vec{d} = \Delta KE$$

$$F = \frac{\Delta KE}{d} = \frac{\frac{1}{2}m(v_f^2 - \cancel{v_0^2}^{\,0})}{d}$$

$$F = \frac{\frac{1}{2}m v_f^2}{d} = \frac{\frac{1}{2}(1250\,kg)(750\,m/s)^2}{15\,m}$$

$$F = 2.3\times10^{7}\,N$$