Black Holes (Ph 161)

An introduction to General Relativity.

Lecture II

Ph 161 Black Holes

Homework Assignment 2

Due Tuesday, February 7, 2006

This should be your own work; do not copy problem solutions.

(1.) Write a few paragraphs discussing the Equivalence Principle and its relationship to geometry. Specifically, describe the experiments which test the relative acceleration of different bodies in a gravitational field (Eötvös experiments) and discuss what it is they measure, how they measure it, and their current precision. Then explain the *geometric* significance of the idea that "everything falls (accelerates)at the same rate in a gravitational field," and the analogy to the geometry on a 2-dimensional curved surface.

(2.) Give an argument as to why there can be no *global* inertial, Minkowski coordinate systems in the presence of a gravitational field.

(3.) With a coordinate transformation from locally inertial (locally Minkowski) coordinates $\{\xi^\alpha\}$ to a laboratory coordinate system $\{x^\mu\}$ (i.e., one at rest on the surface of the earth) derive the geodesic equations. (This is indeed exactly what we did in class!)

(4.) Hartle Chapter 7: problem 7.

Read Chapters 6 and 7 in Hartle's book.

Ph 161 Black Holes

Homework Assignment 3

Due Tuesday, February 14, 2006

This should be your own work; do not copy problem solutions.

(1.) Hartle Chapter 7: problem 9. This is what we discussed in class on Tuesday. Explicitly write out the Taylor expansion involved and show your arguments for the number of degrees of freedom in both the coordinate transformation and the metric. Comment on how the second derivatives of the metric which cannot be set to zero in general are related to tidal forces.

(2.) Hartle Chapter 8: problem 2.

(3.) Hartle Chapter 8: problem 3.

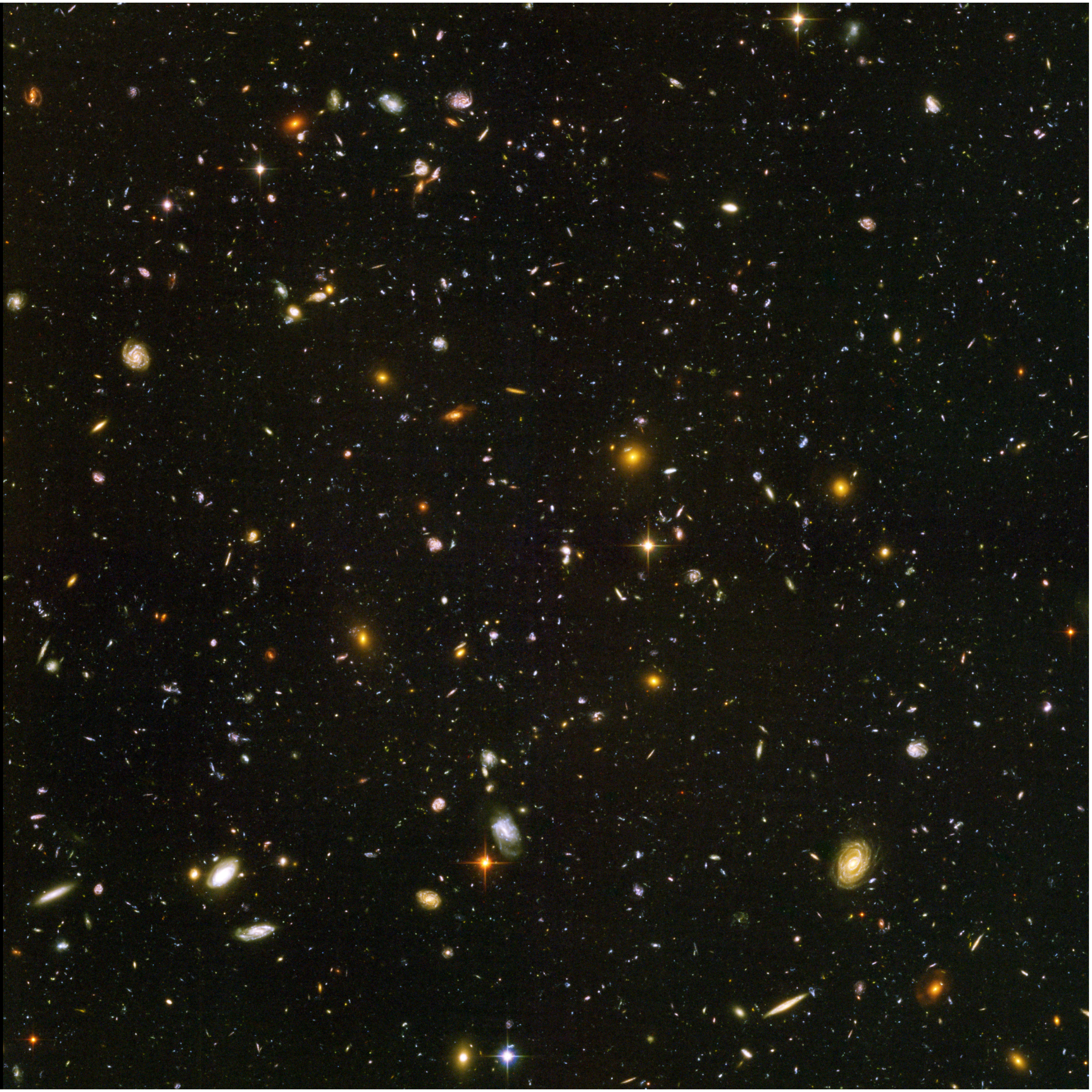
Read Chapter 8 of Hartle

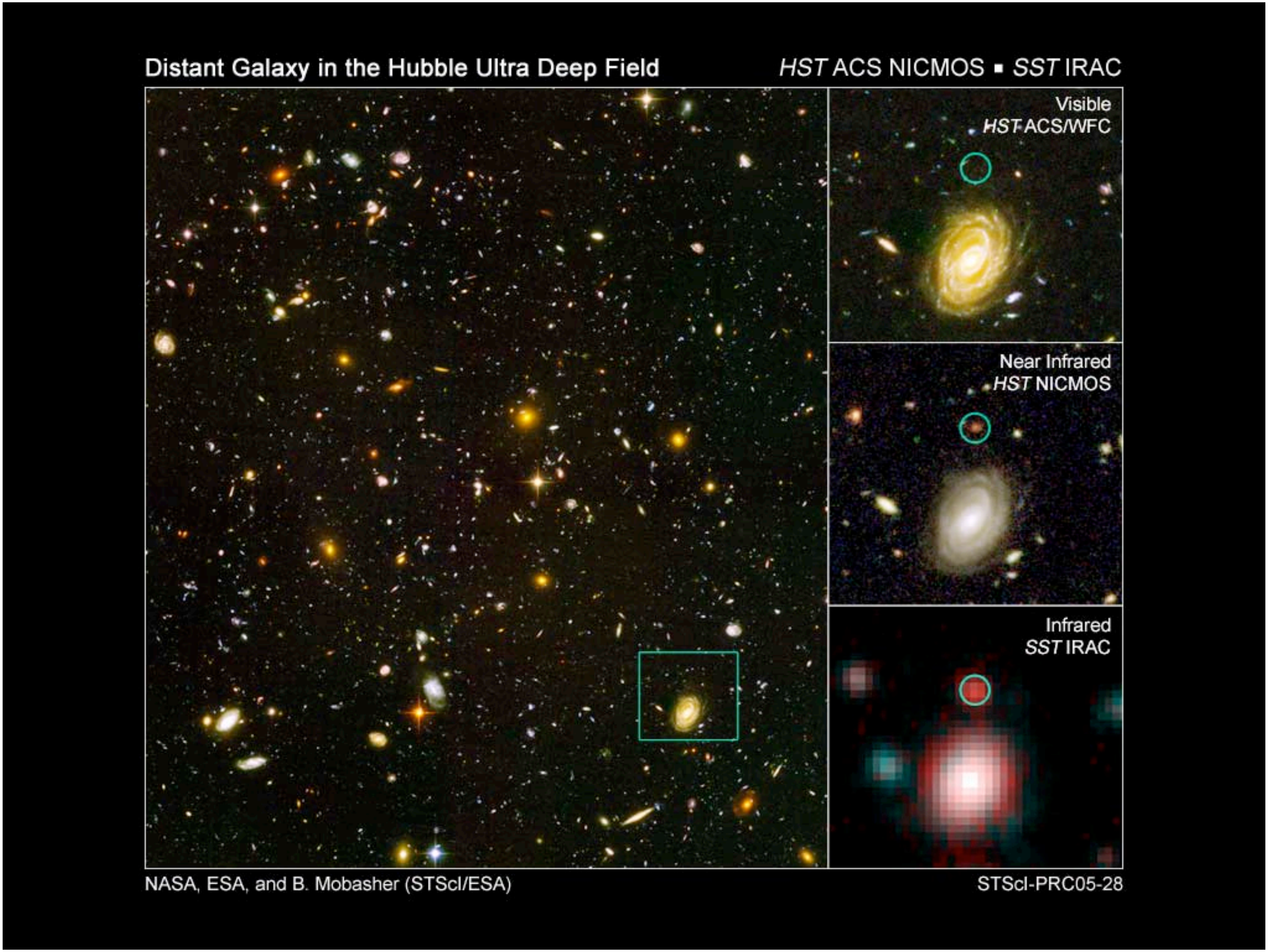
A quick and dirty tour of all of the whole universe

- the large scale structure/evolution of spacetime!

Hubble (HST)
Ultra Deep Field

Some of the first galaxies to form.




Distant Galaxy in the Hubble Ultra Deep Field
HST ACS NICMOS ▪ SST IRAC
Visible
HST ACS/WFC
Near Infrared
HST NICMOS
Infrared
SST IRAC
NASA, ESA, and B. Mobasher (STScI/ESA)
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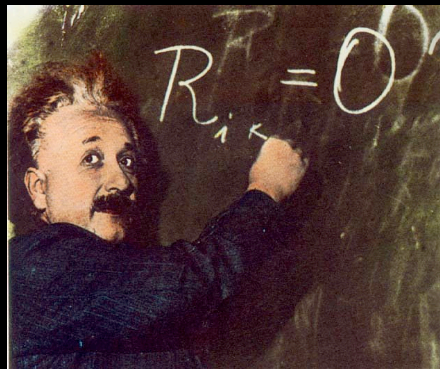


Albert Einstein

George Gamow

A. Friedmann

George LeMaitre

Homogeneity and isotropy of the universe:
implies that *total energy* inside a co-moving spherical surface is constant with time.

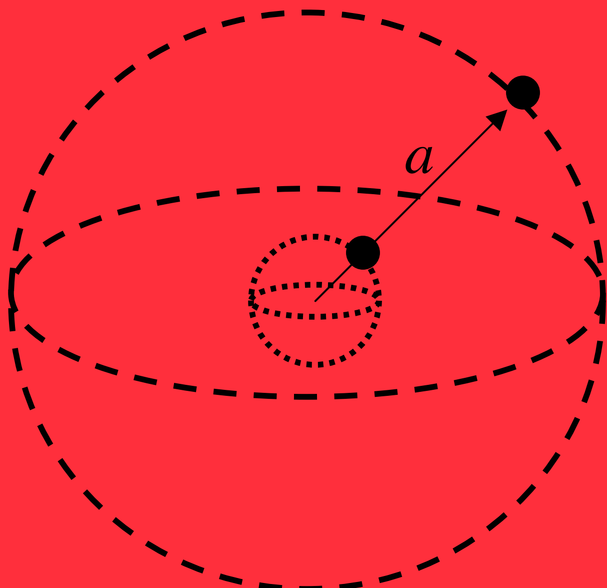
total energy **= (kinetic energy of expansion) + (gravitational potential energy)**

mass-energy density = ρ

test mass = *m*

$$\approx \tfrac{1}{2} m \dot{a}^2$$

$$\approx -\frac{G\left[\frac{4}{3}\pi a^3 \rho\right] m}{a}$$



$$\dot{a}^2 + k = \frac{8}{3}\pi \mathrm{G}\rho\, a^2$$

total energy **> 0 expand forever k = -1**

total energy **= 0 for ρ = ρ_{crit} k = 0**

total energy **< 0 re-collapse k = +1**

$$\Omega = \rho/\rho_{\mathrm{crit}} = \Omega_\gamma + \Omega_\nu + \underbrace{\Omega_{\mathrm{baryon}} + \Omega_{\mathrm{dark\ matter}}}_{\approx 0.3} + \Omega_{\mathrm{vacuum}} \approx 1 \quad (\mathbf{k=0})$$

The key point in our argument was symmetry: specifically, a homogeneous and isotropic distribution of mass and energy!

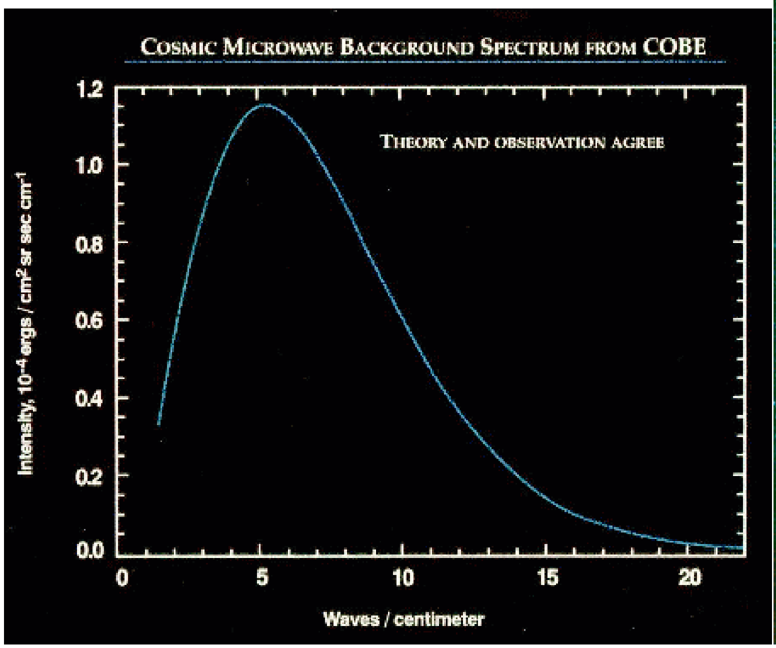
What evidence is there that this is true?

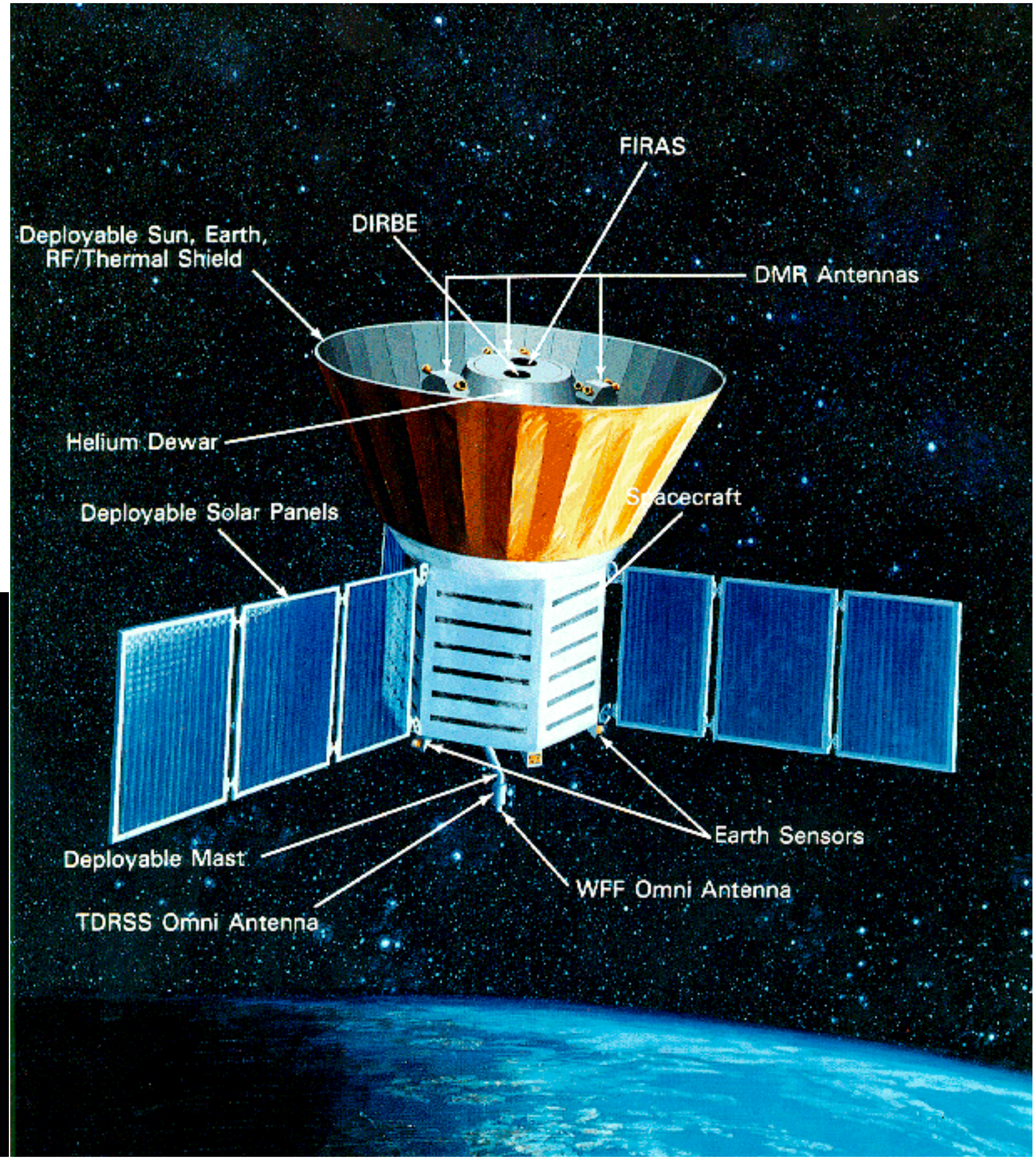
Look around you. This is manifestly NOT true on small scales. The Cosmic Microwave Background Radiation (CMB) represents our best evidence that matter is smoothly and homogeneously distributed on the largest scales.

The COBE satellite - the microwave background radiation

Blackbody radiation

$$T \approx 2.725\,\mathrm{K}$$





Friedman-LeMaitre-Robertson-Walker (FLRW) coordinates

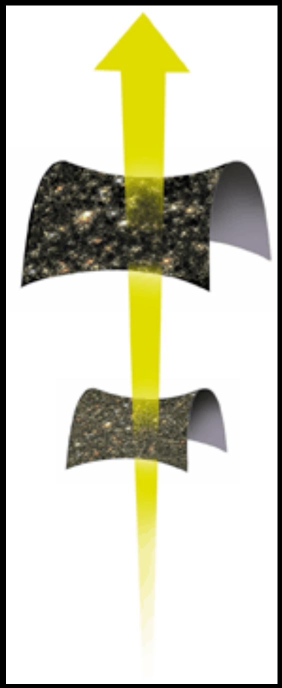
$$(t, r, \theta, \varphi)$$

defined through this metric . . .

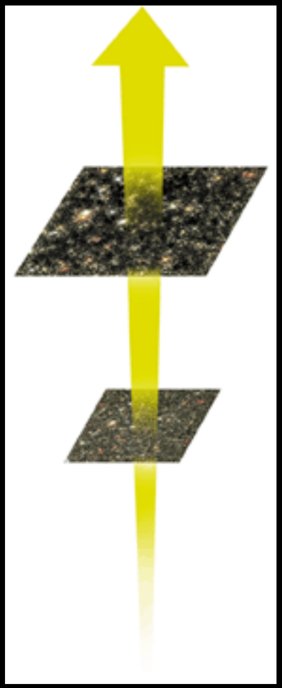
$$\boxed{ds^2 = -dt^2 + a^2\left(t\right)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2\right]}$$

scale factor $a\left(t\right)$

curvature parameter $k = 1,\ 0,\ -1$



$k = -1$



$k = 0$

$k = +1$

How far does a photon travel in the age of the universe?

(causal horizon)

$$\boxed{ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2\right]}$$

Consider a radially-directed photon ($d\theta = d\varphi = 0$)

$$d_{\mathrm{H}}(t) = \int_0^{r_{\mathrm{H}}} \sqrt{g_{rr}}\, dr = a(t) \int_0^{r_{\mathrm{H}}} \frac{dr}{\sqrt{1-kr^2}}$$

photons travel on
null world lines so $ds^2=0$ $\Rightarrow$

$$\frac{dt}{a(t)} = \frac{dr}{\sqrt{1-kr^2}}$$

$$\Rightarrow \quad d_{\mathrm{H}}(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$

Causal (Particle) Horizon

$$d_{\mathrm{H}}(t) = a(t)\int_0^t \frac{dt'}{a(t')} = \begin{cases} 2t & \text{radiation dominated} \quad a(t) \sim t^{1/2} \quad \rho \sim a^{-4} \\ 3t & \text{matter dominated} \quad a(t) \sim t^{2/3} \quad \rho \sim a^{-3} \\ H^{-1}\left[e^{Ht} - 1\right] & \begin{cases} \text{vacuum energy dominated} \\ a(t) = a(t_0)\, e^{H(t-t_0)} \\ H = \dfrac{\dot{a}}{a} = \left(\dfrac{8}{3}\pi G \rho_{\mathrm{vac}}\right)^{1/2} \end{cases} \end{cases}$$

In every case the physical (proper) distance a light signal travels goes to infinity as the value of the timelike coordinate *t* does.

Note, however, that for the vacuum dominated case there is a finite limiting value for the FLRW radial coordinate as *t* goes to infinity . . .

$$\int_0^{r_{\mathrm{H}}} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^t \frac{dt'}{a(t')}$$

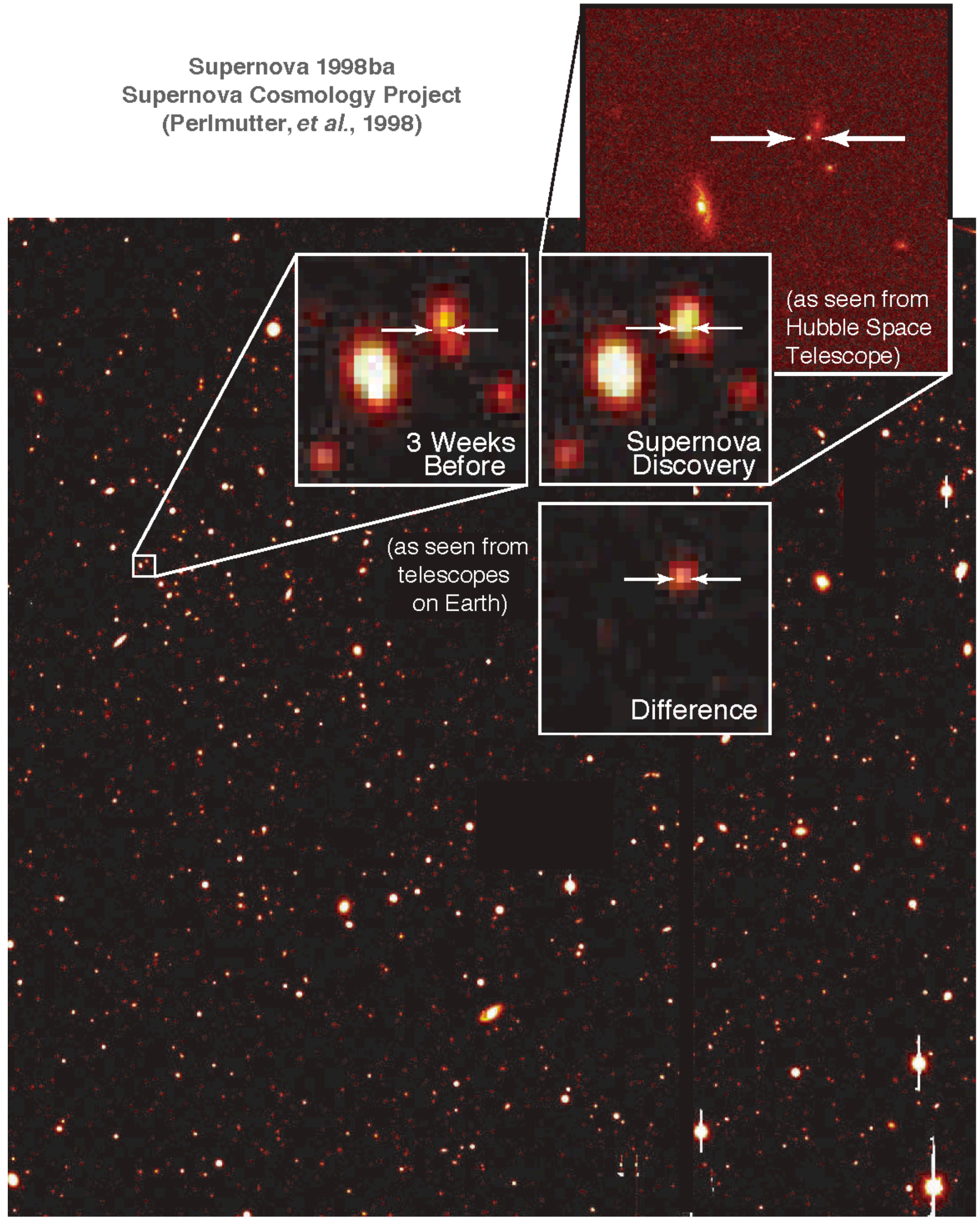
$$r_{\mathrm{H}} \approx H^{-1}\left(1 - e^{-Ht}\right) \to H^{-1} \text{ as } t \to \infty$$

Type Ia supernovae (thermonuclear explosions) serve as “standard candles,” meaning we claim to know their absolute brightness.

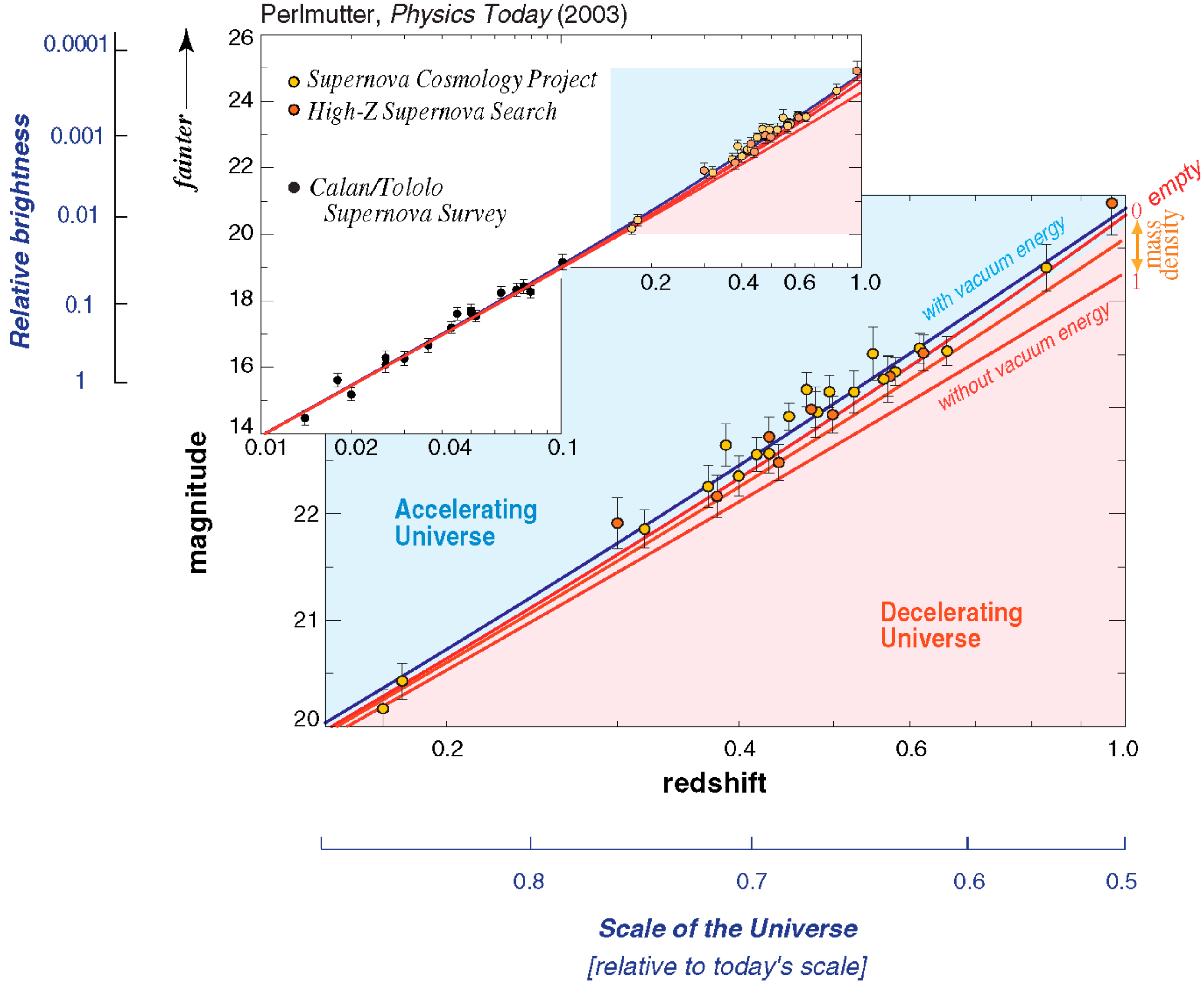
From the measured flux of photons we can get their distance and from their spectra we can get their redshift.

Putting these together we can get distance as a function of redshift and, hence, scale factor as a function of time for redshifts out to Z ~ 1.

LBL supernova cosmology website

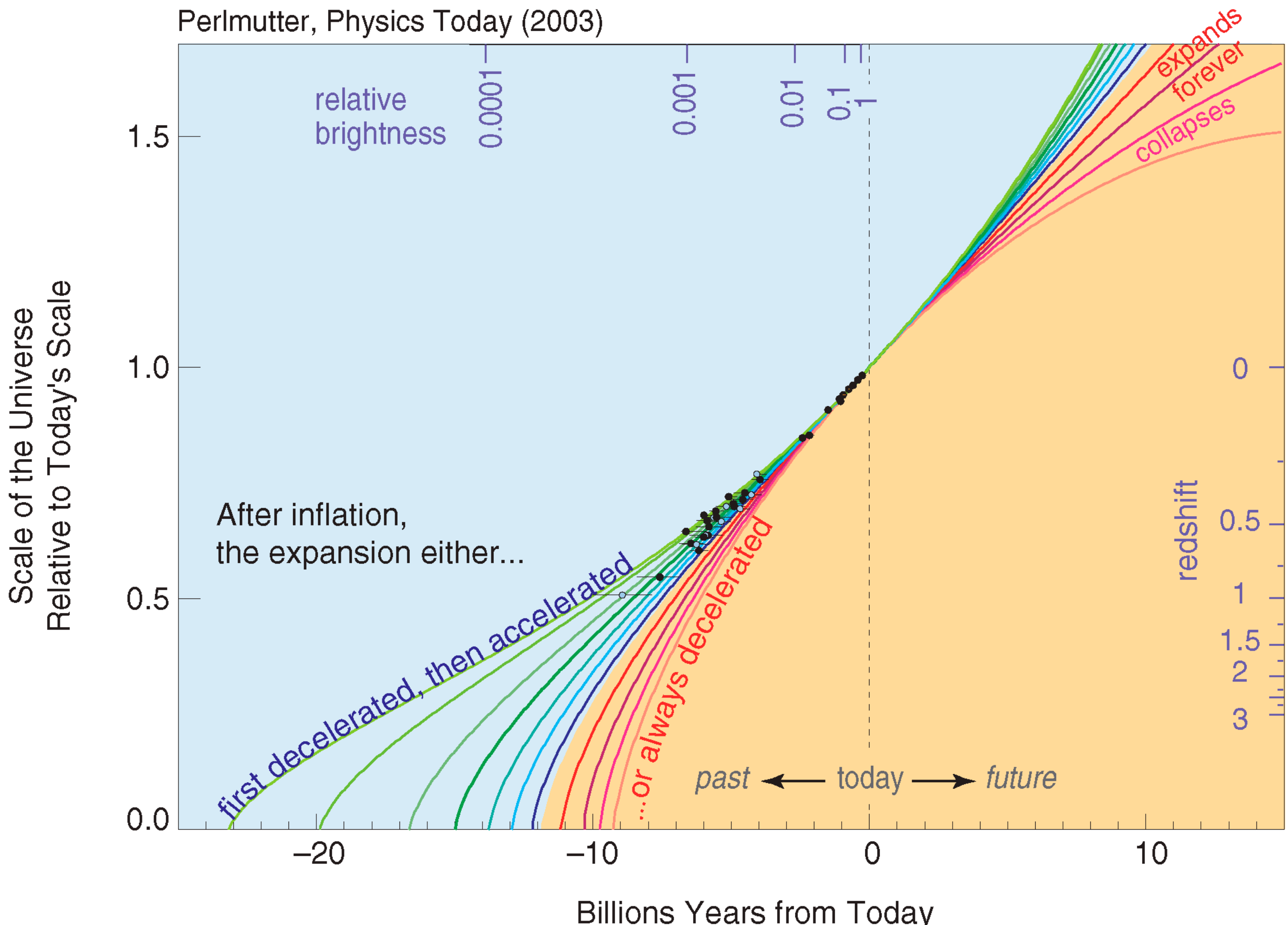


Type Ia Supernovae



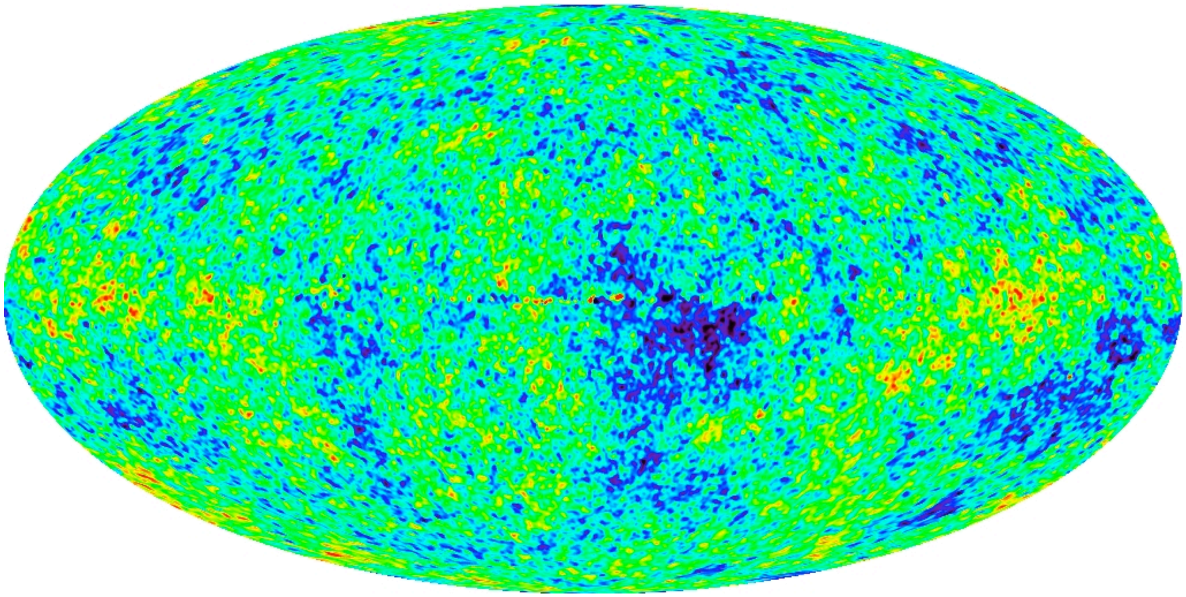
LBL supernova cosmology website

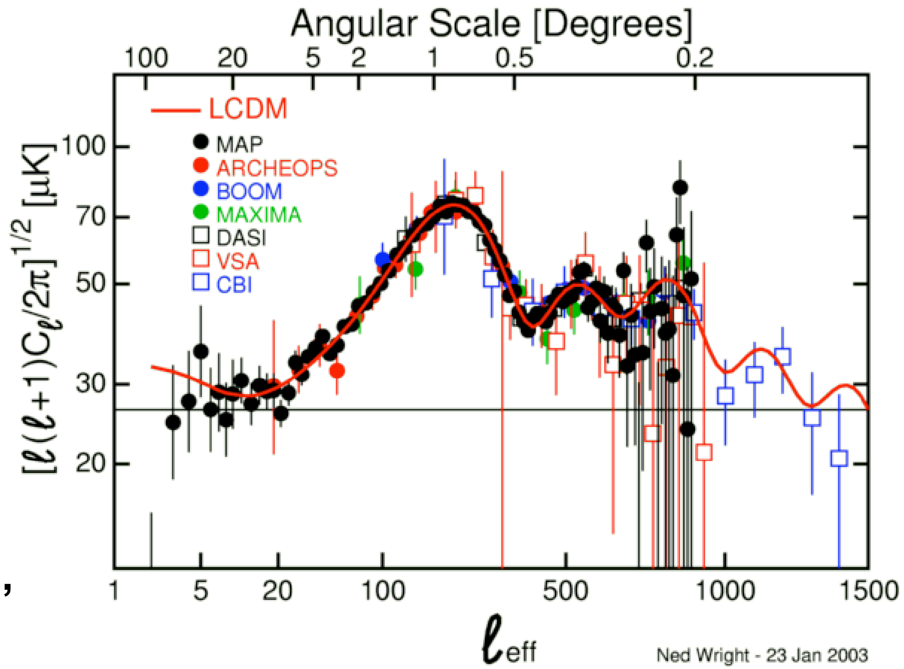
Expansion History of the Universe



LBL supernova cosmology website

WMAP cosmic microwave background satellite

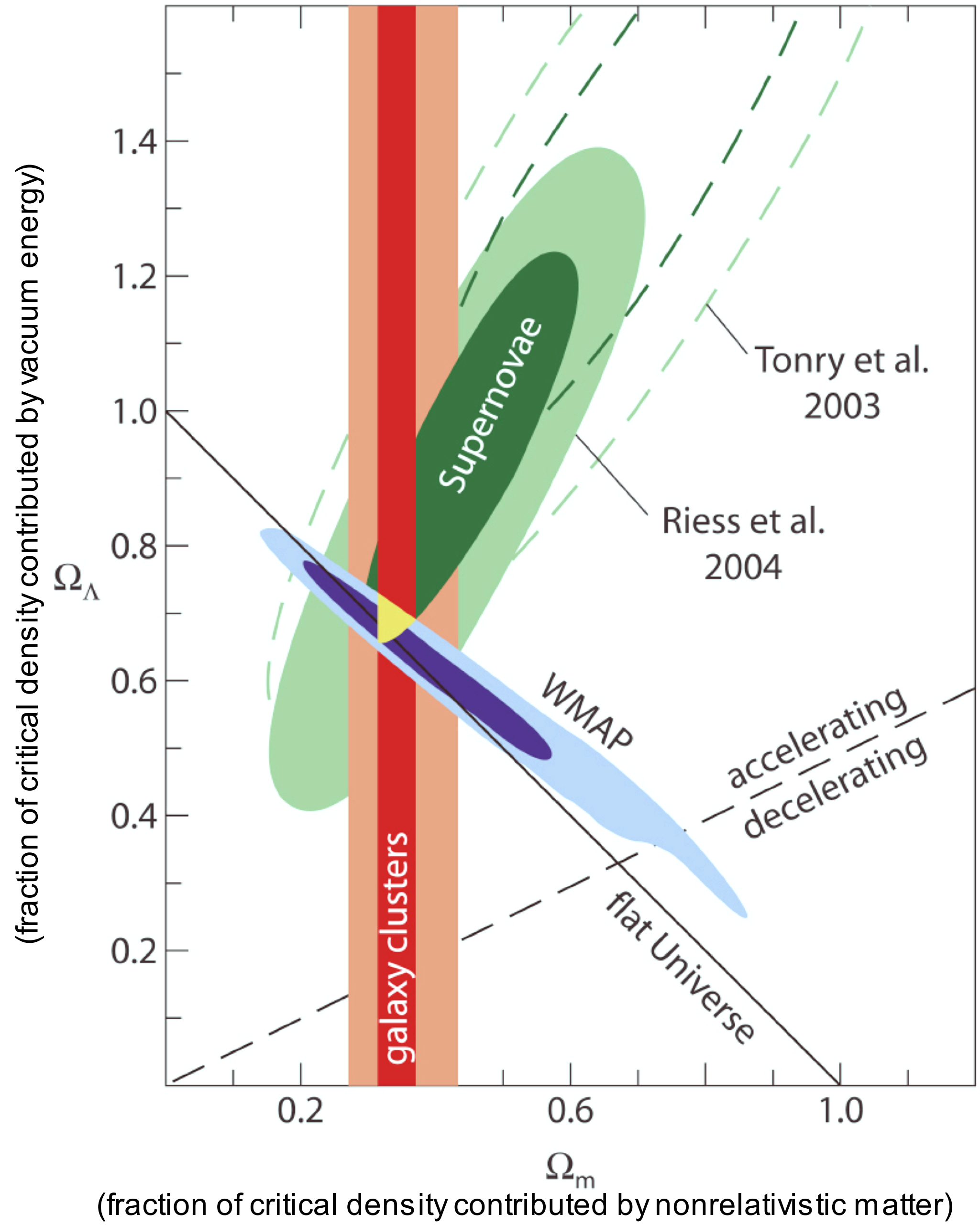




Fluctuations in CMB temperature give Insight into the composition, size, and age of the universe and the large scale character of spacetime.

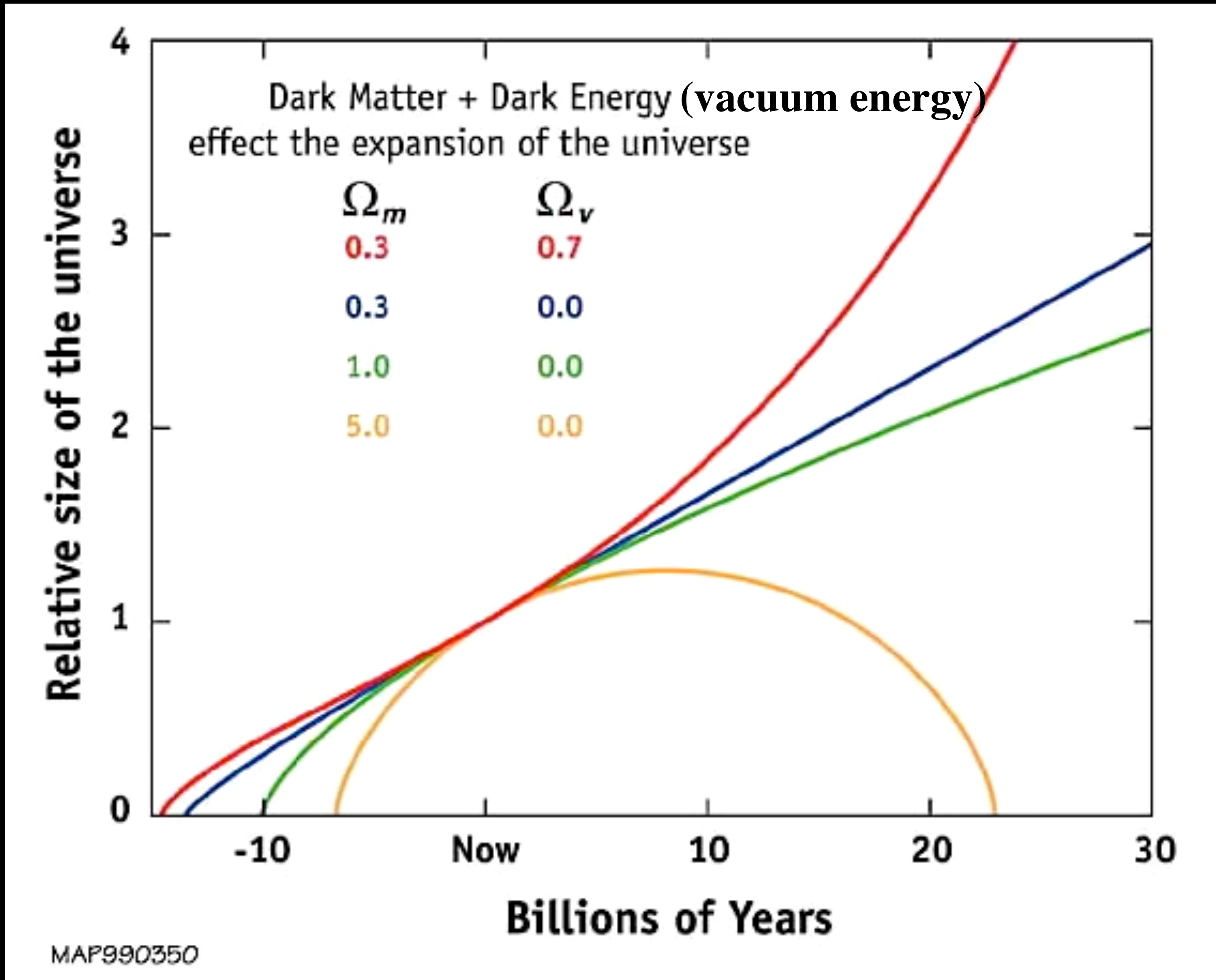
Age = 13.7 Gyr
Spacetime = "flat" (meaning k=0)
Composition = 23% unknown nonrelativistic matter, 73% unknown vacuum energy (dark energy), 4% ordinary baryons.

observational constraints on the content of
of nonrelativistic matter and vacuum energy (dark energy) in the universe

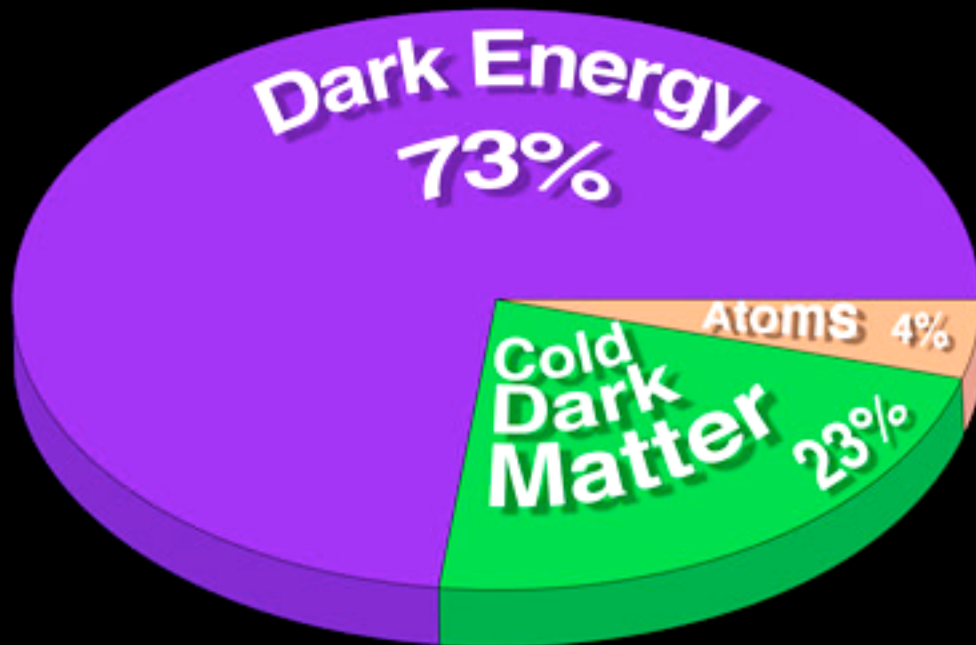


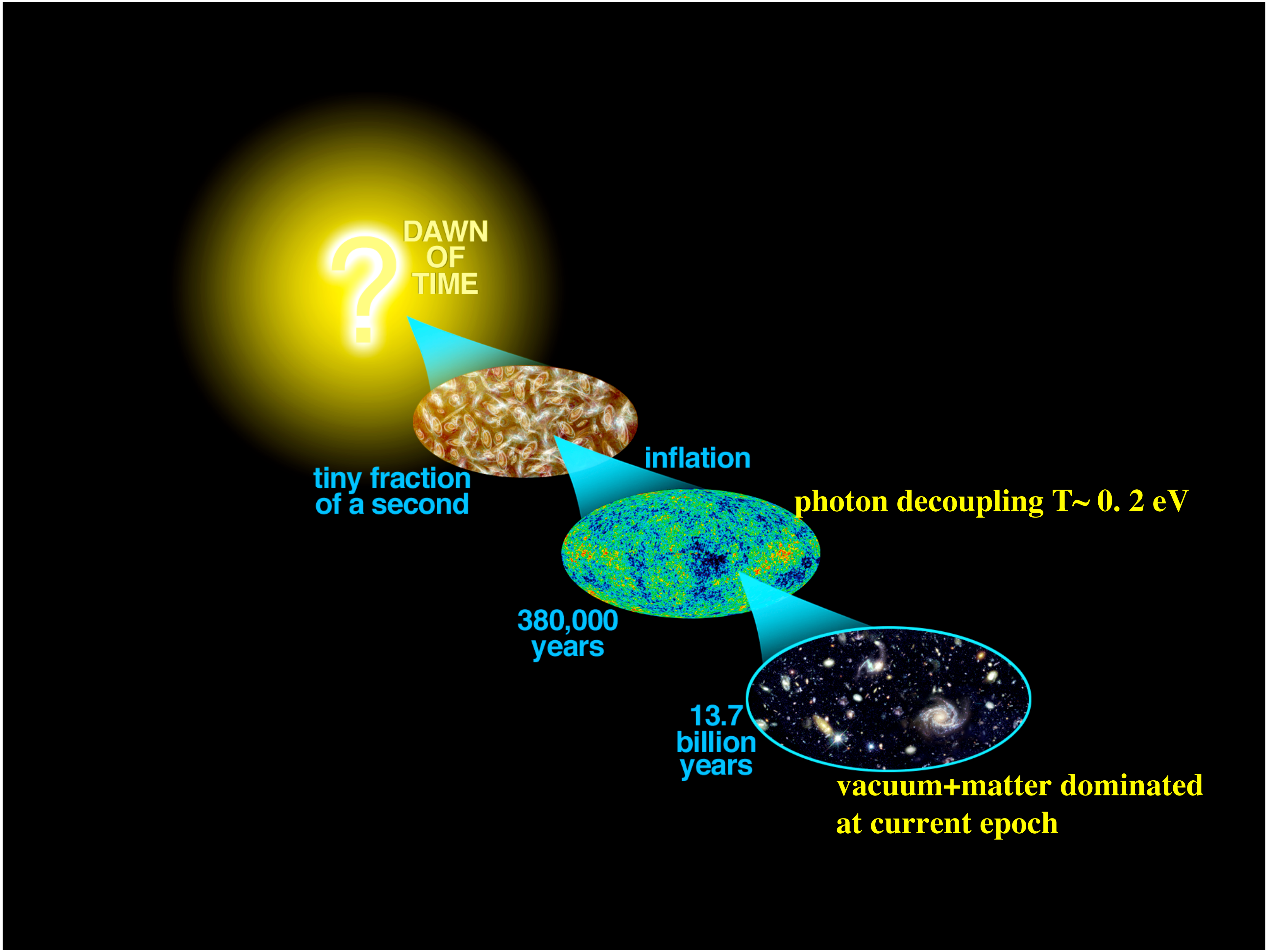
Constraining the Cosmological Parametres

ESO PR Photo 18d/04 (3 June 2004)


Dark Matter + Dark Energy (vacuum energy)
effect the expansion of the universe
Ω_m Ω_v
0.3 0.7
0.3 0.0
1.0 0.0
5.0 0.0
Relative size of the universe
4
3
2
1
0
-10
Now
10
20
30
Billions of Years
MAP990350

We live in a k = 0, critically closed universe.



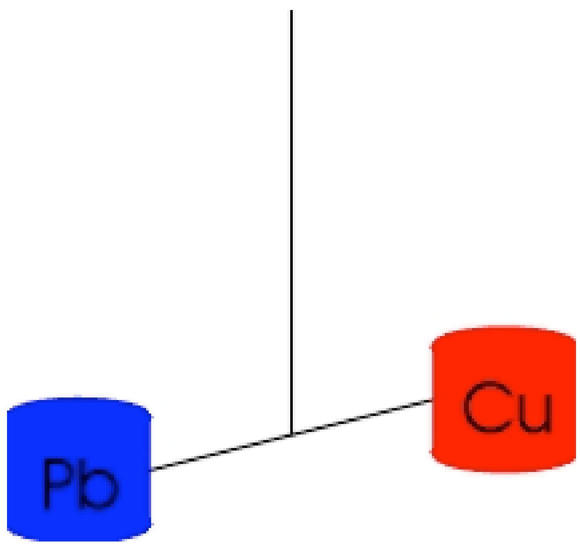

DAWN OF TIME
tiny fraction of a second
inflation
photon decoupling T~ 0. 2 eV
380,000 years
13.7 billion years
vacuum+matter dominated at current epoch

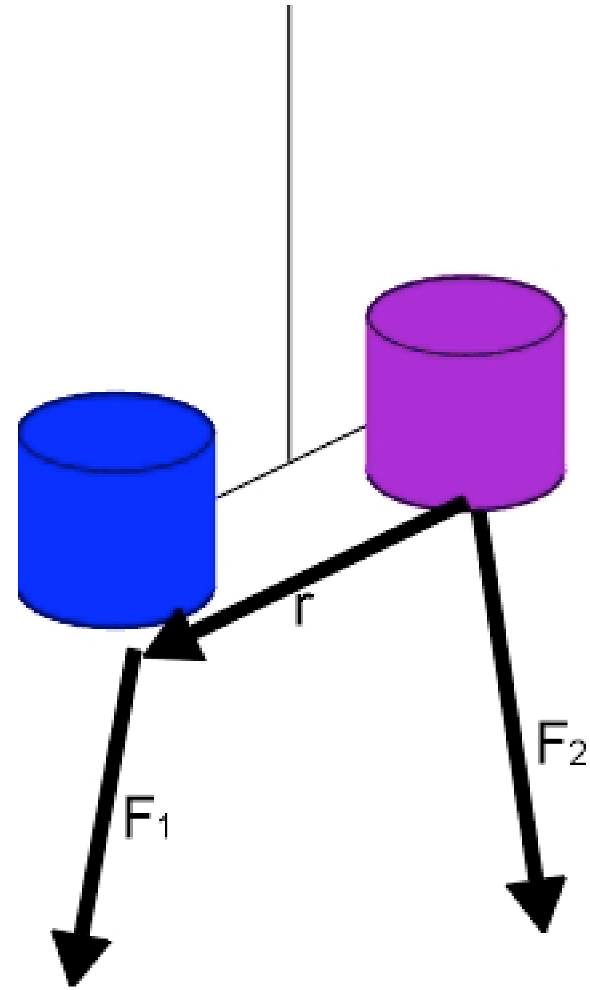
The Equivalence Principle

- Eotvos experiments
- meaning for freely falling bodies
- geometric implications
- geodesics

The EÖTVÖS Experiment

torsion balance

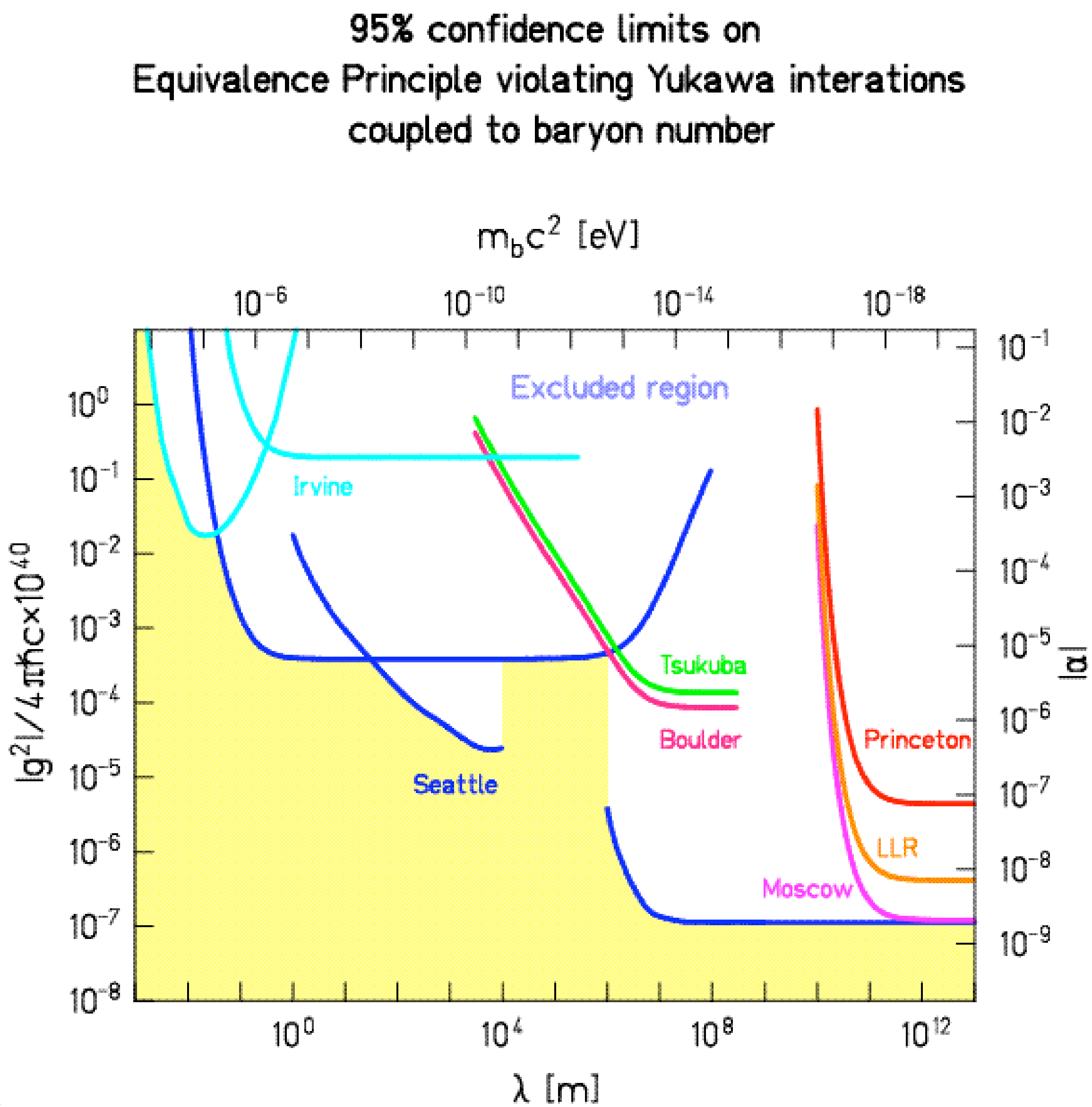




see www.npl.washington.edu/eotwash

Magnitude of torque on fiber: $T = \left[\mathbf{r} \cdot (\mathbf{F}_1 \times \mathbf{F}_2)\right] / |\mathbf{F}_1 + \mathbf{F}_2|$

EotWash lab's results: sensitivity for long range forces is at about 1 part in 10^{13}



www.npl.washington.edu/eotwash/

OK, what does this mean?

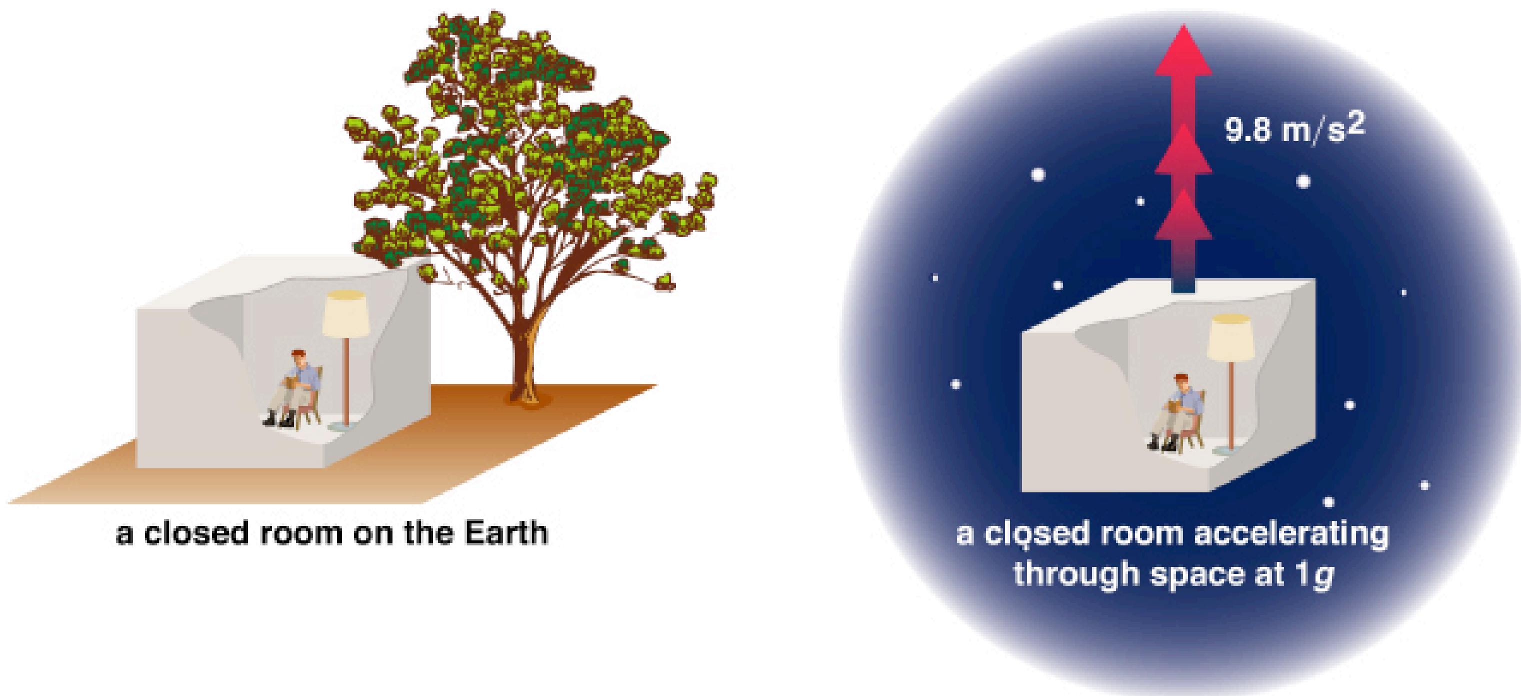
Everything falls at the same rate!

Apollo 15 astronaut
David R. Scott drops a hammer
and a feather . . . Guess what happens?

www.hq.nasa.gov/ . . ./History/SP-4214/cover.html

One begins to get a creepy feeling that the acceleration produced by "gravity" has nothing to do with what the bodies in question are made out of, but rather is a property of space (spacetime) itself!

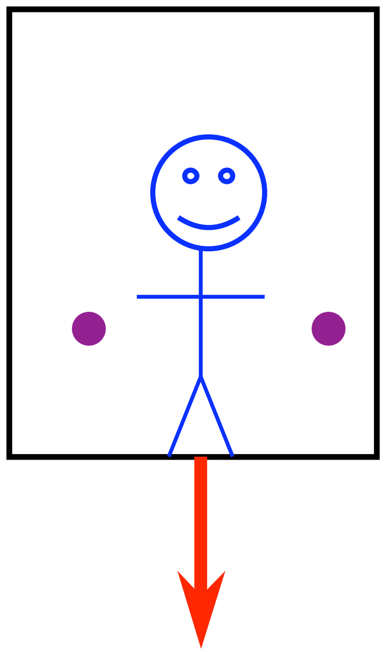
equivalence of inertial and gravitational mass:



cse.ssl.berkeley.edu/bmendez/ay10/2002/notes/pics

elevators in free fall . . . & the E. P.

Someone cuts your elevator's cable and you release the two balls that you have in your hands . . . What happens?



1 g = 9.8 m s^{-2}

If we make the elevator small enough, it looks to us as if THERE IS NO GRAVITY !!!

Cannot tell the difference between an elevator in free fall and the *absence* of gravitation.

Gravitation as Geometry

Statement of the Equivalence Principle:

In a *sufficiently small* region of space & time we can find a freely falling (locally Minkowski) coordinate system in which the effects of gravitation are absent - the laws of physics are the same as they are in a Minkowski coordinate system with no gravitation.

Statement of the fundamental theorem of differential geometry for 2-D surfaces:

In a *sufficiently small* region on any 2-D surface, the geometry is locally flat and Cartesian. (We can pass a tangent plane through any point on the surface. In a *sufficiently small* region around where this tangent plane touches the surface, the geometry will be flat, like a Cartesian plane.)

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$dl^2 = dx^2 + dy^2$$

Coordinate Transformations

Follows from the chain rule: view coordinates in one system as functions of the coordinates in the other frame.

e.g., consider these four functions: $x^\mu\left(\xi^0, \xi^1, \xi^2, \xi^3\right)$

with $\mu = 0, 1, 2, 3$

chain rule gives:

$$dx^\mu = \frac{\partial x^\mu}{\partial \xi^\alpha} \cdot d\xi^\alpha$$

$$= \frac{\partial x^\mu}{\partial \xi^0} \cdot d\xi^0 + \frac{\partial x^\mu}{\partial \xi^1} \cdot d\xi^1 + \frac{\partial x^\mu}{\partial \xi^2} \cdot d\xi^2 + \frac{\partial x^\mu}{\partial \xi^3} \cdot d\xi^3$$

Coordinate transformation is these 16 functions:

$$dx^\mu = \Lambda^\mu{}_\alpha \, d\xi^\alpha$$

$$\boxed{\Lambda^\mu{}_\alpha = \frac{\partial x^\mu}{\partial \xi^\alpha}}$$

In locally inertial (Minkowski) coordinates $\xi^\alpha(x)$ the particle is unaccelerated, and moving on a straight line

$$\frac{d^2\xi^\alpha}{d\tau^2} = 0$$

$$\frac{d}{d\tau}\left(\frac{\partial\xi^\alpha}{\partial x^\mu}\frac{dx^\mu}{d\tau}\right) = 0 \qquad\qquad \frac{\partial\xi^\alpha}{\partial x^\mu}\frac{d^2x^\mu}{d\tau^2} + \frac{\partial^2\xi^\alpha}{\partial x^\mu\,\partial x^\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

multiply both sides by $\frac{\partial x^\rho}{\partial\xi^\alpha}$ and sum:

$$\frac{\partial x^\rho}{\partial\xi^\alpha}\cdot\frac{\partial\xi^\alpha}{\partial x^\mu}\cdot\frac{d^2x^\mu}{d\tau^2} + \frac{\partial x^\rho}{\partial\xi^\alpha}\cdot\frac{\partial^2\xi^\alpha}{\partial x^\mu\,\partial x^\nu}\cdot\frac{dx^\mu}{d\tau}\cdot\frac{dx^\nu}{d\tau} = 0$$

$$\underbrace{\left(\frac{\partial x^\rho}{\partial\xi^\alpha}\cdot\frac{\partial\xi^\alpha}{\partial x^\mu}\right)}_{\delta^\rho{}_\mu}\cdot\frac{d^2x^\mu}{d\tau^2} + \underbrace{\left[\frac{\partial x^\rho}{\partial\xi^\alpha}\cdot\frac{\partial^2\xi^\alpha}{\partial x^\mu\,\partial x^\nu}\right]}_{\Gamma^\rho_{\mu\,\nu}}\cdot\frac{dx^\mu}{d\tau}\cdot\frac{dx^\nu}{d\tau} = 0$$

$$\boxed{\frac{d^2x^\rho}{d\tau^2} + \Gamma^\rho_{\mu\,\nu}\cdot\frac{dx^\mu}{d\tau}\cdot\frac{dx^\nu}{d\tau} = 0}$$

Geodesic Equation

$$\boxed{\frac{d^2x^\rho}{d\tau^2} + \Gamma^\rho_{\mu\,\nu} \cdot \frac{dx^\mu}{d\tau} \cdot \frac{dx^\nu}{d\tau} = 0}$$

So in this coordinate system (x) it looks as if the particle experiences a “gravitational force” and is accelerated.

Note that another way to write this equation is

$$\frac{du^\rho}{d\tau} + \Gamma^\rho_{\mu\,\nu}\, u^\mu\, u^\nu = 0$$

Connection Coefficients (Christoffel symbols)

$$\Gamma^{\rho}_{\mu\,\nu}$$

These are obviously related to how the locally Minkowski coordinates differ from our lab coordinates x

$$\Gamma^{\rho}_{\mu\,\nu} = \frac{\partial x^{\rho}}{\partial \xi^{\alpha}} \cdot \frac{\partial^2 \xi^{\alpha}}{\partial x^{\mu}\,\partial x^{\nu}}$$

Now, in considering these two coordinate systems, the locally Minkowski coordinates $\{\xi^{\alpha}\}$, and the "lab" coordinates $\{x^{\mu}\}$, we will demand that the spacetime interval is always the same:

$$
\begin{aligned}
ds^2 &= \eta_{\alpha\,\beta}\, d\xi^{\alpha}\, d\xi^{\beta} \\
&= \eta_{\alpha\,\beta} \left(\frac{\partial \xi^{\alpha}}{\partial x^{\mu}}\, dx^{\mu} \right) \left(\frac{\partial \xi^{\beta}}{\partial x^{\nu}}\, dx^{\nu} \right) \\
&= \underbrace{\left(\eta_{\alpha\,\beta}\, \frac{\partial \xi^{\alpha}}{\partial x^{\mu}}\, \frac{\partial \xi^{\beta}}{\partial x^{\nu}} \right)}_{g_{\mu\,\nu}} dx^{\mu}\, dx^{\nu} \\
&= g_{\mu\,\nu}\, dx^{\mu}\, dx^{\nu}
\end{aligned}
$$

Length, Area, and Volume and the Metric

Watch out! Actual *physical* (or *proper*) lengths, areas, and volumes are not the same as *coordinate* values of the same quantity.

It must be kept in mind that the spacetime interval is preserved under coordinate transformations. Think about the invariant *interval* corresponding to an infinitesimal coordinate increment:

$$\sqrt{ds^2} = \sqrt{g_{ii}}\,dx^i$$

More Examples . . .

The components of the metric tensor in freely falling, locally Minkowski coordinates $\{\xi^\alpha\}$ are $\eta_{\alpha\beta}$

The components of the metric tensor in the “lab” coordinate system $\{x^\mu\}$ are $g_{\mu\nu}$

The Metric Tensor and "General" Coordinates

$$g_{\bar{\alpha}\,\bar{\beta}} = \Lambda^{\mu}{}_{\bar{\alpha}}\,\Lambda^{\nu}{}_{\bar{\beta}}\,g_{\mu\,\nu}$$

. . . this is how the "components" of the metric transform under a coordinate transformation, where the transformation matrix elements are (for a so-called "coordinate basis")

$$\Lambda^{\mu}{}_{\bar{\alpha}} = \frac{\partial x^{\mu}}{\partial x^{\bar{\alpha}}}$$

for the two different coordinate systems $\{x^{\bar{\alpha}}\}$ **and** $\{x^{\mu}\}$

The Equivalence Principle says that at any event in spacetime it is always possible to find a transformation to locally Minkowski coordinates.

The general metric tensor field is $g_{\mu\,\nu}\left(\mathbf{x}\right)$

It is symmetric: $g_{\mu\,\nu} = g_{\nu\,\mu}$ Why?

Therefore, there are 10 independent functions $g_{\mu\,\nu}\left(\mathbf{x}\right)$ at any event (point) in spacetime.

The metric tensor *defines* a coordinate system and *vice versa* through the line element

$$\begin{aligned} ds^2 &= g_{\mu\,\nu}\,dx^\mu\,dx^\nu \\ &= g_{0\,0}\,dx^0\,dx^0 + 2g_{0\,1}\,dx^0\,dx^1 + g_{1\,1}\,dx^1\,dx^1 \\ &\quad +2g_{1\,2}\,dx^1\,dx^2 + g_{2\,2}\,dx^2\,dx^2 + 2g_{2\,3}\,dx^2\,dx^3 + g_{3\,3}\,dx^3\,dx^3 \end{aligned}$$

It will turn out that the Christoffel symbols can be written in terms of the inverse metric and the partial derivatives of the metric as . . .

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\nu} \left[\frac{\partial g_{\beta\nu}}{\partial x^{\gamma}} + \frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\nu}} \right]$$

where the inverse metric is so-named because . . .

$$g^{\mu\rho}\, g_{\rho\nu} = \delta^{\mu}{}_{\nu}$$

We will not use this expression for the Christoffel symbols very often . . . usually there are easier ways to get them!

General Vectors

$$\mathbf{A} = A^{\mu}\,\mathbf{e}_{\mu}$$

$$\mathbf{B} = B^{\lambda}\,\mathbf{e}_{\lambda}$$

$$\mathbf{A}\cdot\mathbf{B} = g_{\mu\,\nu}A^{\mu}B^{\nu}$$

. . . where the metric tensor components are the inner products of the general, curvilinear (coordinate) basis vectors . . .

$$\mathbf{e}_{\mu}\cdot\mathbf{e}_{\nu} = g_{\mu\,\nu}$$

Examples in 3-D: flat Cartesian coordinates and spherical polar coordinates.

$$
\begin{aligned}
ds^2 &= dx^2 + dy^2 + dz^2 \\
&= dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2
\end{aligned}
$$

(x, y, z)

$$g_{ij} = \delta_{ij}$$

$$g_{xx} = g_{yy} = g_{zz} = 1 \text{ all others } = 0$$

(r, θ, φ)

$$g_{rr} = 1,\ g_{\theta\theta} = r^2,\ g_{\varphi\varphi} = r^2 \sin^2\theta$$

"flat space" means Minkowski coordinates (t, x, y, z)

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

the metric for which is just $\eta_{\mu\,\nu}$

What about coordinates ($t,\ r,\ \theta,\ \varphi$) ?

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

$$g_{00} = -1,\ \ g_{rr} = 1,\ \ g_{\theta\theta} = r^2,\ \ g_{\varphi\varphi} = r^2 \sin^2\theta$$

BOTH COORDINATE SYSTEMS DESCRIBE THE SAME GEOMETRY

How do we tell the difference between between coordinates that imply curvature (gravitational effects) and just plain old flat space masquerading itself with "*curvilinear coordinates*"?

The answer can be found in the Equivalence Principle which says that physics is invariant under coordinate transformations, *all* coordinate transformations. We are free to choose a coordinate transformation any way we want:

The E.P. gives us enough freedom to choose coordinates at any event (point) to transform the metric components to be those of the Minkowski metric and the first derivatives of the metric to be zero, thereby making the Christoffel symbols zero as well.

Expand our "lab" coordinates in a Taylor series about point $\mathbf{x}'_{\mathcal{P}}$ in the desired coordinates (which will of course be the locally inertial, Minkowski coordinates) . . .

$$x^\alpha\left(x'^\beta\right) = x^\alpha\left(x'^\beta_{\mathcal{P}}\right) + \left(\frac{\partial x^\alpha}{\partial x'^\beta}\right)\bigg|_{x'_{\mathcal{P}}}\left(x'^\beta - x'_{\mathcal{P}}{}^\beta\right) + \frac{1}{2}\left(\frac{\partial^2 x^\alpha}{\partial x'^\beta \partial x'^\gamma}\right)\bigg|_{x'_{\mathcal{P}}}\left(x'^\beta - x'_{\mathcal{P}}{}^\beta\right)\left(x'^\gamma - x'_{\mathcal{P}}{}^\gamma\right) + \ldots$$

Similarly expand the metric functions and use

$$g'_{\gamma\delta} = g_{\alpha\beta}\frac{\partial x^\alpha}{\partial x'^\gamma}\frac{\partial x^\beta}{\partial x'^\delta}$$

We would like to transform the metric to the flat space, Minkowski metric and we would like to get rid of as many derivatives of the metric functions as possible . . . What can we do with our freedom to choose the coordinate transformation?

$10 \text{ independent numbers } g_{\mu\nu}\left(\mathbf{x}'_{\mathcal{P}}\right)$ $\quad$ $16 \text{ independent numbers } \Lambda^{\mu}{}_{\beta}\left(\mathbf{x}'_{\mathcal{P}}\right)$

$40 \text{ independent numbers } \frac{\partial g_{\mu\nu}}{\partial x'^{\rho}}\left(\mathbf{x}'_{\mathcal{P}}\right)$ $\quad$ $40 \text{ independent numbers } \frac{\partial \Lambda^{\mu}{}_{\beta}}{\partial x'^{\rho}}\left(\mathbf{x}'_{\mathcal{P}}\right)$

$$100 \text{ independent numbers } \frac{\partial^2 g_{\mu\nu}}{\partial x'^{\rho}\partial x'^{\gamma}}\left(\mathbf{x}'_{\mathcal{P}}\right)$$

$$\text{but only } 80 \text{ independent numbers } \frac{\partial^2 \Lambda^{\mu}{}_{\beta}}{\partial x'^{\rho} x'^{\lambda}}\left(\mathbf{x}'_{\mathcal{P}}\right)$$

20 second derivatives of the metric which cannot in general be set to zero with the coordinate freedom given by the E. P.

It turns out that in a weak gravitational field the time-time component of the metric is related to the Newtonian gravitational potential by . . .

$$g_{0\,0} \approx -1 - 2\varphi$$

Where the Newtonian gravitational potential is $\varphi \approx -\frac{G\,M}{R}$

$$G \equiv \frac{1}{m_{\rm pl}^2}$$

$$m_{\rm pl} \approx 1.221 \times 10^{22}\,\mathrm{MeV}$$

$$\hbar\,c \approx 197.33\,\mathrm{MeV\,fm}$$

$$1\,\mathrm{fm} = 10^{-13}\,\mathrm{cm}$$

$$1\,\mathrm{MeV} \approx 1.6022 \times 10^{-13}\,\mathrm{Joules}$$

$$\varphi \approx -\frac{M\,\hbar\,c}{m_{\rm pl}^2\,R}$$

dimensionless !

Characteristic Metric Deviation

OBJECT	MASS (solar masses)	RADIUS (cm)	Newtonian Gravitational Potential
earth	3×10^{-6}	6.4×10^{8}	$\sim 10^{-9}$
sun	1	6.9×10^{10}	$\sim 10^{-6}$
white dwarf	~1	5×10^{8}	$\sim 10^{-4}$
neutron star	~1	10^{6}	~0.1 to 0.2

A convenient coordinate system for
***weak* & *static* (no time dependence) gravitational fields**
is given by the following coordinate system/metric:

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\varphi)\left(dx^2 + dy^2 + dz^2\right)$$

This would be a decent description of the spacetime geometry and gravitational effects around the earth, the sun, and white dwarf stars, but not near the surfaces of neutron stars.

We will explore this metric with variational principles later.